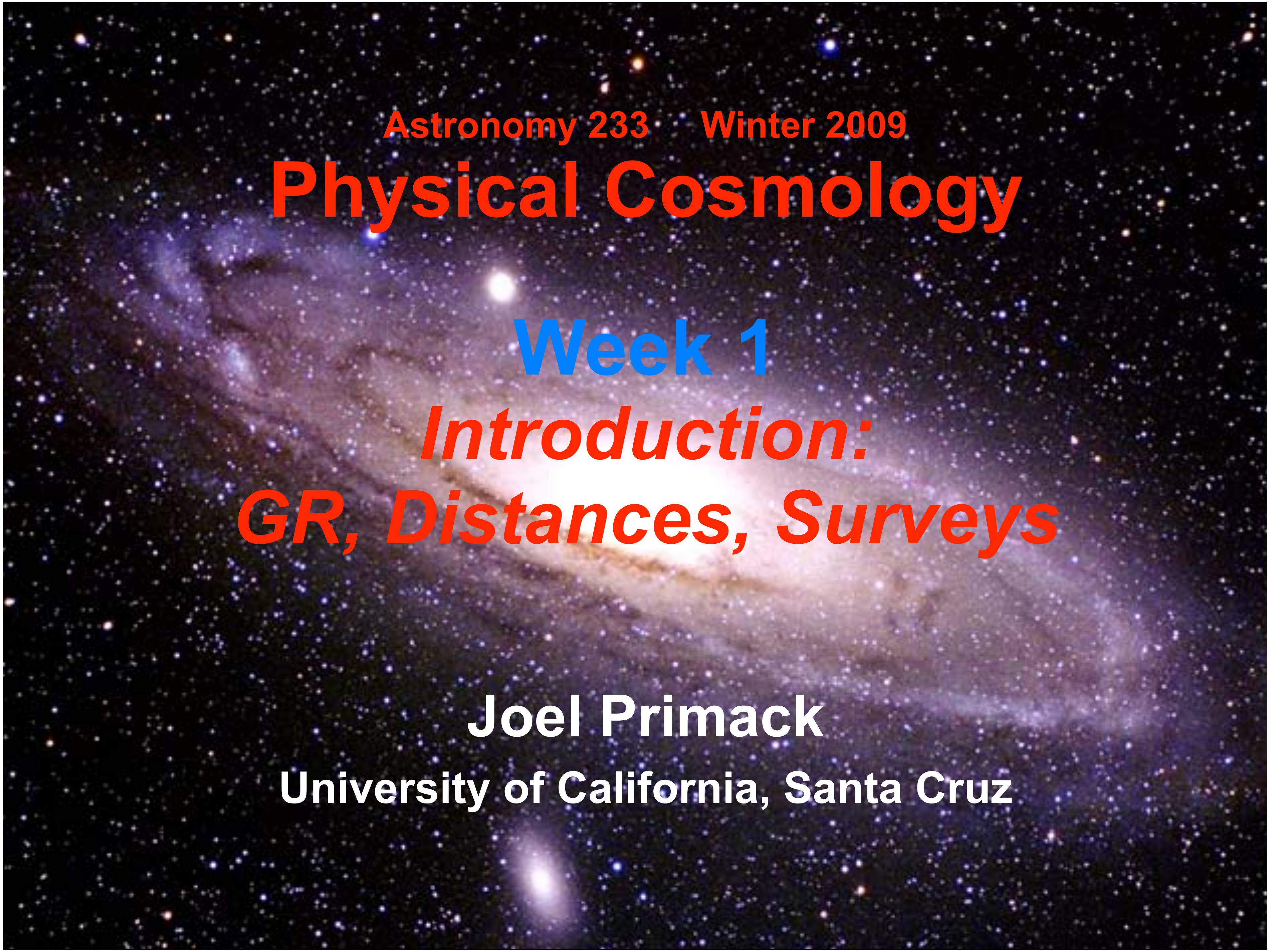
Astronomy 233 Winter 2009
Physical Cosmology
Week 1
Introduction:
GR, Distances, Surveys
Joel Primack
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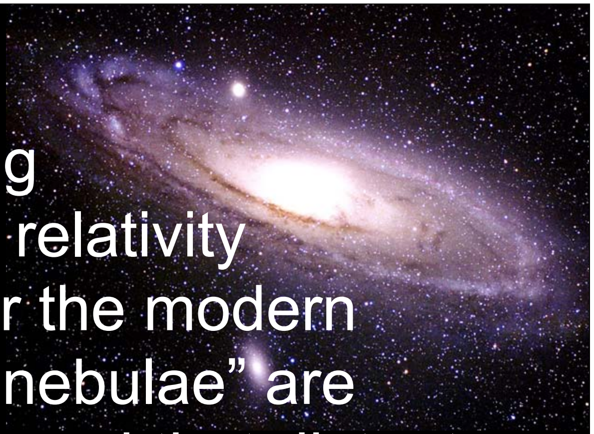
Modern Cosmology



A series of major discoveries has laid a lasting foundation for cosmology. Einstein's general relativity (1916) provided the conceptual foundation for the modern picture. Then Hubble discovered that "spiral nebulae" are large galaxies like our own Milky Way (1922), and that distant galaxies are receding from the Milky Way with a speed proportional to their distance (1929), which means that we live in an expanding universe. The discovery of the cosmic background radiation (1965) showed that the universe began in a very dense, hot, and homogeneous state: the Big Bang. This was confirmed by the discovery that the cosmic background radiation has exactly the same spectrum as heat radiation (1989), and the measured abundances of the light elements agree with the predictions of Big Bang theory if the abundance of ordinary matter is about 4% of critical density. Most of the matter in the universe is invisible particles which move very sluggishly in the early universe ("Cold Dark Matter").

Experimental and Historical Sciences

both make predictions about new knowledge, whether from experiments or from the past

Historical Explanation Is Always Inferential

Our age cannot look back to earlier things
Except where reasoning reveals their traces Lucretius

Patterns of Explanation Are the Same in the Historical Sciences as in the Experimental Sciences

Specific conditions + General laws ⇒ Particular event

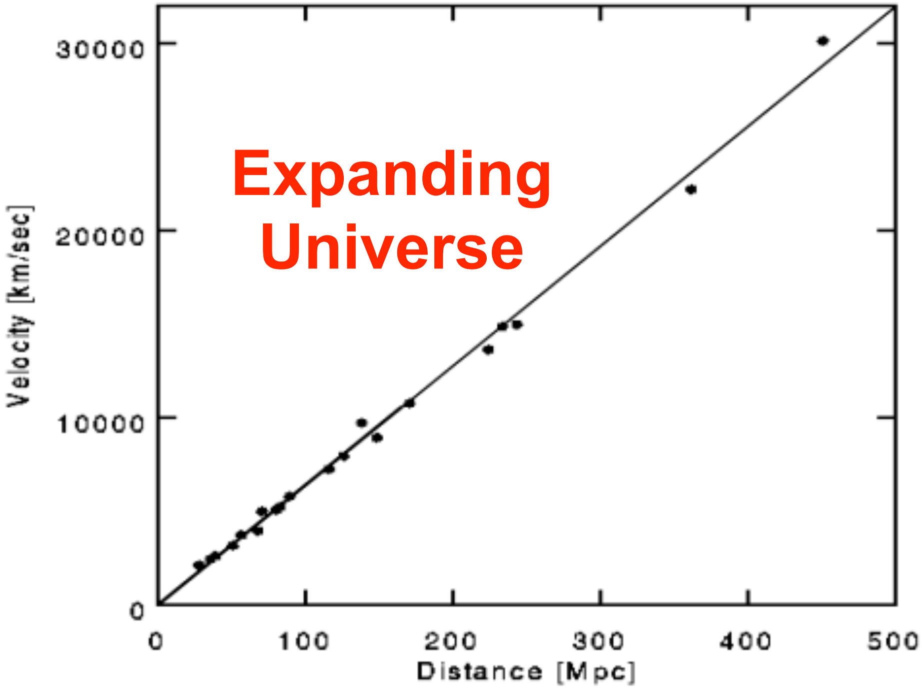
In history as anywhere else in empirical science, the explanation of a phenomenon consists in subsuming it under general empirical laws; and the criterion of its soundness is … exclusively whether it rests on empirically well confirmed assumptions concerning initial conditions and general laws.

C.G. Hempel, *Aspects of Scientific Explanation* (1965), p. 240.

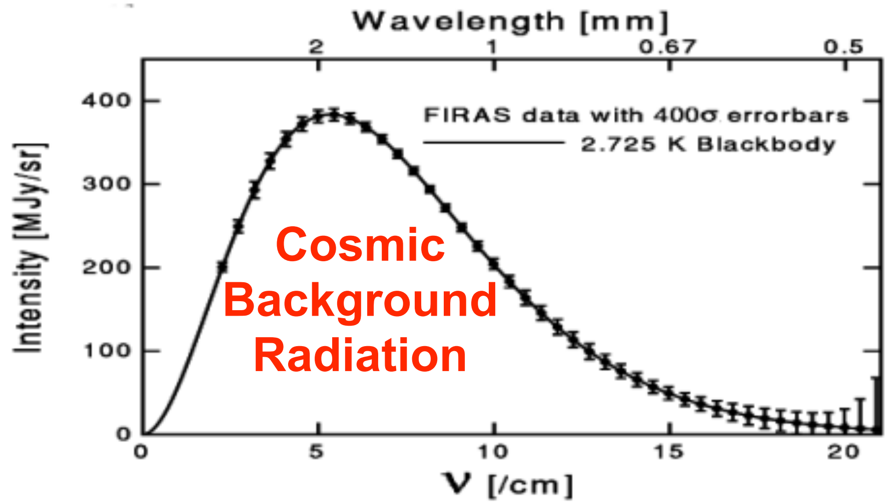
Successful Predictions of the Big Bang

First Prediction	First Confirmation
Expansion of the Universe	
Friedmann 1922, Lemaitre 1927 based on Einstein 1916	Hubble 1929
Cosmic Background Radiation	
Existence of CBR Gamow, Alpher, Hermann 1948	Penzias & Wilson 1965
CBR Thermal Spectrum Peebles 1966	COBE 1989
CBR Fluctuation Amplitude Cold Dark Matter theory 1984	COBE 1992
CBR Acoustic Peak	BOOMERANG 2000 MAXIMA 2000
Light Element Abundances	
Peebles 1966, Wagoner 1967	D/H Tytler et al. 1997

Three Pillars of the Big Bang



A modern illustration of Hubble's Law, displaying the increase of recession speed of galaxies growing in direct proportion to their distance.

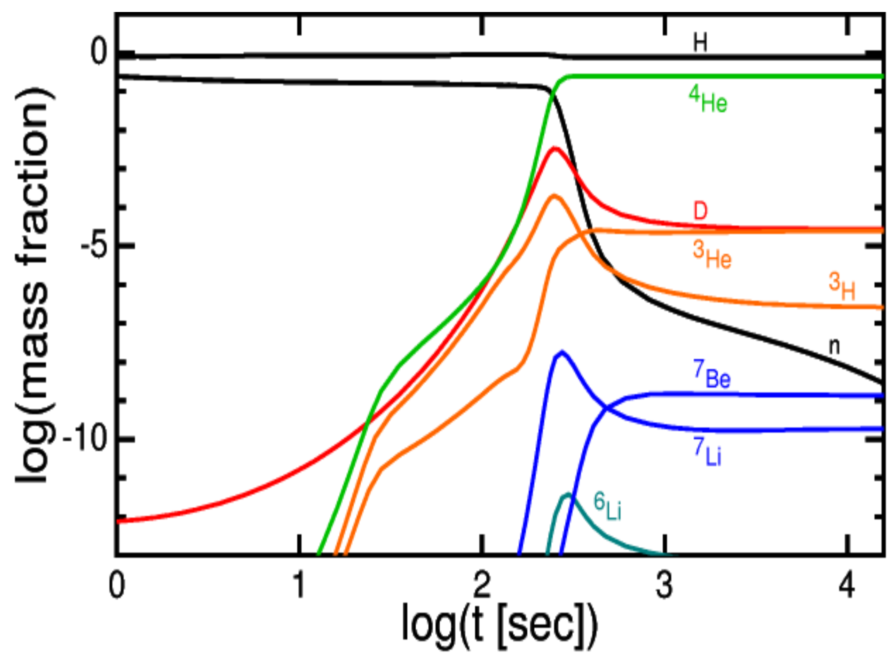


The variation of the intensity of the microwave background radiation with its frequency, as observed by the COBE satellite from above the Earth's atmosphere. The observations (boxes) display a perfect fit with the (solid) curve expected from pure heat radiation with a temperature of 2.73°K.

Big Bang Nucleosynthesis

The detailed production of the lightest elements out of protons and neutrons during the first three minutes of the universe's history. The nuclear reactions occur rapidly when the temperature falls below a billion degrees Kelvin. Subsequently, the reactions are shut down, because of the rapidly falling temperature and density of matter in the expanding universe.

Caution: ^{7}Li may now be discordant



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Dynamical effects of the cosmological constant

Ofer Lahav,[1] Per B. Lilje,[2] Joel R. Primack[3] and Martin J. Rees[1]

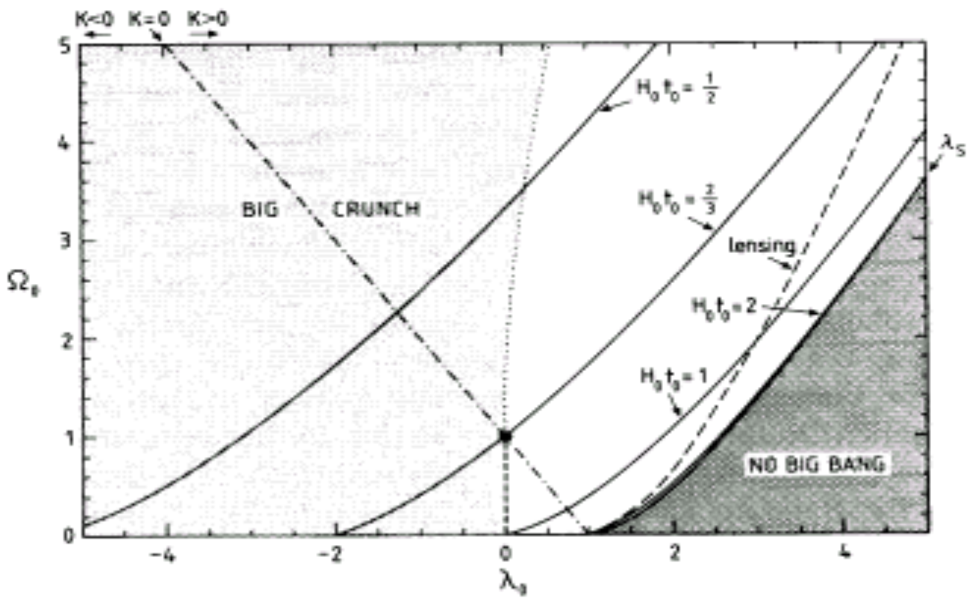
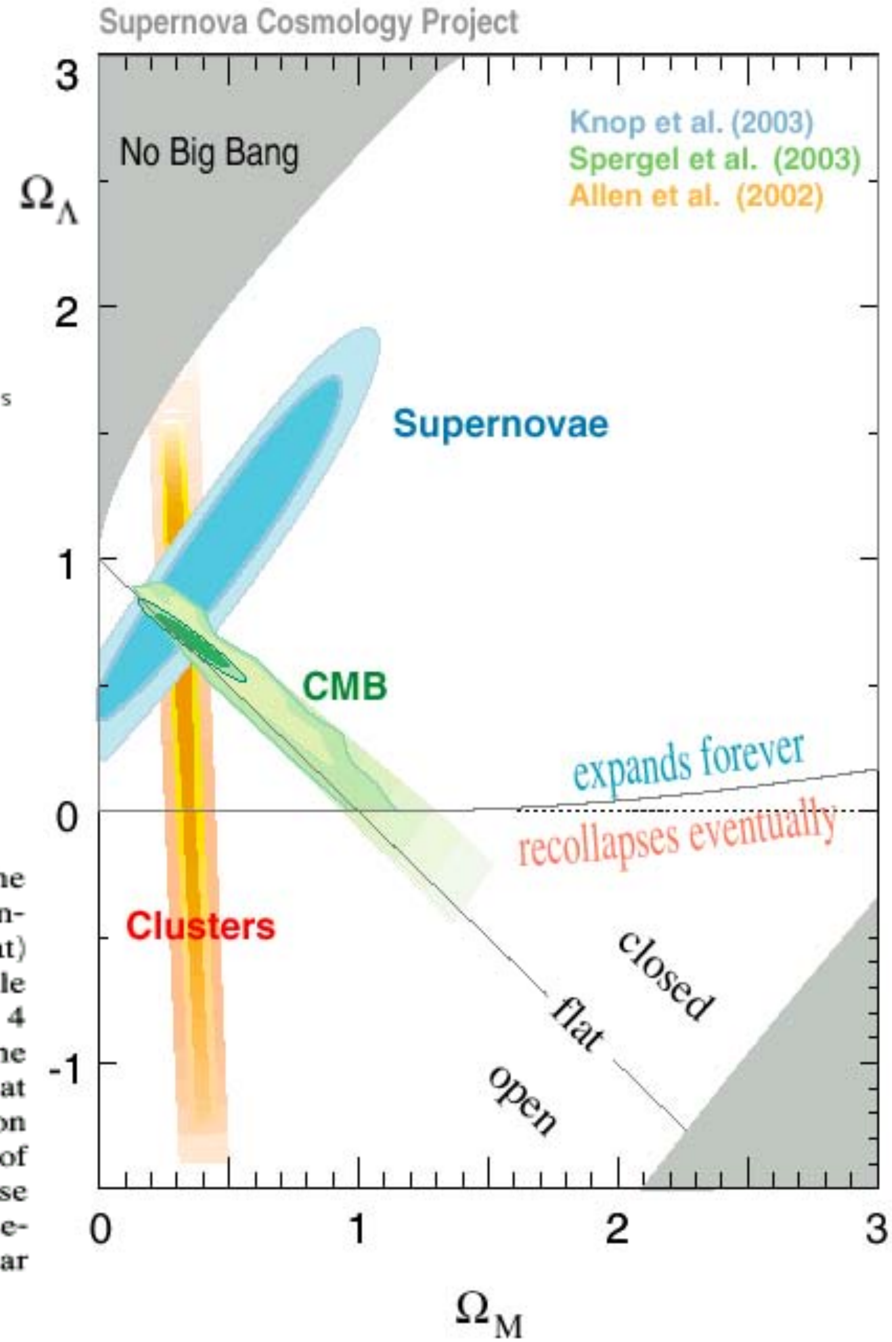


Figure 1. The phase-space of the density parameter Ω_0 and the cosmological constant $\lambda_0 = \Lambda/(3H_0^2)$ with various fundamental constraints. The dashed-dotted line indicates an inflationary (i.e. flat) universe. Note that some open models will have a Big Crunch, while some closed models will expand forever. The solid lines show 4 values for the age of the universe H_0t_0, and the dashed line is the constraint of Gott *et al.* (1989) from a normally lensed quasar at $z = 3.27$. The boundary (λ_s) of the shaded 'No Big Bang' region corresponds to a coasting phase in the past, while the boundary of the 'Big Crunch' (for $\Omega_0 > 1$) region corresponds to a coasting phase in the future. We see that the permitted range in the ($\lambda_0 - \Omega_0$) phase-space is fairly small, but allows values different from the popular point ($\Omega_0 = 1$, $\lambda_0 = 0$).



General Relativity and Cosmology

GR: MATTER TELLS SPACE HOW TO CURVE

$$R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 8\pi G T^{\mu\nu} + \Lambda g^{\mu\nu}$$

CURVED SPACE TELLS MATTER HOW TO MOVE

$$\frac{du^{\mu}}{ds} + \Gamma^{\mu}_{\alpha\beta} u^{\alpha} u^{\beta} = 0$$

Cosmological Principle: on large scales, space is uniform and isotropic. COBE-Copernicus Theorem: If all observers observe a nearly-isotropic Cosmic Background Radiation (CBR), then the universe is locally nearly homogeneous and isotropic – i.e., is approximately described by the Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t) \left[dr^2 (1 - kr^2)^{-1} + r^2 d\Omega^2\right]$$

with curvature constant $k = -1, 0$, or $+1$. Substituting this metric into the Einstein equation at left above, we get the Friedmann eq.

Friedmann-Robertson-Walker Framework

(homogeneous, isotropic universe)

FRW E(00) $\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$ ⟵ Friedmann equation

FRW E(ii) $\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p - \frac{k}{a^2} + \Lambda$

$$H_0 \equiv 100h\,\mathrm{km\,s^{-1}Mpc^{-2}} \equiv 70h_{70}\,\mathrm{km\,s^{-1}Mpc^{-2}}$$

$$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda \text{ with } H \equiv \frac{\dot{a}}{a},\ a_0 \equiv 1,\ \Omega_0 \equiv \frac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2},$$

$$\rho_{c,0} \equiv \frac{3H_0^2}{8\pi G} = 1.36 \times 10^{11} h_{70}^2 M_\odot \mathrm{Mpc}^{-3}$$

$$E(ii) - E(00) \Rightarrow \frac{2\ddot{a}}{a} = -\frac{8\pi}{3}G\rho - 8\pi G p + \frac{2}{3}\Lambda$$

$$\text{Divide by } 2E(00) \Rightarrow \quad q_0 \equiv -\left(\frac{\ddot{a}}{a}\frac{a^2}{\dot{a}^2}\right)_0 = \frac{\Omega_0}{2} - \Omega_\Lambda$$

$$E(00) \Rightarrow t_0 = \int_0^1 \frac{da}{a}\left[\frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}\right]^{-\frac{1}{2}} = H_0^{-1}\int_0^1 \frac{da}{a}\left[\frac{\Omega_0}{a^3} - \frac{k}{H_0^2 a^2} + \Omega_\Lambda\right]^{-\frac{1}{2}}$$

$$t_0 = H_0^{-1} f(\Omega_0, \Omega_\Lambda) \qquad H_0^{-1} = 9.78h^{-1}\mathrm{Gyr} = 13.97\ h_{70}^{-1}\ \mathrm{Gyr}$$

$$f(1,0) = \tfrac{2}{3}, \quad f(0,0) = 1, \quad f(0,1) = \infty, \quad f(0.3, 0.7) = 0.964$$

$$[E(00)a^3]' \text{ vs. } E(ii) \Rightarrow \quad \frac{\partial}{\partial a}(\rho a^3) = -3pa^2 \ (\text{“continuity”})$$

Given eq. of state $p = p(\rho)$, integrate to determine $\rho(a)$,
integrate $E(00)$ to determine $a(t)$

Matter: $p = 0 \Rightarrow \rho = \rho_0 a^{-3}$ (assumed above in q_0, t_0 eqs.)

Radiation: $p = \frac{\rho}{3}$, $k = 0 \Rightarrow \rho \propto a^{-4}$