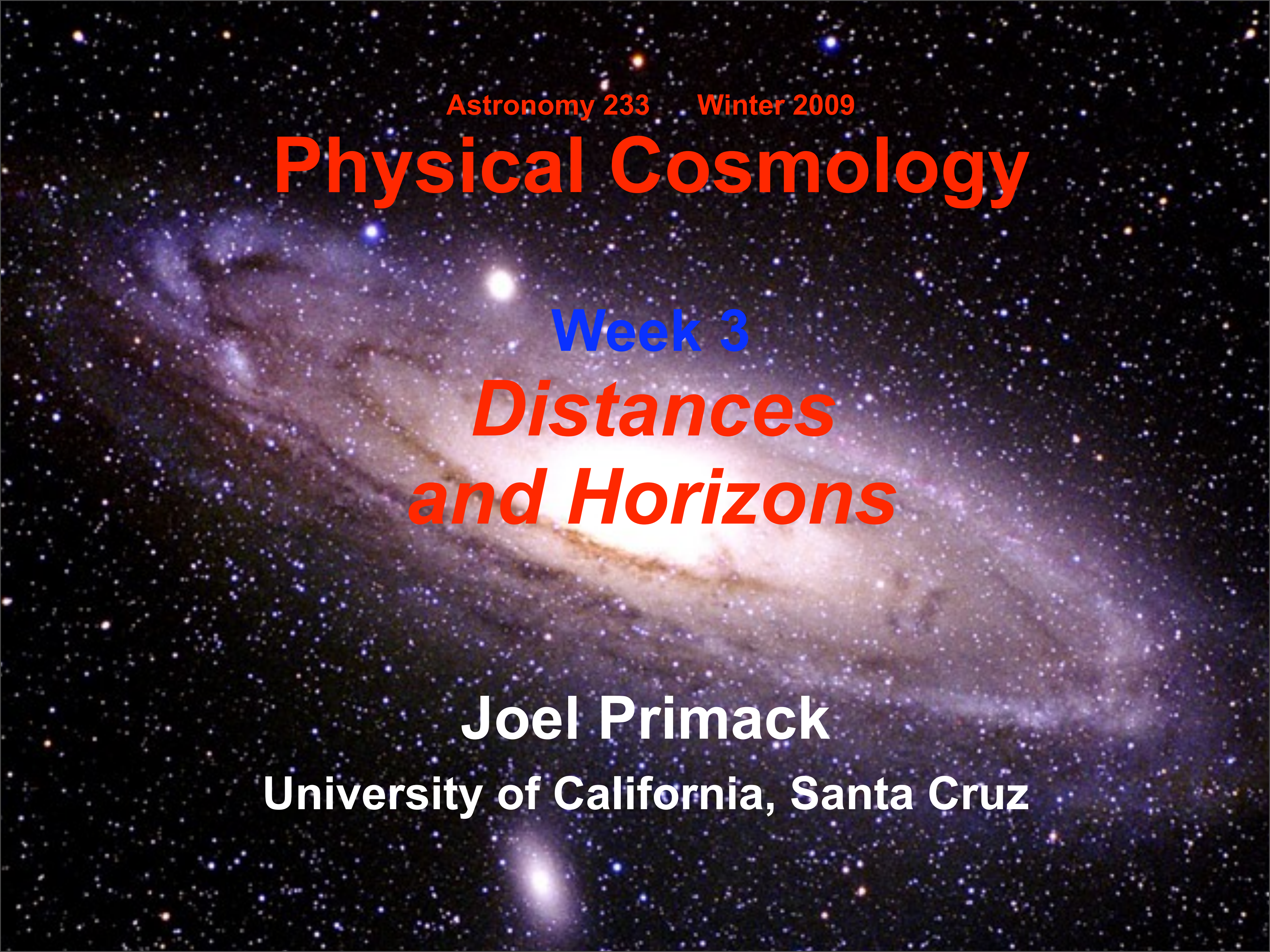


Astronomy 233 Winter 2009

Physical Cosmology

Week 3

Distances and Horizons

Joel Primack

University of California, Santa Cruz

Astronomy 233 – Physical Cosmology – Winter 2009

Class meets MW 2-3:45PM, ISB 231
Website: **http://physics.ucsc.edu/~joel/Ay/233/**
Instructor: Joel Primack – email: **joel@physics.ucsc.edu** – ISB 318 – phone 831 459 2580
Office hours: Thursdays 2:30-3:30 or by appointment

Catalog description: Survey of modern physical cosmology, including Newtonian cosmology, curved space-times, observational tests of cosmology, the early universe, inflation, nucleosynthesis, dark matter, and the formation of structure in the universe. Prerequisite(s): course 202. Offered in alternate academic years.

Textbook: Steven Weinberg, *Cosmology* (Oxford University Press, 2008). This is an excellent, up to date book. It has far more material than we can cover in this course, but it does not cover galaxy formation in very much detail. You should think of Weinberg's

Cosmology as a reference book where you can find more information on subjects covered in lectures. I do not plan to follow Weinberg's book in detail, and you may find other books to be of equal value in learning the material of this course. As I mentioned in the first lecture, another highly recommended book is Scott Dodelson, *Modern Cosmology* (Academic Press, 2003).

The organization of the course will depend on the interests and previous knowledge of the students. From the questionnaire I see that most students are either especially interested in inflation, the very early universe, and the particle physics aspects of cosmology, or else especially interested in galaxy formation and evolution. Ay233 will emphasize the former – students interested in the latter topic should attend Ay214 (see next paragraph). Many of the weekly Astronomy colloquia and seminars this quarter are relevant to cosmology. You may also be interested in Astronomy 214, which is a seminar course on galaxies in Winter Quarter 2009. It meets MW 12:15-1:45 in ISB 102. During January 21 through February 23, Prof. Avishai Dekel will be lecturing in Ay214 on the formation of galaxies – important material for anyone who wants to understand modern cosmology.

Friedmann-Robertson-Walker Framework (homogeneous, isotropic universe)

FRW E(00) $\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$ ← Friedmann equation

FRW E(ii) $\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p - \frac{k}{a^2} + \Lambda$

$$\begin{aligned} H_0 &\equiv 100h\,\mathrm{km\,s^{-1}Mpc^{-2}} \\ &\equiv 70h_{70}\,\mathrm{km\,s^{-1}Mpc^{-2}} \end{aligned}$$

$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda$ with $H \equiv \frac{\dot{a}}{a}$, $a_0 \equiv 1$, $\Omega_0 \equiv \frac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}$,

$$\rho_{c,0} \equiv \frac{3H_0^2}{8\pi G} = 1.36 \times 10^{11} h_{70}^2 M_\odot \mathrm{Mpc}^{-3}$$

$E(ii) - E(00) \Rightarrow \frac{2\ddot{a}}{a} = -\frac{8\pi}{3}G\rho - 8\pi G p + \frac{2}{3}\Lambda$

Divide by $2E(00) \Rightarrow \quad q_0 \equiv -\left(\frac{\ddot{a}}{a}\frac{a^2}{\dot{a}^2}\right)_0 = \frac{\Omega_0}{2} - \Omega_\Lambda$

$$E(00) \Rightarrow t_0 = \int_0^1 \frac{da}{a}\left[\frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}\right]^{-\frac{1}{2}} = H_0^{-1}\int_0^1 \frac{da}{a}\left[\frac{\Omega_0}{a^3} - \frac{k}{H_0^2 a^2} + \Omega_\Lambda\right]^{-\frac{1}{2}}$$

$t_0 = H_0^{-1} f(\Omega_0, \Omega_\Lambda)$ $\qquad H_0^{-1} = 9.78h^{-1}\mathrm{Gyr}$

$= 13.97\ h_{70}^{-1}$ Gyr

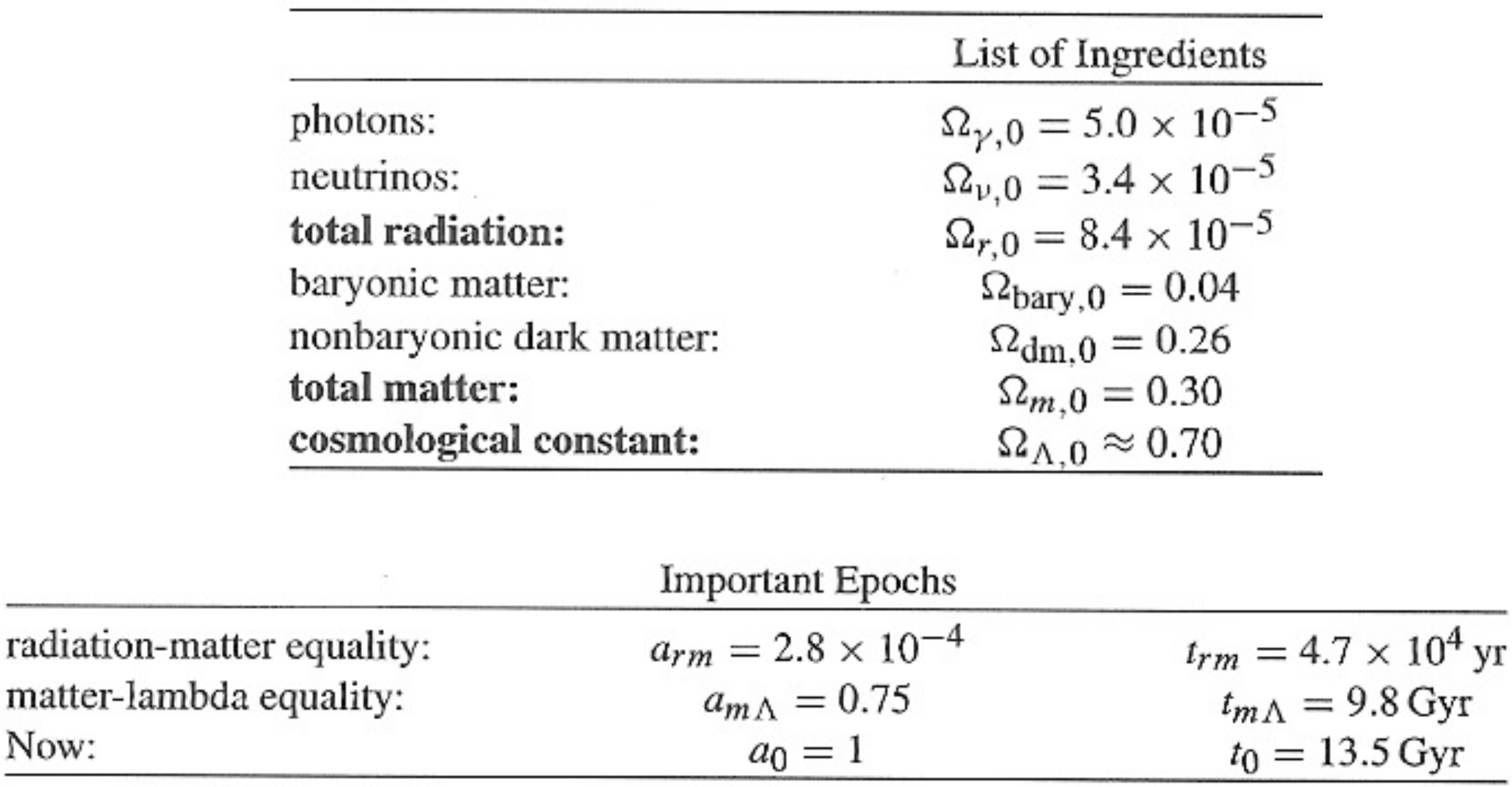
$f(1,0) = \frac{2}{3}$
$f(0,0) = 1$
$f(0,1) = \infty$
$f(0.3, 0.7) = 0.964$

$[E(00)a^3]'$ vs. $E(ii) \Rightarrow \quad \frac{\partial}{\partial a}(\rho a^3) = -3pa^2$ ("continuity")

Given eq. of state $p = p(\rho)$, integrate to determine $\rho(a)$,
integrate $E(00)$ to determine $a(t)$

Matter: $p = 0 \Rightarrow \rho = \rho_0 a^{-3}$ (assumed above in q_0, t_0 eqs.)
Radiation: $p = \frac{\rho}{3}$, $k = 0 \Rightarrow \rho \propto a^{-4}$

LCDM Benchmark Cosmological Model: Ingredients & Epochs



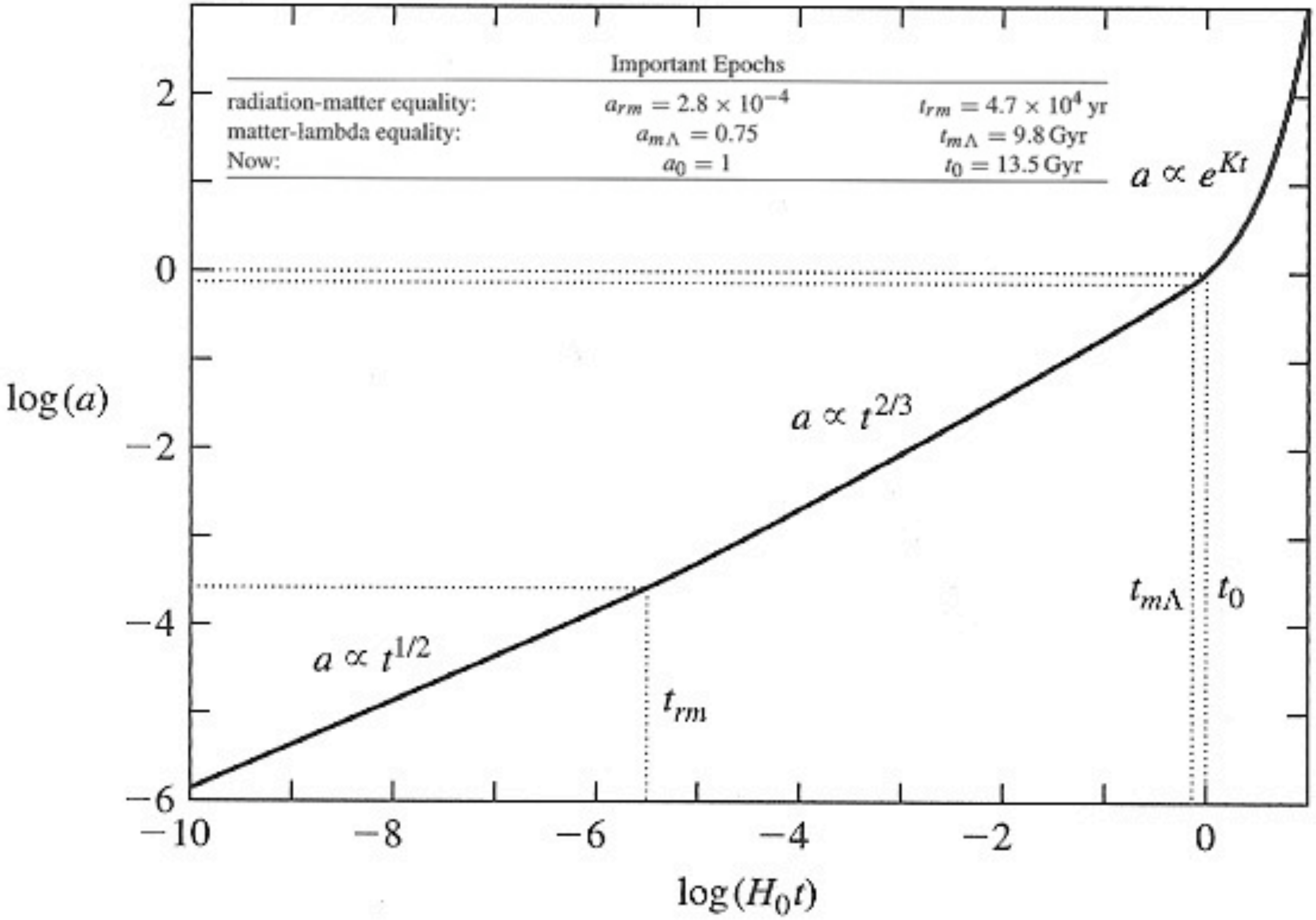
	List of Ingredients
photons:	$\Omega_{\gamma,0} = 5.0 \times 10^{-5}$
neutrinos:	$\Omega_{\nu,0} = 3.4 \times 10^{-5}$
total radiation:	$\Omega_{r,0} = 8.4 \times 10^{-5}$
baryonic matter:	$\Omega_{\mathrm{bary},0} = 0.04$
nonbaryonic dark matter:	$\Omega_{\mathrm{dm},0} = 0.26$
total matter:	$\Omega_{m,0} = 0.30$
cosmological constant:	$\Omega_{\Lambda,0} \approx 0.70$

Important Epochs

radiation-matter equality:	$a_{rm} = 2.8 \times 10^{-4}$	$t_{rm} = 4.7 \times 10^{4}$ yr
matter-lambda equality:	$a_{m\Lambda} = 0.75$	$t_{m\Lambda} = 9.8$ Gyr
Now:	$a_0 = 1$	$t_0 = 13.5$ Gyr

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

Benchmark Model: Scale Factor vs. Time



Important Epochs

radiation-matter equality:	$a_{rm} = 2.8 \times 10^{-4}$	$t_{rm} = 4.7 \times 10^4$ yr
matter-lambda equality:	$a_{m\Lambda} = 0.75$	$t_{m\Lambda} = 9.8$ Gyr
Now:	$a_0 = 1$	$t_0 = 13.5$ Gyr

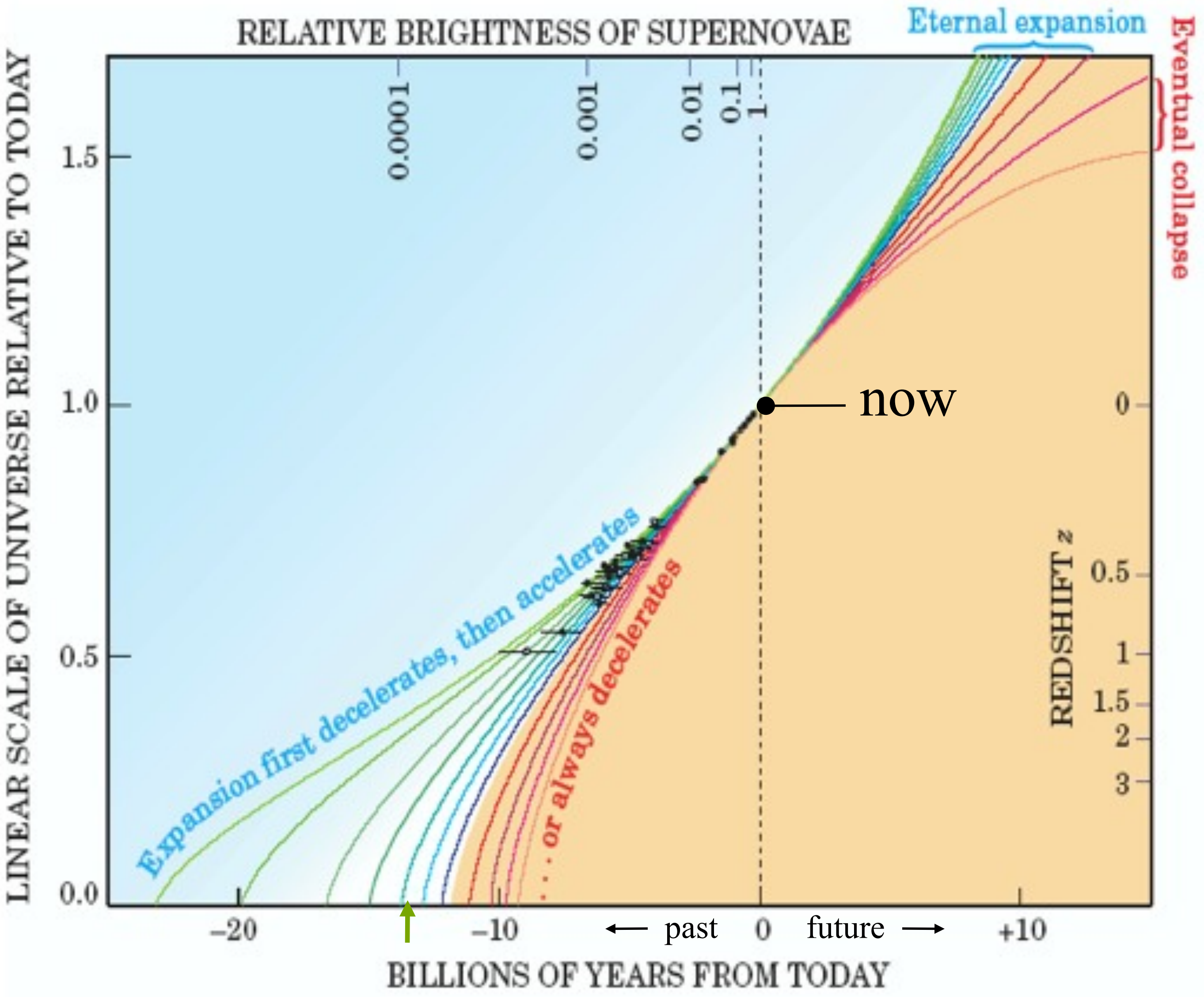
FIGURE 6.5 The scale factor a as a function of time t (measured in units of the Hubble time), computed for the Benchmark Model. The dotted lines indicate the time of radiation-matter equality, $a_{rm} = 2.8 \times 10^{-4}$, the time of matter-lambda equality, $a_{m\Lambda} = 0.75$, and the present moment, $a_0 = 1$.

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

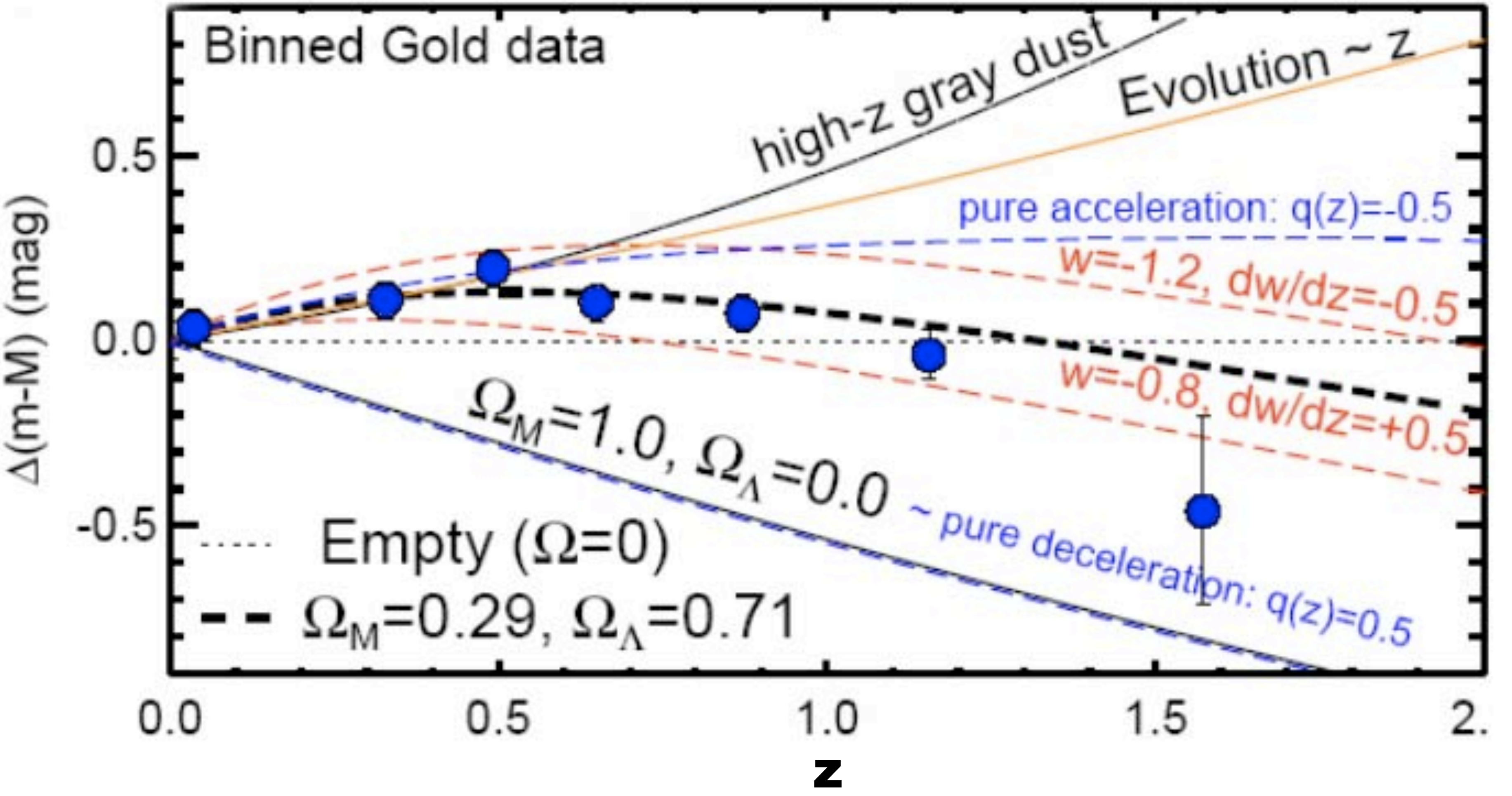
History of Cosmic Expansion for $\Omega_\Lambda = 1 - \Omega_M$

With $\Omega_\Lambda = 0$ the age of the decelerating universe would be only 9 Gyr, but $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$ gives an age of 14Gyr, consistent with stellar and radioactive decay ages

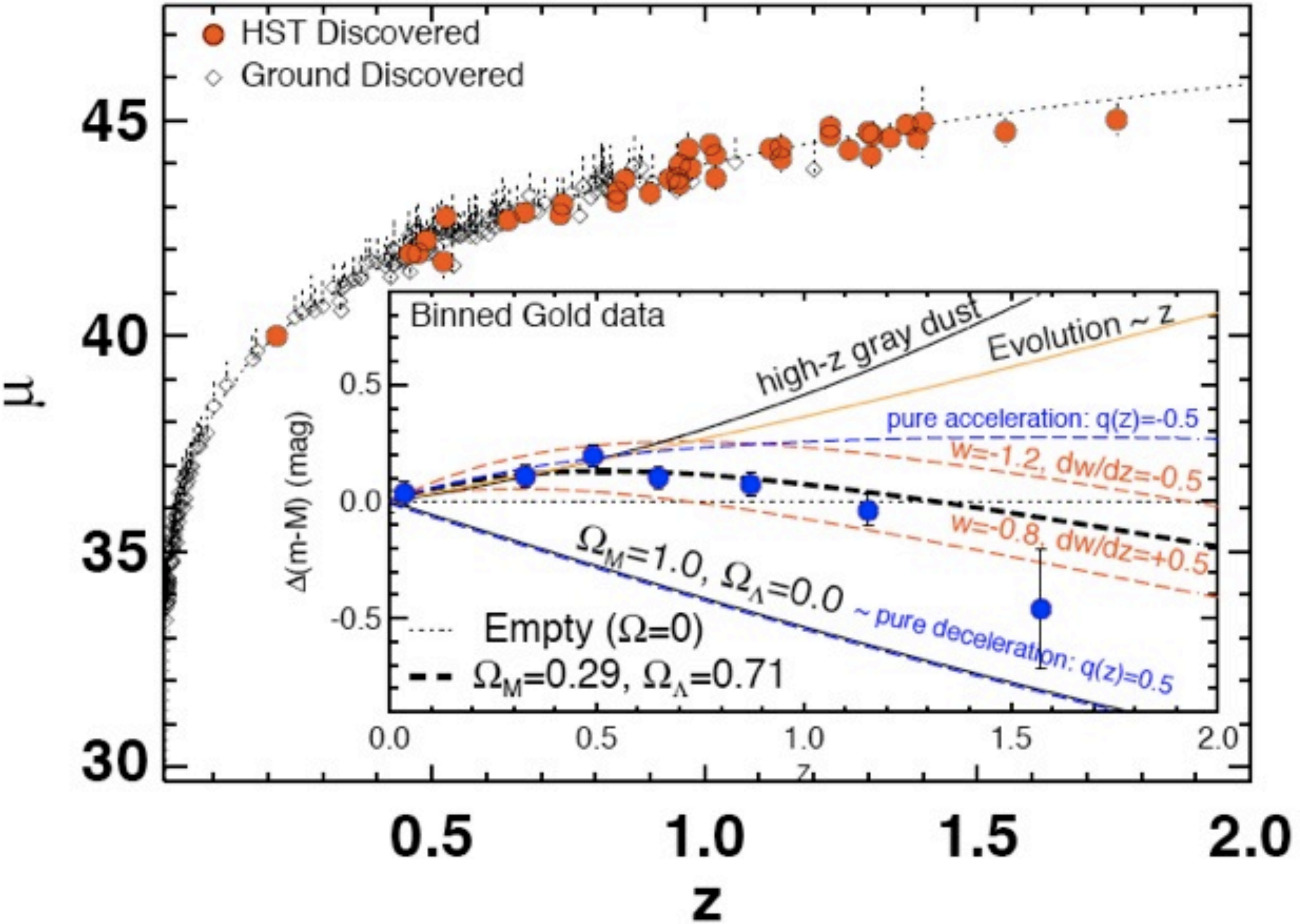
Figure 4. The history of cosmic expansion, as measured by the high-redshift supernovae (the black data points), assuming flat cosmic geometry. The scale factor *R* of the universe is taken to be 1 at present, so it equals $1/(1 + z)$. The curves in the blue shaded region represent cosmological models in which the accelerating effect of vacuum energy eventually overcomes the decelerating effect of the mass density. These curves assume vacuum energy densities ranging from 0.95 ρ_c (top curve) down to 0.4 ρ_c. In the yellow shaded region, the curves represent models in which the cosmic expansion is always decelerating due to high mass density. They assume mass densities ranging (left to right) from 0.8 ρ_c up to 1.4 ρ_c. In fact, for the last two curves, the expansion eventually halts and reverses into a cosmic collapse.



Saul Perlmutter, ***Physics Today***, Apr 2003



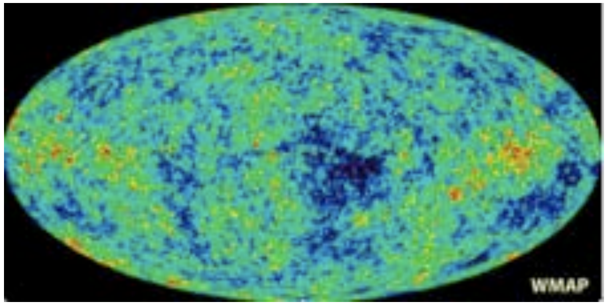
From the A. Riess et al. 2007, ApJ, 659, 98 Abstract: **The unique leverage of the HST high-redshift SNe Ia provides the first meaningful constraint on the dark energy equation-of-state parameter at $z \geq 1$. The result remains consistent with a cosmological constant ($w(z) = -1$), and rules out rapidly evolving dark energy ($dw/dz >> 1$). The defining property of dark energy, its negative pressure, appears to be present at $z > 1$, in the epoch preceding acceleration, with ⌋ 98% confidence in our primary fit. Moreover, the $z > 1$ sample-averaged spectral energy distribution is consistent with that of the typical SN Ia over the last 10 Gyr, indicating that any spectral evolution of the properties of SNe Ia with redshift is still below our detection threshold.**



SNe Ia from ground-based discoveries in the Gold sample are shown as diamonds, HST-discovered SNe Ia are shown as filled symbols. Overplotted is the best fit for a flat cosmology: M = 0.27, = 0.73. **Inset**: Residual Hubble diagram and models after subtracting empty Universe model. The Gold sample is binned in equal spans of nΔz = 6 where n is the number of SNe in a bin and z is the redshift range of the bin. Fig. 6 of A. Riess et al. 2007, ApJ, 659, 98.

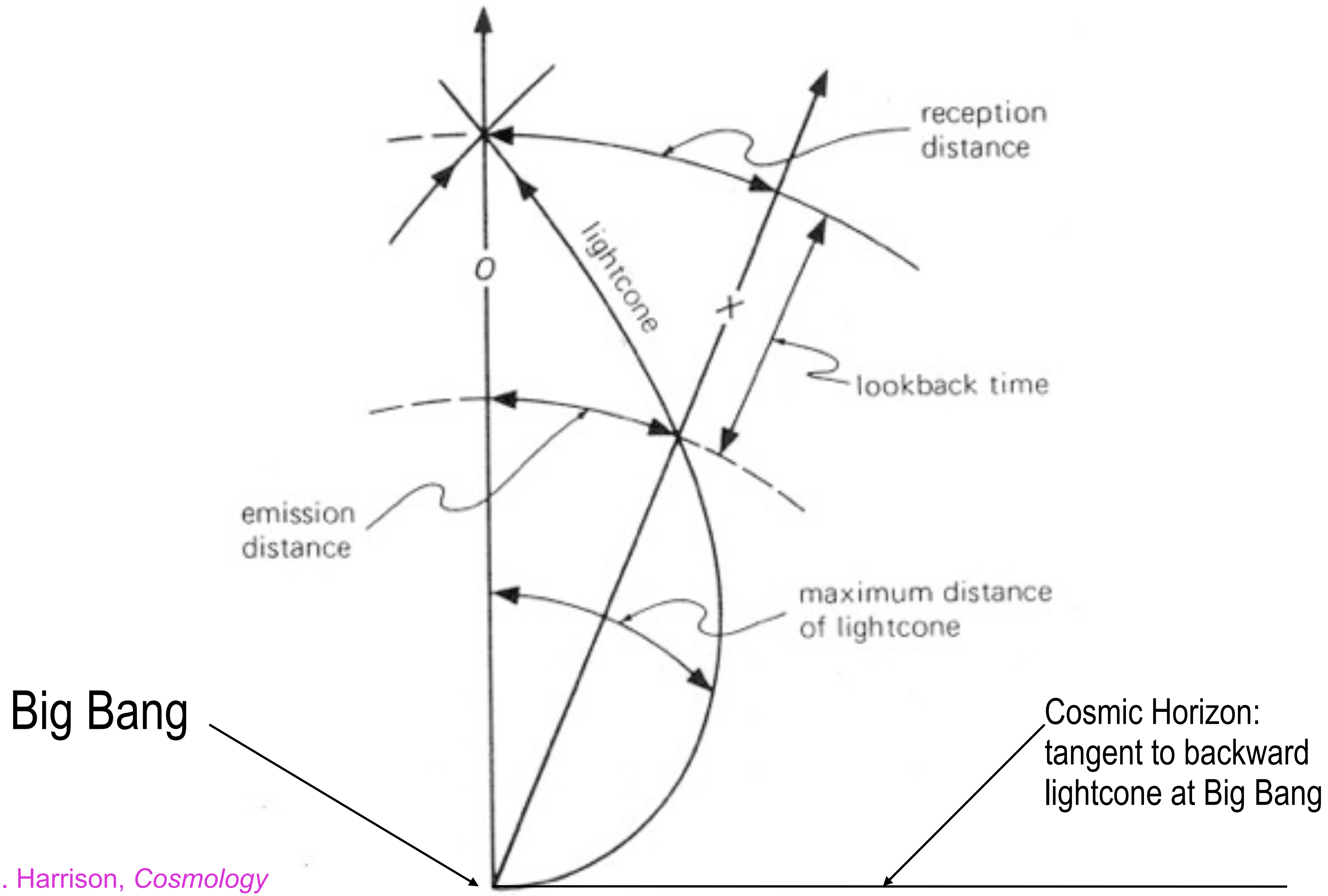
Brief History of the Universe

- Cosmic Inflation generates density fluctuations
- Symmetry breaking: more matter than antimatter
- All antimatter annihilates with almost all the matter (1s)
- Big Bang Nucleosynthesis makes light nuclei (10 min)
- Electrons and light nuclei combine to form atoms, and the cosmic background radiation fills the newly transparent universe (380,000 yr)



- Galaxies and larger structures form (~1 Gyr)
- Carbon, oxygen, iron, ... are made in stars
- Earth-like planets form around 2nd generation stars
- Life somehow starts (~4 Gyr ago) and evolves on earth

Picturing the History of the Universe: The Backward Lightcone

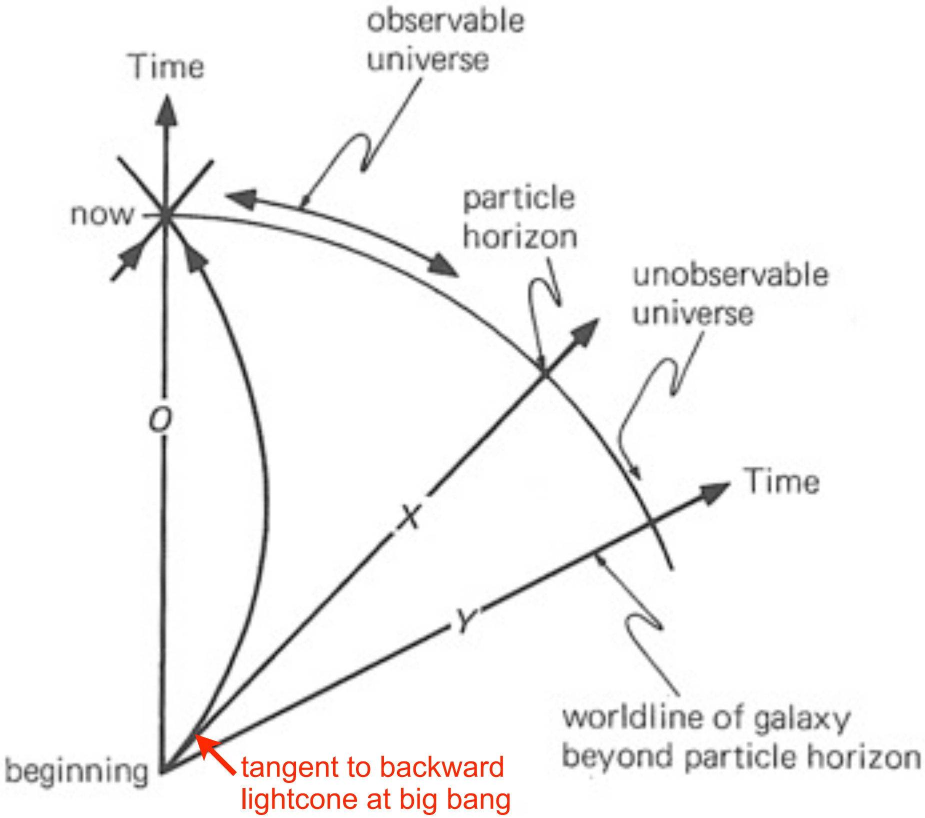


Big Bang

Cosmic Horizon:
tangent to backward
lightcone at Big Bang

From E. Harrison, *Cosmology*
(Cambridge UP, 2000).

Picturing the History of the Universe: The Backward Lightcone



tangent to backward lightcone at big bang

Figure 21.11. At the instant labeled "now" the particle horizon is at worldline X. In a big bang universe, all galaxies at the particle horizon have infinite redshift.

From E. Harrison, *Cosmology* (Cambridge UP, 2000).

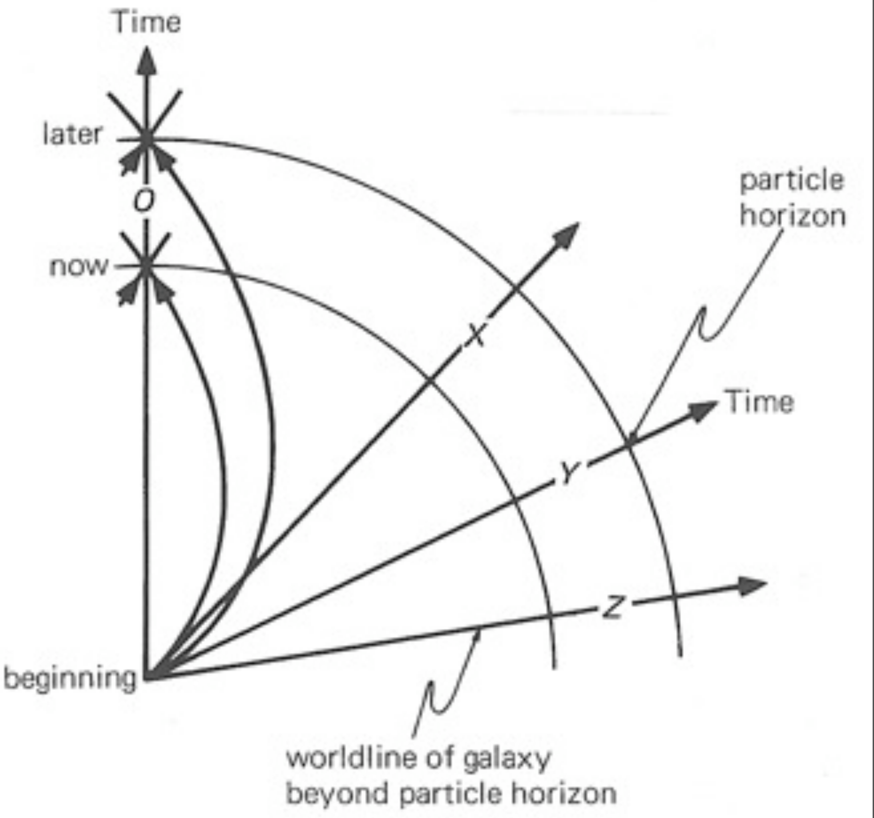


Figure 21.12. At the instant labeled "later" the particle horizon has receded to world line Y. Notice the distance of the particle horizon is always a reception distance, and the particle horizon always overtakes the galaxies and always the fraction of the universe observed increases.

Horizons

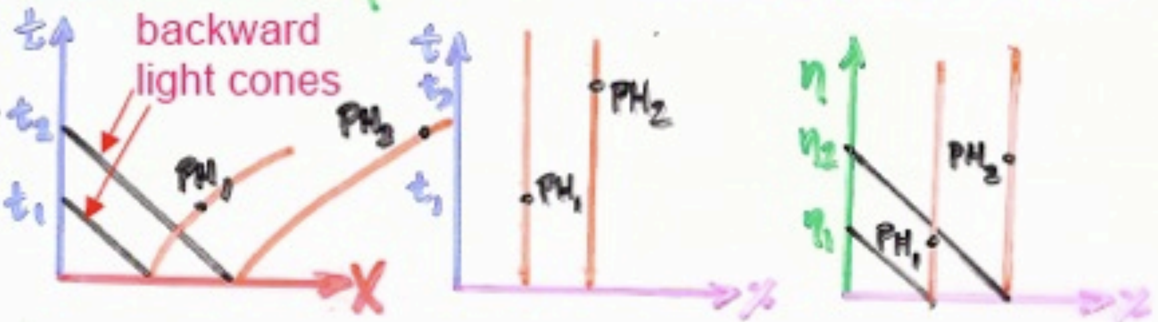
$$ds^2 = dt^2 - dX^2 = dt^2 - R^2 dx^2 = R^2(d\eta^2 - dx^2)$$

conformal time $d\eta = dt/R$ comoving coord. $dx = dX/R$

PARTICLE HORIZON

Spherical surface that at time t separates *worldlines* into observed vs. unobserved

FRW

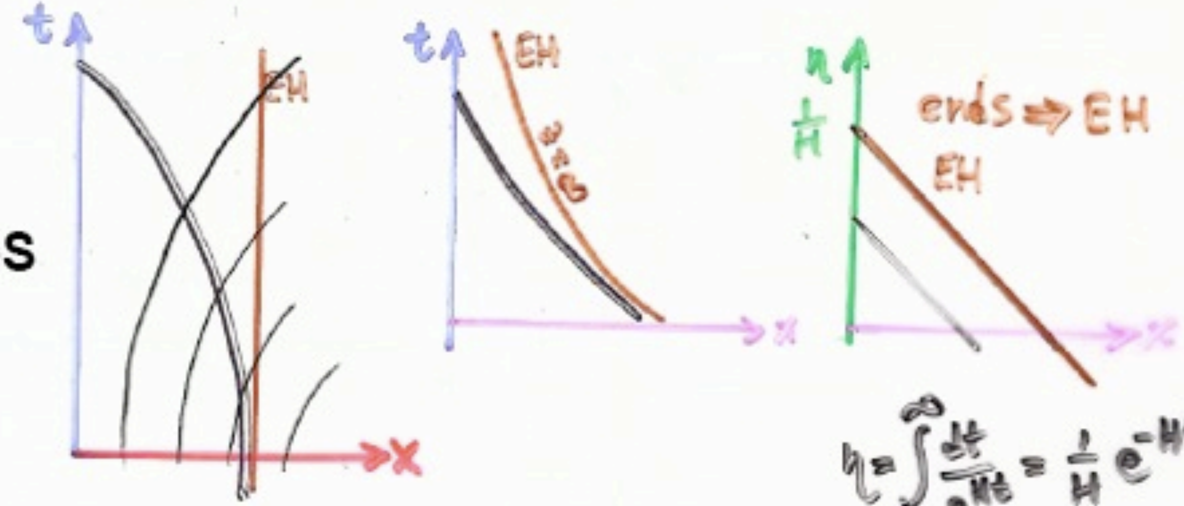


backward light cones

EVENT HORIZON

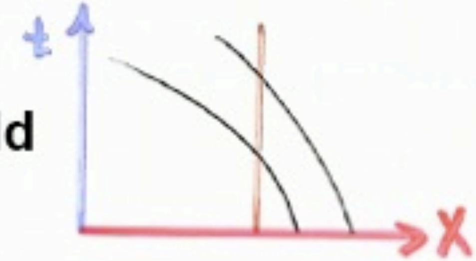
Backward lightcone that separates *events* that will someday be observed from those never observed

deS



$$\eta = \int_{t_0}^{\infty} \frac{dt}{e^{Ht}} = \frac{1}{H} e^{-Ht_0} \le \frac{1}{H}$$

Schwarzschild



See Harrison, *Cosmology*
Rindler, *Relativity*

Distances in an Expanding Universe

Proper distance = physical distance = d_p

$d_p(t_0)$ = (physical distance at t_0) = $a(t_0)\, r_e = r_e$

$\chi(t_e)$ = (comoving distance of galaxy emitting at time t_e)

$$\chi(t_e) = \int_0^{r_e} dr = r_e = c\int_{t_e}^{t_0} dt/a = c\int_{a_e}^{1} da/(a^2 H)$$

because

$$dt = (dt/da)\, da = (a\, dt/da)\, da/a$$
$$= da/(aH)$$

$d_p(t_e)$ = (physical distance at t_e) = $a(t_e)\, r_e = a_e\, r_e$

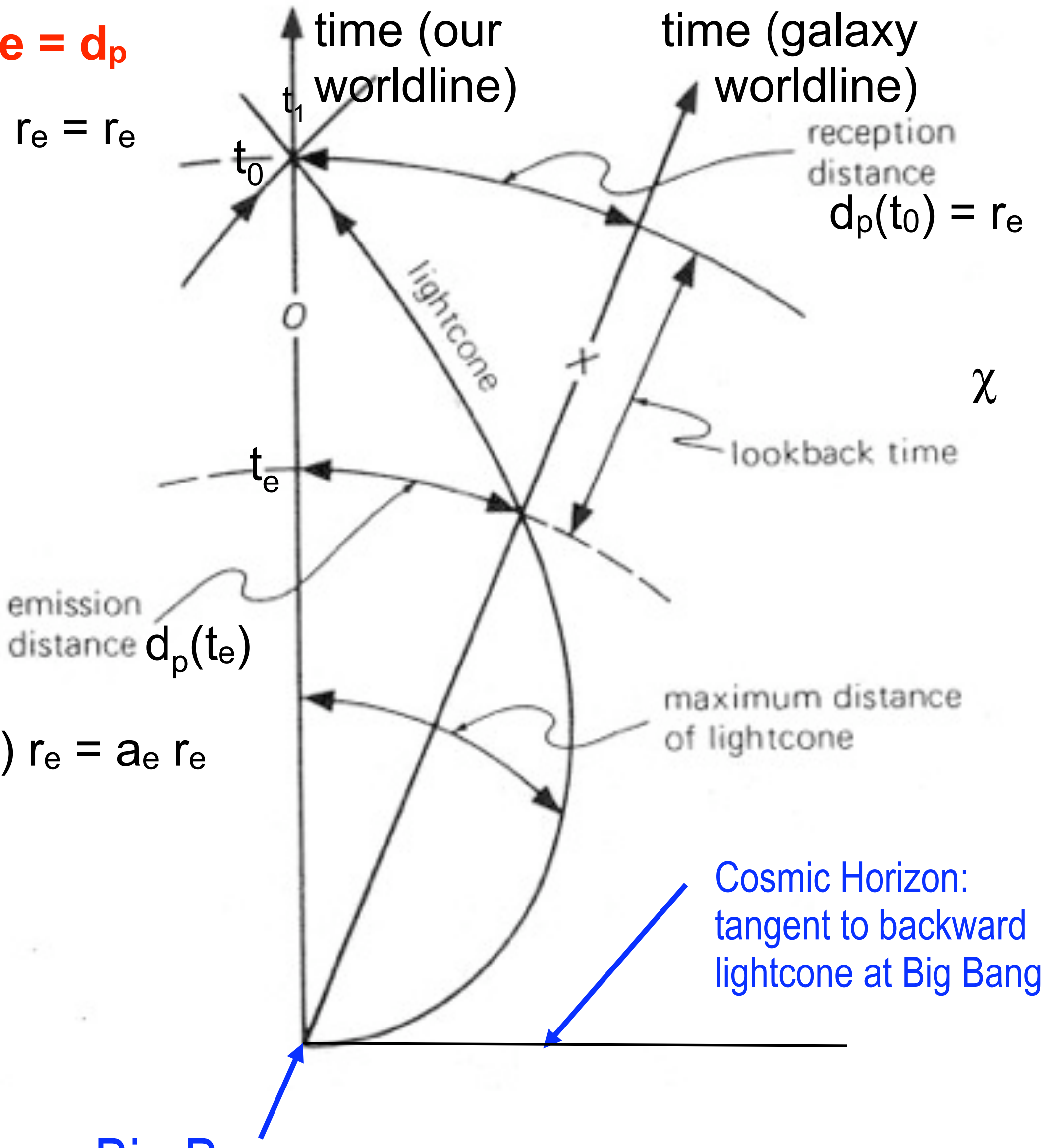
The Hubble radius $d_H = c\, H_0^{-1}$=

$= 4.29\, h_{70}^{-1}$ Gpc $= 13.97 h_{70}^{-1}$ Glyr

For E-dS, where $H = H_0\, a^{-3/2}$,

$$\chi(t_e) = r_e = d_p(t_0) = 2 d_H\, (1 - a_e^{1/2})$$

$$d_p(t_e) = 2 d_H\, a_e\, (1 - a_e^{1/2})$$



Cosmic Horizon: tangent to backward lightcone at Big Bang

Big Bang

From E. Harrison, *Cosmology* (Cambridge UP, 2000).