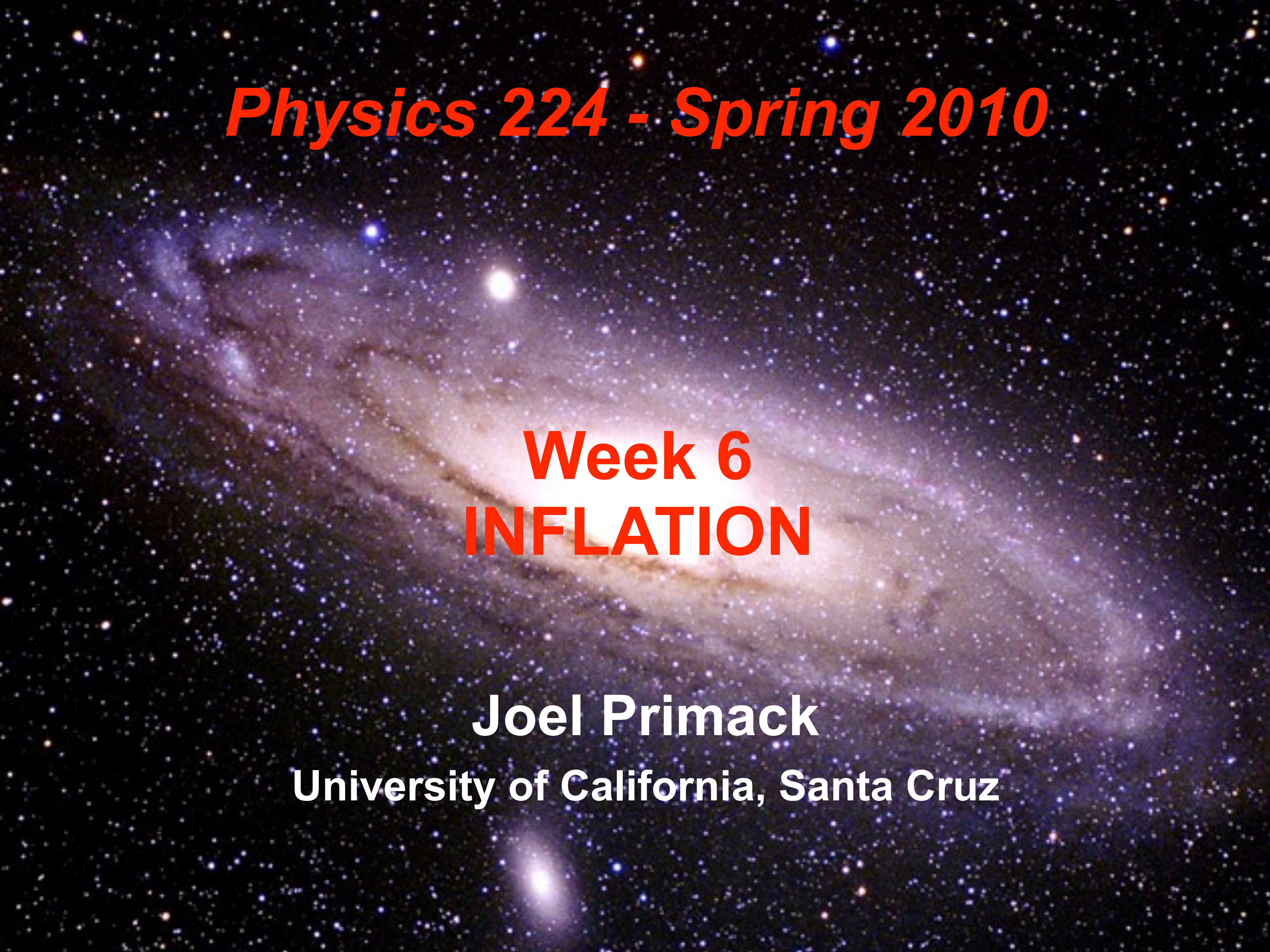

Physics 224 - Spring 2010

Week 6
INFLATION

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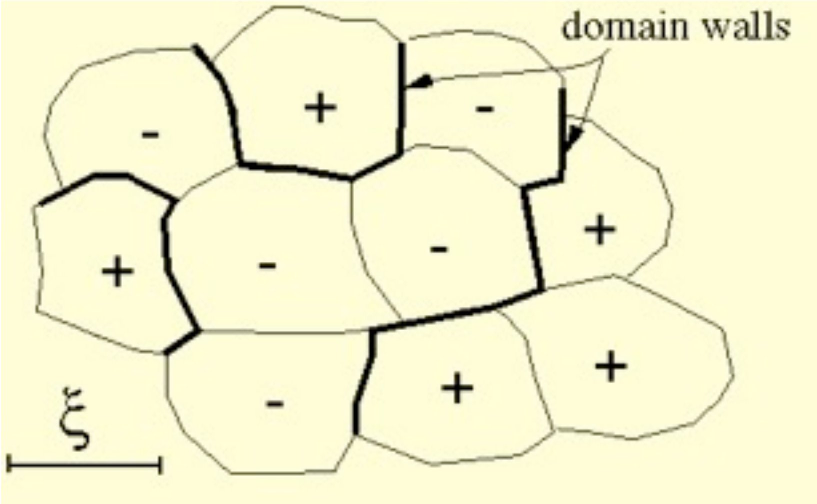
Why do cosmic topological defects form?

If cosmic strings or other topological defects *can* form at a cosmological phase transition then they *will* form. This was first pointed out by Kibble and, in a cosmological context, the defect formation process is known as the **Kibble mechanism**.

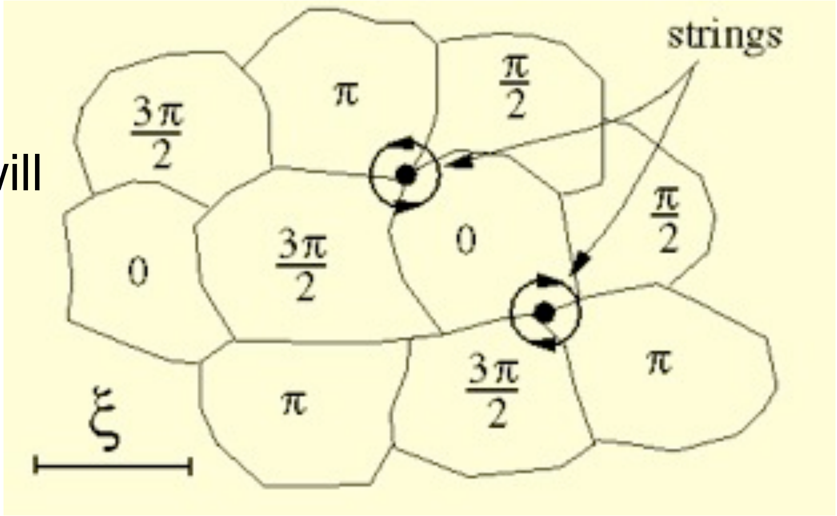
The simple fact is that causal effects in the early universe can only propagate (as at any time) at the speed of light c. This means that at a time t, regions of the universe separated by more than a distance $d=ct$ can know nothing about each other. In a symmetry breaking phase transition, different regions of the universe will choose to fall into different minima in the set of possible states (this set is known to mathematicians as the vacuum manifold). Topological defects are precisely the “boundaries” between these regions with different choices of minima, and their formation is therefore an inevitable consequence of the fact that different regions cannot agree on their choices.

For example, in a theory with two minima, plus + and minus -, then neighboring regions separated by more than ct will tend to fall randomly into the different states (as shown below). Interpolating between these different minima will be a **domain wall**.



Cosmic strings will arise in slightly more complicated theories in which the minimum energy states possess `holes'. The strings will simply correspond to non-trivial `windings' around these holes (as illustrated at right).

The Kibble mechanism for the formation of cosmic strings.



Topological defects can provide a unique link to the physics of the very early universe. Furthermore, they can crucially affect the evolution of the universe, so their study is an unavoidable part of any serious attempt to understand the early universe. The **cosmological consequences** vary with the type of defect considered. Domain walls and monopoles are cosmologically catastrophic. Any cosmological model in which they form will evolve in a way that contradicts the basic observational facts that we know about the universe. Such models must therefore be ruled out! Cosmic inflation was invented to solve this problem.

Cosmic strings and textures are (possibly) much more benign. Among other things, the were until recently thought to be a possible source of the fluctuations that led to the formation of the large-scale structures we observe today, as well as the anisotropies in the Cosmic Microwave Background. However, the CMB anisotropies have turned out not to agree with the predictions of this theory.

Cosmic String Dynamics and Evolution

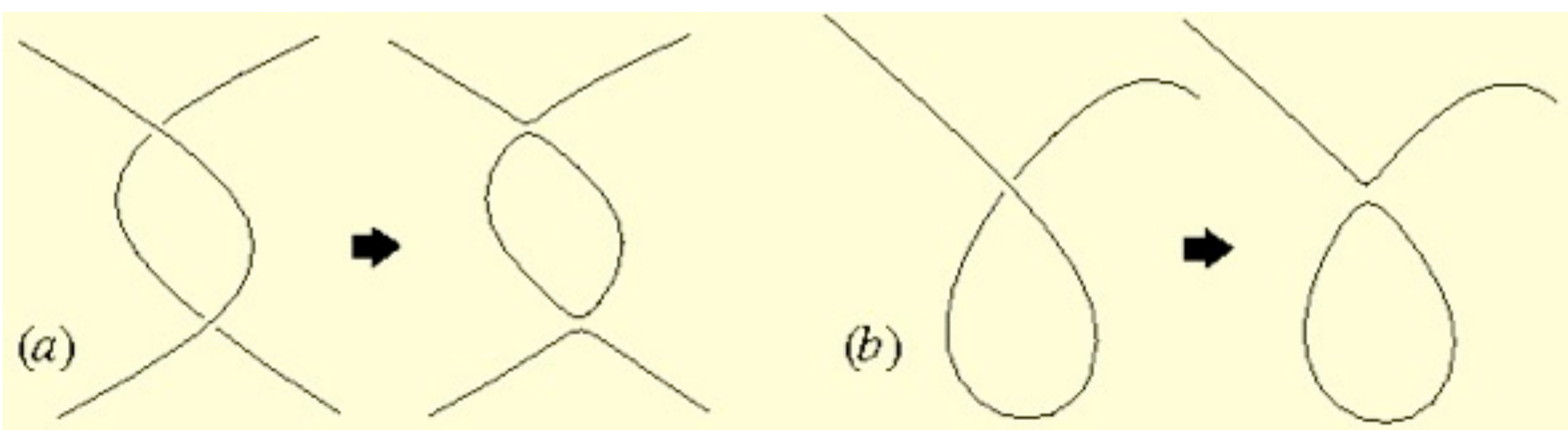
The evolution of cosmic string network is the relatively complicated result of only three rather simple and fundamental processes: cosmological expansion, intercommuting & loop production, and radiation.

Cosmological expansion

The overall expansion of the universe will ‘stretch’ the strings, just like any other object that is not gravitationally bound. You can easily undertand this through the well-known analogy of the expanding balloon. If you draw a line of the surface of the balloon and then blow it up, you will see that the length of your `string' will grow at the same rate as the radius of the balloon.

Intercommuting & loop production

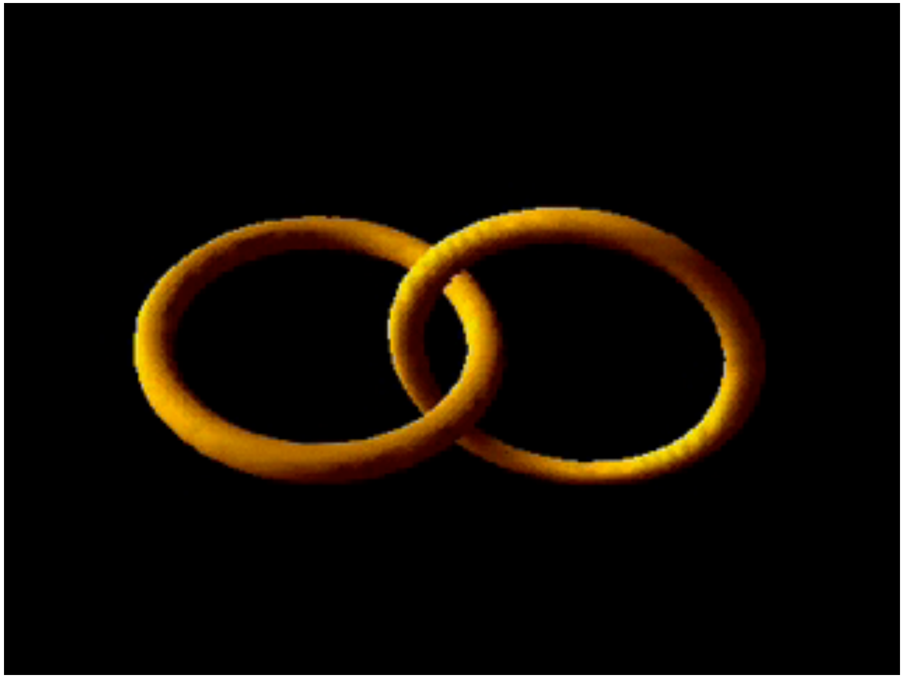
Whenever two long strings cross each other, they exchange ends, or `intercommute' (case (a) in the figure below). In particular, a long string can intercommute with itself, in which case a loop will be produced (this is case (b) below).



Radiation from strings

Both long cosmic strings and small loops will emit radiation. In most cosmological scenarios this will be **gravitational radiation**, but electromagnetic radiation or axions can also be emitted in some cases (for some specific phase transitions).

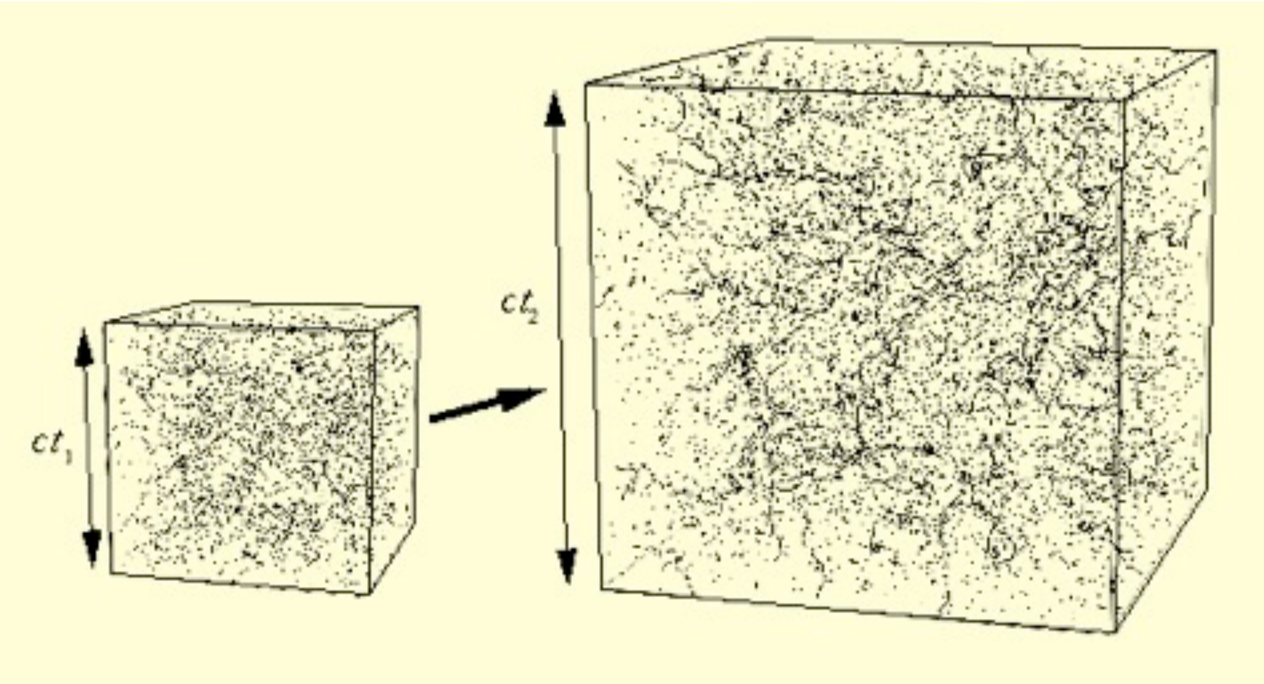
The effect of radiation is much more dramatic for loops, since they lose all their energy this way, and eventually disappear. Here you can see what happens in the case of two interlocked loops. This configuration is unlikely to happen in a cosmological setting, but it is nevertheless quite enlightening. Notice the succession of complicated dynamic processes before the loop finally disappears!



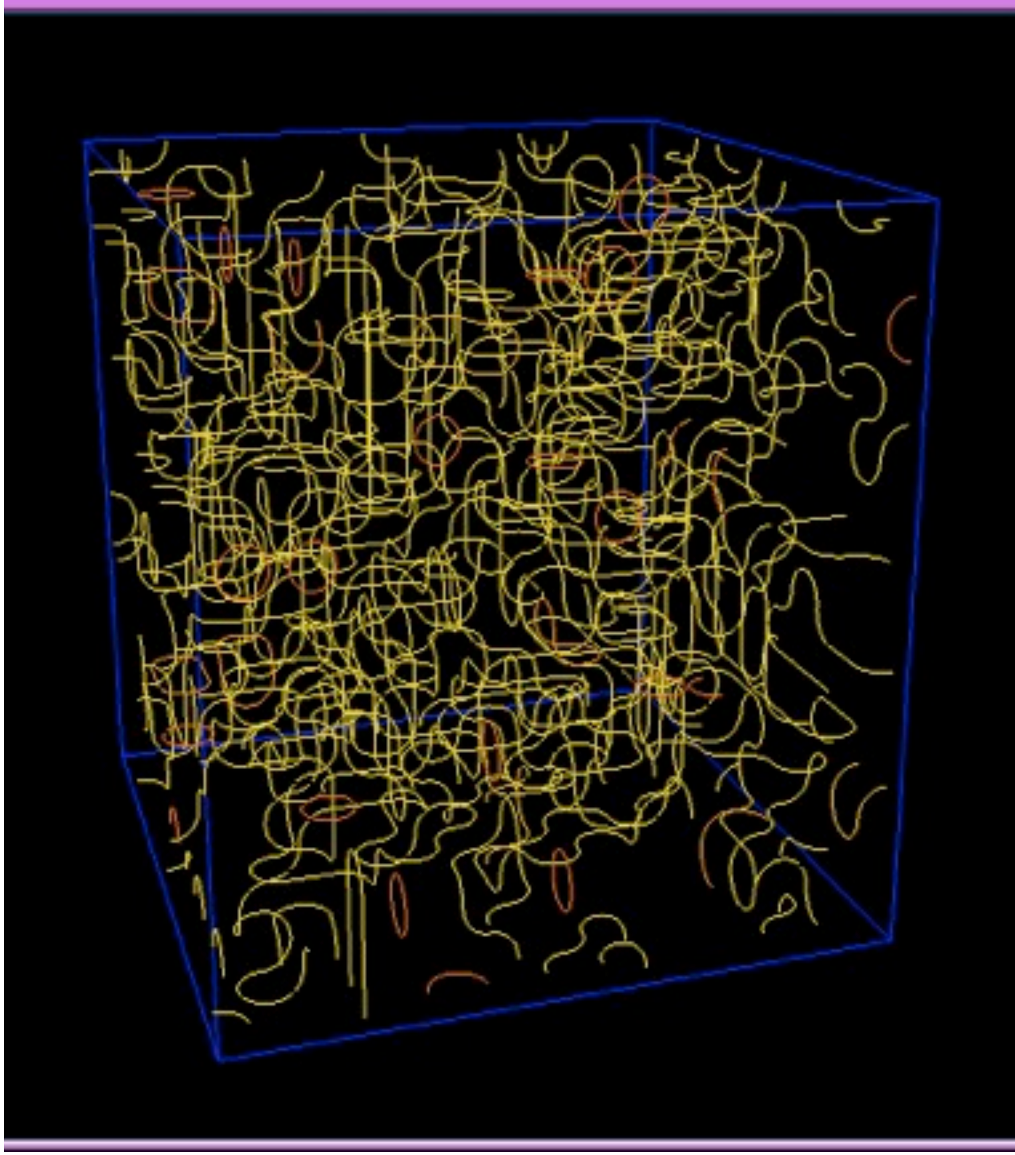
After formation, an initially high density string network begins to chop itself up by producing small loops. These loops oscillate rapidly (relativistically) and decay away into gravitational waves.

The net result is that the strings become more and more dilute with time as the universe expands. From an enormous density at formation, mathematical modelling suggests that today there would only be about 10 long strings stretching across the observed universe, together with about a thousand small loops!

In fact the network dynamics is such that the string density will eventually stabilize at an exactly constant level relative to the rest of the radiation and matter energy density in the universe. Thus the string evolution is described as `**scaling**' or scale-invariant, that is, the properties of the network look the same at any particular time t if they are scaled (or multiplied) by the change in the time. This is schematically represented below:

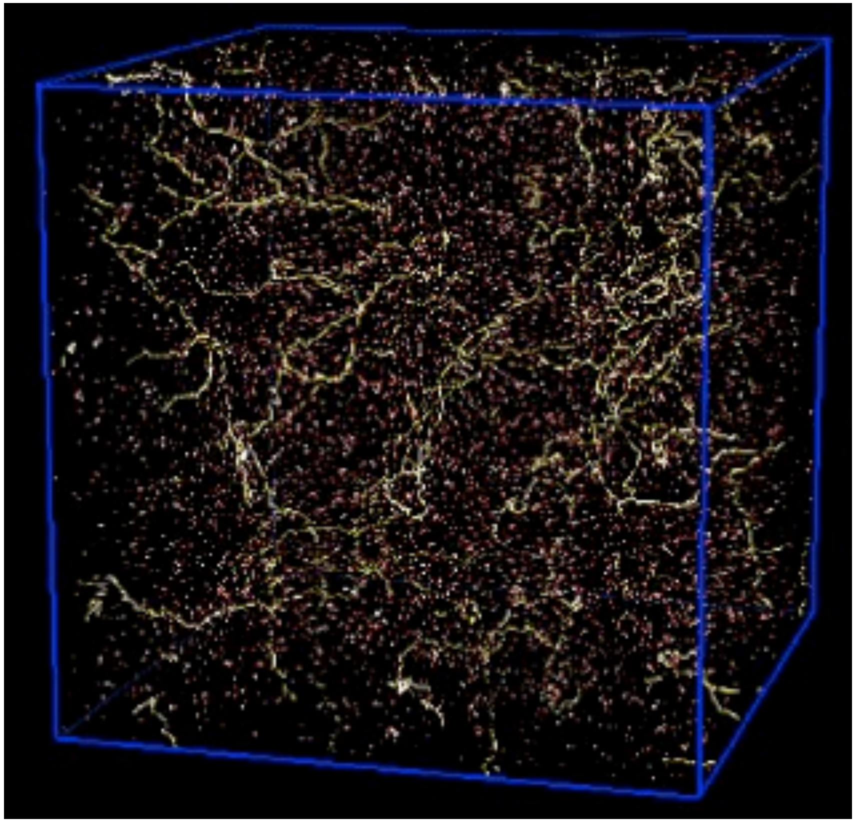


Because strings are extremely complex non-linear objects, the only rigorous way to study their evolution and cosmological consequences is to simulate in on the computer. One of the aims of performing numerical simulations of the evolution of cosmic string networks is to subsequently use the resulting information as an input to build (relatively) simpler semianalytic models that reproduce (in an averaged sense) the crucial properties of these objects. One starts by generating an initial “box of stings” containing a configuration of strings such as one would expect to find after a phase transition in the early universe. Then one evolves this initial box, by using the laws of motion of the strings.

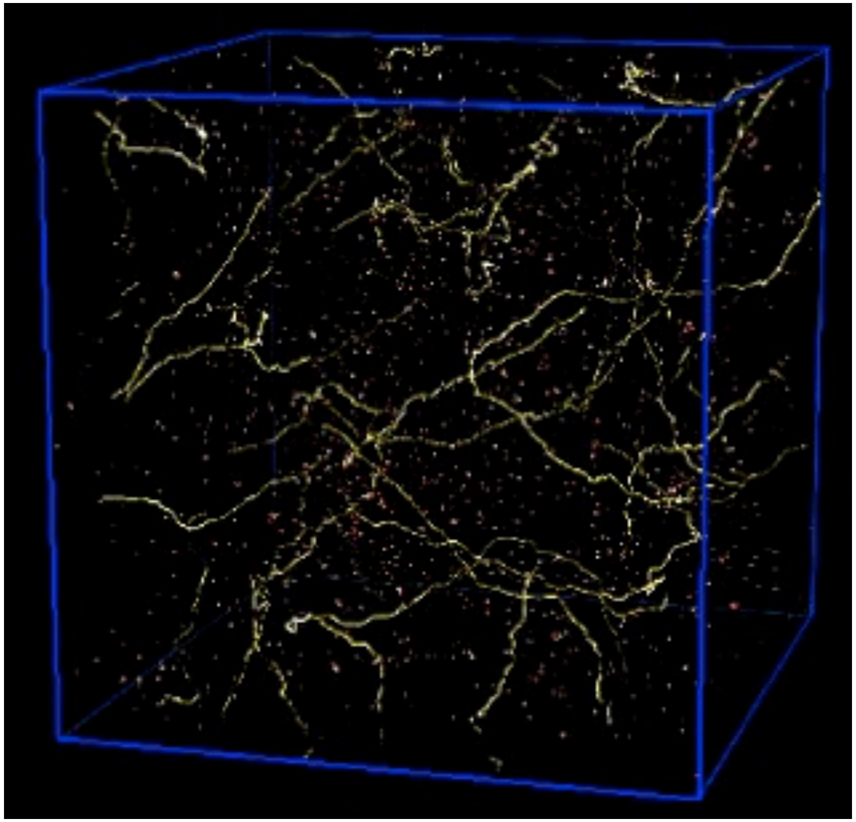


In this and all other pictures and movies below long strings are shown in yellow, while small loops have a color code going from yellow to red according to their size (red loops being the smallest).

Why do the two boxes below look different? Because the rate at which the universe is expanding is different.

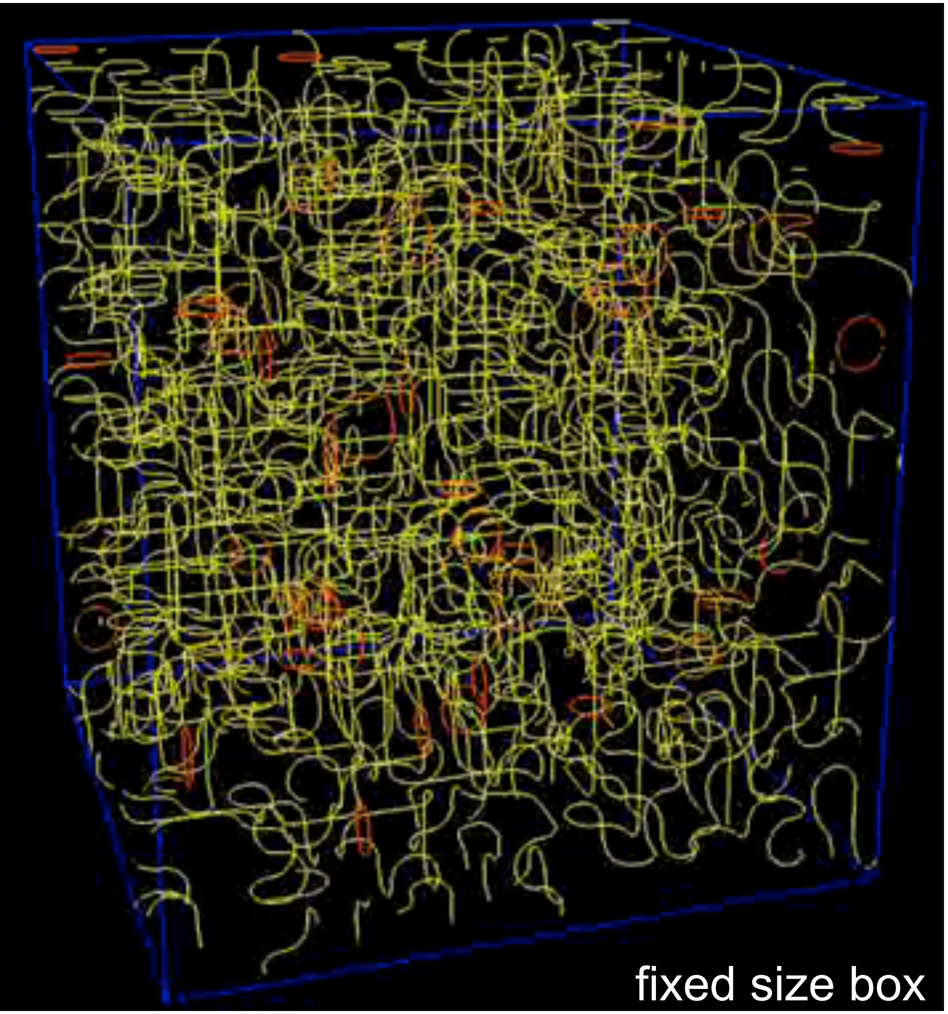


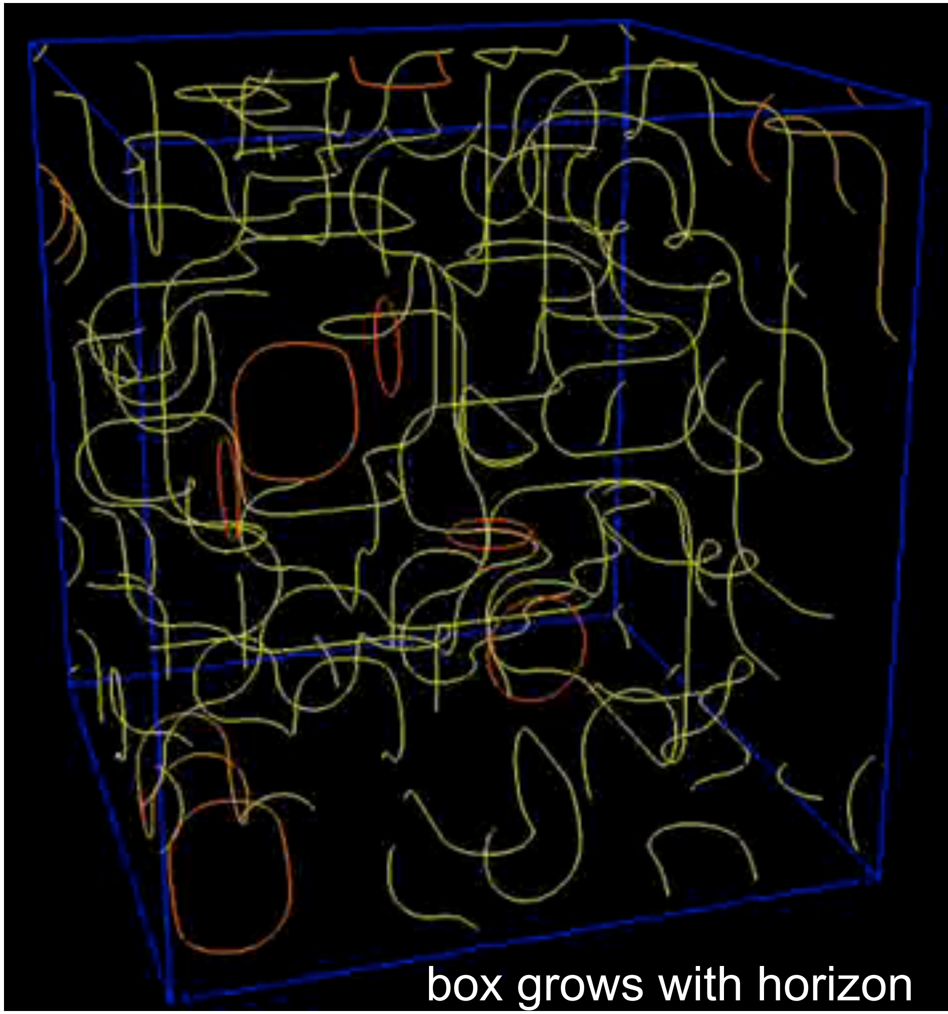
Snapshot of a string network in the radiation era. Note the high density of small loops and the `wiggliness' of the long strings in the network. The box size is about 2ct. (B. Allen & E. P. Shellard)



Snapshot of a string network in the matter era. Compare with the radiation case at left. Notice the lower density of both long strings and loops, as well as the lower `wiggliness' of the former. The box size is again about 2ct.

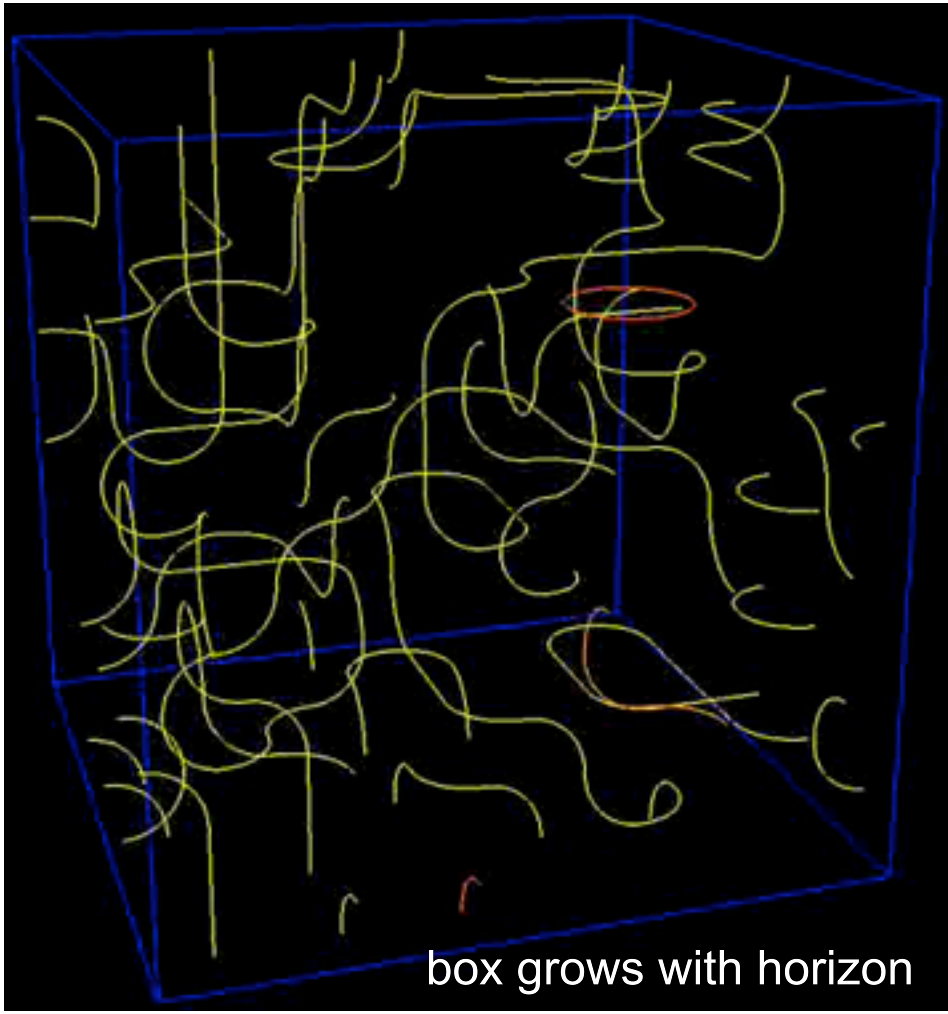
Two movies of the evolution of a cosmic string network in the radiation era. In the movie on the left the box has a fixed size (so you will see fewer and fewer strings as it evolves), while in the one on the right it grows as the comoving horizon. (C. Martins & E. P. Shellard)





Notice that the number of long strings in the box that grows with the horizon remains roughly constant, in agreement with the scaling hypothesis. This is because the additional length in strings is quickly converted into small loops.

Two movies of the evolution of a cosmic string network in the matter era. In the movie on the left the box has a fixed size (so you will see fewer and fewer strings as it evolves), while in the one on the right it grows as the comoving horizon. (C. Martins & E. P. Shellard)



Notice that the number of long strings in the box that grows with the horizon again remains roughly constant, in agreement with the scaling hypothesis. This is because the additional length in strings is quickly converted into

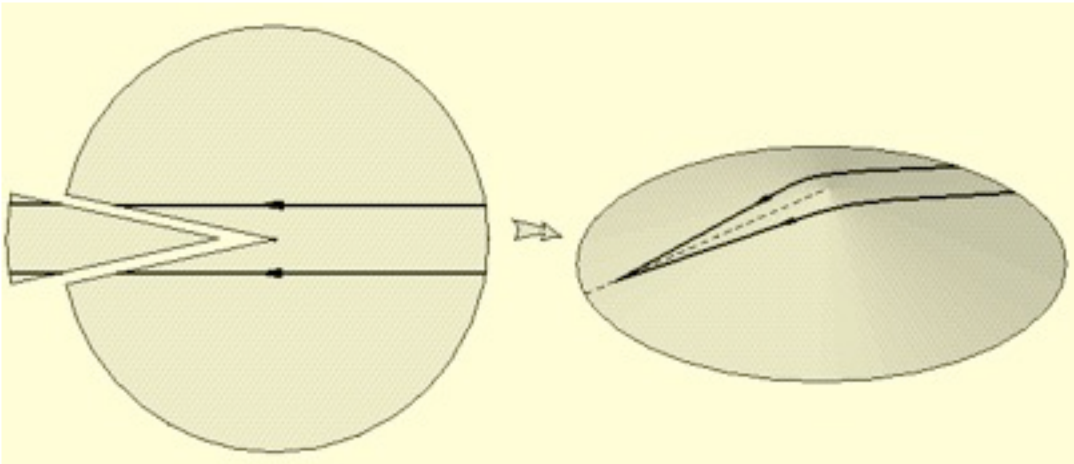
When strings evolve, scaling from smaller scales to larger ones, they create perturbations in the matter energy density of the universe. Because of their tension, cosmic strings pull straight as they come inside the horizon. Although there is no gravitational force from a static string, such moving cosmic strings produce wakes toward which matter falls, thus serving as seeds for structure formation. For a static string along the z axis of mass μ per unit length, the energy momentum tensor is

$$T^{\mu\nu} = \mu\, \mathrm{diag}(1,0,0,-1)\delta(x)\delta(y)$$

and the metric is

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}z^2 - \mathrm{d}R^2 - (1-4G\mu)^2 R^2 \mathrm{d}\varphi^2$$

$G\mu \approx (M_{GUT}/M_{Pl})^2 \approx 10^{-6}$ is just the magnitude needed for GUT string structure formation. There is an **angular defect** of $8\pi G\mu$ = 5.18" (10^6 $G\mu$). This implies that the geodesic path of light is curved towards a string when light passes by it. Two copies of a galaxy near a cosmic string will appear to observers on the other side of the string.



A. Vilenkin, E. P. S. Shellard, **Cosmic Strings and Other Topological Defects** (Cambridge U P, 1994)

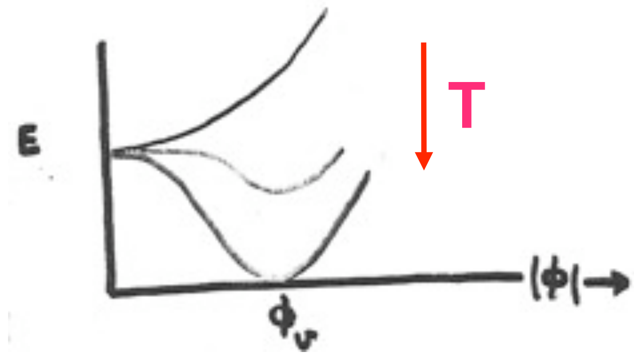
Cosmic Strings Summary

Cosmic strings arise in spontaneously broken (SB) gauge theories

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |D_\mu \phi|^2 - \lambda(|\phi|^2 - \phi_v^2)^2$$

as a consequence of causality in the expanding universe.

As the temperature T falls, a complex scalar field ϕ gets a nonzero expectation value



$$\phi(x) = \phi_v \, e^{\,i\theta(x)}$$

The phase θ will inevitably be different in regions separated by distances greater than the horizon size when the SB phase transition occurred. If θ runs over $0 \rightarrow 2\pi$ as x goes around a loop in space, the loop encloses a string.

By 2000, it was clear that **cosmic defects are not the main source of the CMB anisotropies**.

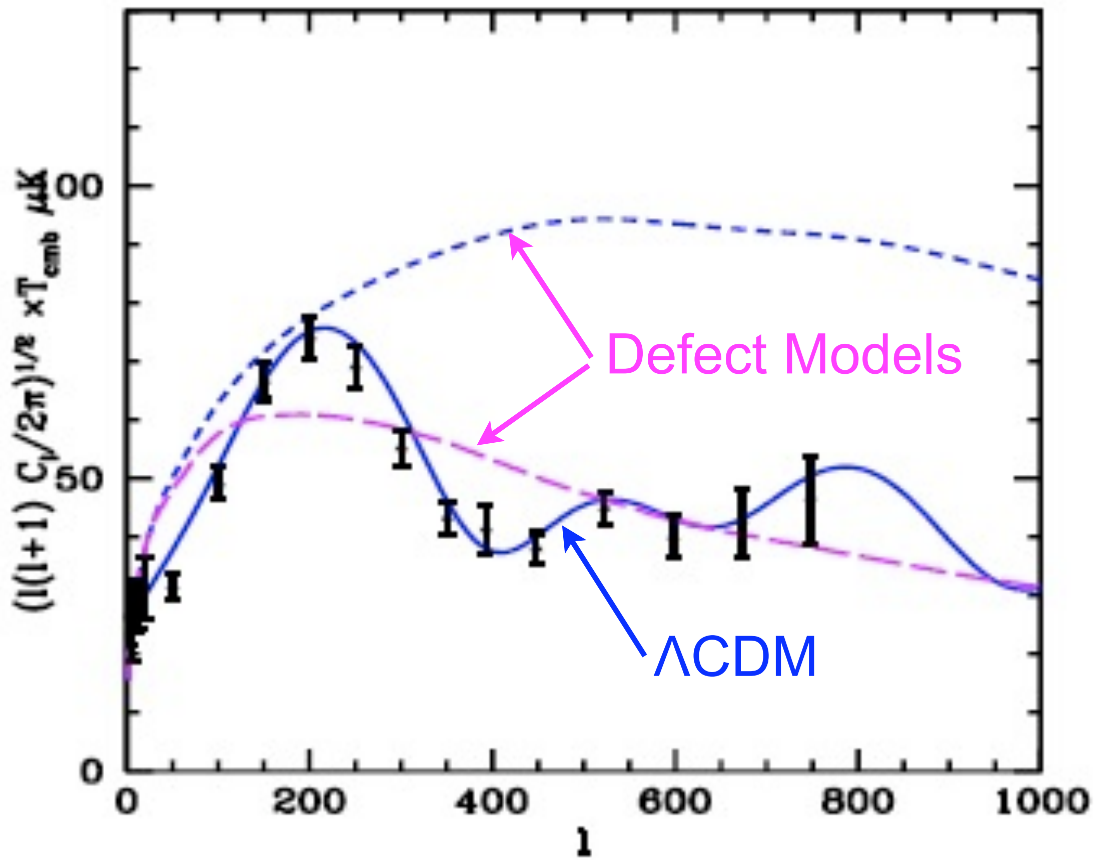


Figure 3: Current data (as complied by Knox[22]) with two defect models (dashed) and an inflation-based model (solid). The upper defect model has a standard ionization history and the lower model has an ionization history specifically designed to produce a sharper, shifted peak.

Andreas Albrecht, Defect models of cosmic structure in light of the new CMB data, XXXVth Rencontres de Moriond ``Energy Densities in the Universe'' (2000).

Fitting CMB data with cosmic strings and inflation, by Neil Bevis, Mark Hindmarsh, Martin Kunz, and Jon Urrestilla 2008 PRL100.021301

The inflationary paradigm is successful in providing a match to measurements of the cosmic microwave background (CMB) radiation, and it appears that **any successful theory of high energy physics must be able to incorporate inflation**. While ad hoc single-field inflation can provide a match to the data, **more theoretically motivated models commonly predict the existence of cosmic strings**. These strings are prevalent in supersymmetric inflation models and occur frequently in grand-unified theories (GUTs). String theory can also yield strings of cosmic extent. Hence the observational consequences of cosmic strings are important, including their sourcing of additional anisotropies in the CMB radiation. In this letter we present a multi-parameter fit to CMB data for models incorporating cosmic strings. It is the first such analysis to use simulations of a fully dynamical network of local cosmic strings, and the first to incorporate their microphysics with a field theory. It yields conclusions which differ in significant detail from previous analyses based upon simplified models: **we find that the CMB data moderately favor a 10% contribution from strings to the temperature power spectrum** measured at multipole $\ell = 10$ with a corresponding spectral index of primordial scalar perturbations $n_s \simeq 1$. There are also important implications for models of inflation with blue power spectra ($n_s > 1$). These are disfavored by CMB data under the concordance model (power-law CDM which gives $n_s = 0.951{+0.015}\,{-0.019}$) and previous work seemed to show that this remains largely the case even if cosmic strings are allowed. However with our more complete CMB calculations, we find that the CMB puts no pressure on such models if they produce cosmic strings. Our conclusions are slightly modified when additional non-CMB data are included, with the preference for strings then reduced.

GUT Monopoles

A simple SO(3) GUT illustrates how nonsingular monopoles arise. The Lagragian is

$$\mathcal{L} = \frac{1}{2}D_\mu\Phi^a D^\mu\Phi^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{8}\lambda(\Phi^a\Phi^a - \sigma^2)^2,$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - e\varepsilon_{abc}A^b_\mu A^c_\nu,$$

$$D_\mu\Phi^a = \partial_\mu\Phi^a - e\varepsilon_{abc}A^b_\mu\Phi^c.$$

The masses of the resulting charged vector and Higgs bosons after spontaneous symmetry breaking are

$$M_V^2 = e^2\sigma^2,$$

$$M_S^2 = \lambda\sigma^2.$$

If the Higgs field Φ^a happens to rotate about a sphere in SO(3) space as one moves around a sphere about any particular point in $\mathbf{x}$-space, then it must vanish at the particular point. Remarkably, if we identify the massless vector field as the photon, this configuration corresponds to a nonsingular magnetic monopole, as was independently discovered by 'tHooft and Polyakov. The monopole has magnetic charge twice the minimum Dirac value, $g = 2\pi/e = (4\pi/e^2)(e/2) \approx 67.5\ e$.

The singular magnetic field is cut off at scale σ, and as a result the GUT monopole has mass $M_{monopole} \approx M_V/\alpha \approx M_{GUT}/\alpha \approx 10^{18}$ GeV.

The first accurate calculation of the mass of the ‘t Hooft - Polyakov non-singular monopole was Bais & Primack (Phys. Rev. D13:819,1976).

GUT Monopole Problem

The Kibble mechanism produces ~ one GUT monopole per horizon volume when the GUT phase transition occurs. These GUT monopoles have a number density over entropy

$$n_M/s \sim 10^2 (T_{GUT}/M_{Pl})^3 \sim 10^{-13}$$

(compared to $n_B/s \sim 10^{-9}$ for baryons) Their annihilation is inefficient since they are so massive, and as a result they are about as abundant as gold atoms but 10^{16} times more massive, so they “overclose” the universe. This catastrophe must be avoided! **This was Alan Guth’s initial motivation for inventing cosmic inflation.**

I will summarize the key ideas of inflation theory, following my lectures at the Jerusalem Winter School, published as the first chapter in Avishai Dekel & Jeremiah Ostriker, eds., *Formation of Structure in the Universe* (Cambridge University Press, 1999), and Dierck-Ekkehard Liebscher, *Cosmology* (Springer, 2005) (available online through the UCSC library).

Motivations for Inflation

	Problem Solved
Horizon	Homogeneity, Isotropy, Uniform T
Flatness/Age	Expansion and gravity balance
"Dragons"	Monopoles, doman walls,... banished
Structure	Small fluctuations to evolve into galaxies, clusters, voids

Cosmological constant $\Lambda > 0 \Rightarrow$ space repels space, so the more space the more repulsion, $\Rightarrow$ de Sitter exponential expansion $a \propto e^{\sqrt{\Lambda}t}$.

Inflation is exponentially accelerating expansion caused by effective cosmological constant ("false vacuum" energy) associated with hypothetical scalar field ("inflaton").

		Forces of Nature	Spin
Known	{	Gravity	2
		Strong, weak, and electromagnetic	1
Goal of LHC		Mass (Higgs Boson)	0
Early universe		Inflation (Inflaton)	0

Inflation lasting only $\sim 10^{-32}$s suffices to solve all the problems listed above. Universe must then convert to ordinary expansion through conversion of false to true vacuum ("re-"heating).

Joel Primack, in *Formation of Structure in the Universe*, ed. Dekel & Ostriker (Cambridge Univ Press, 1999)

Inflation Basics

The basic idea of inflation is that before the universe entered the present adiabatically expanding Friedmann era, it underwent a period of de Sitter exponential expansion of the scale factor, termed *inflation* (Guth 1981). Actually, inflation is never precisely de Sitter, and any superluminal (faster-than-light) expansion is now called inflation. Inflation was originally invented to solve the problem of too many GUT monopoles, which, as mentioned in the previous section, would otherwise be disastrous for cosmology.

The de Sitter cosmology corresponds to the solution of Friedmann's equation in an empty universe (i.e., with $\rho = 0$) with vanishing curvature ($k = 0$) and positive cosmological constant ($\Lambda > 0$). The solution is $a = a_o e^{Ht}$, with constant Hubble parameter $H = (\Lambda/3)^{1/2}$. There are analogous solutions for $k = +1$ and $k = -1$ with $a \propto \cosh Ht$ and $a \propto \sinh Ht$ respectively. The scale factor expands exponentially because the positive cosmological constant corresponds effectively to a negative pressure. de Sitter space is discussed in textbooks on general relativity (for example, Rindler 1977, Hawking & Ellis 1973) mainly for its geometrical interest. Until cosmological inflation was considered, the chief significance of the de Sitter solution in cosmology was that it is a limit to which all indefinitely expanding models with $\Lambda > 0$ must tend, since as $a \to \infty$, the cosmological constant term ultimately dominates the right hand side of the Friedmann equation.

Joel Primack, in *Formation of Structure in the Universe*, (Cambridge Univ Press, 1999)

As Guth (1981) emphasized, the de Sitter solution might also have been important in the very early universe because the vacuum energy that plays such an important role in spontaneously broken gauge theories also acts as an effective cosmological constant. A period of de Sitter inflation preceding ordinary radiation-dominated Friedmann expansion could explain several features of the observed universe that otherwise appear to require very special initial conditions: the horizon, flatness/age, monopole, and structure formation problems. (See Table 1.6.)

Let us illustrate how inflation can help with the horizon problem. At recombination ($p^+ + e^- \rightarrow H$), which occurs at $a/a_o \approx 10^{-3}$, the mass encompassed by the horizon was $M_H \approx 10^{18} M_\odot$, compared to $M_{H,o} \approx 10^{22} M_\odot$ today. Equivalently, the angular size today of the causally connected regions at recombination is only $\Delta\theta \sim 3°$. Yet the fluctuation in temperature of the cosmic background radiation from different regions is very small: $\Delta T/T \sim 10^{-5}$. How could regions far out of causal contact have come to temperatures that are so precisely equal? This is the "horizon problem". With inflation, it is no problem because the entire observable universe initially lay inside a single causally connected region that subsequently inflated to a gigantic scale. Similarly, inflation exponentially dilutes any preceeding density of monopoles or other unwanted relics (a modern version of the "dragons" that decorated the unexplored borders of old maps).

In the first inflationary models, the dynamics of the very early universe was typically controlled by the self-energy of the Higgs field associated with the breaking of a Grand Unified Theory (GUT) into the standard 3-2-1 model: GUT$\rightarrow SU(3)_{color} \otimes [SU(2) \otimes U(1)]_{electroweak}$. This occurs when the cosmological temperature drops to the unification scale $T_{GUT} \sim 10^{14}$ GeV at about 10^{-35} s after the Big Bang. Guth (1981) initially considered a scheme in which inflation occurs while the universe is trapped in an unstable state (with the GUT unbroken) on the wrong side of a maximum in the Higgs potential. This turns out not to work: the transition from a de Sitter to a Friedmann universe never finishes (Guth & Weinberg 1981). The solution in the "new inflation" scheme (Linde 1982; Albrecht and Steinhardt 1982) is for inflation to occur *after* barrier penetration (if any). It is necessary that the potential of the scalar field controlling inflation ("*inflaton*") be nearly flat (i.e., decrease very slowly with increasing inflaton field) for the inflationary period to last long enough. This nearly flat part of the potential must then be followed by a very steep minimum, in order that the energy contained in the Higgs potential be rapidly shared with the other degrees of freedom ("reheating"). A more general approach, "chaotic" inflation, has been worked out by Linde (1983, 1990) and others; this works for a wide range of inflationary potentials, including simple power laws such as $\lambda\phi^4$. However, for the amplitude of the fluctuations to be small enough for consistency with observations, it is necessary that the inflaton self-coupling be very small, for example $\lambda \sim 10^{-14}$ for the ϕ^4 model. This requirement prevents a Higgs field from being the inflaton, since Higgs fields by definition have gauge couplings to the gauge field (which are expected to be of order unity), and these would generate self-couplings of similar magnitude even if none were present.

It turns out to be necessary to inflate by a factor $\gtrsim e^{66}$ in order to solve the flatness problem, i.e., that $\Omega_0 \sim 1$. (With $H^{-1} \sim 10^{-34}$ s during the de Sitter phase, this implies that the inflationary period needs to last for only a relatively small time $\tau \gtrsim 10^{-32}$ s.) The "flatness problem" is essentially the question why the universe did not become curvature dominated long ago. Neglecting the cosmological constant on the assumption that it is unimportant after the inflationary epoch, the Friedmann equation can be written

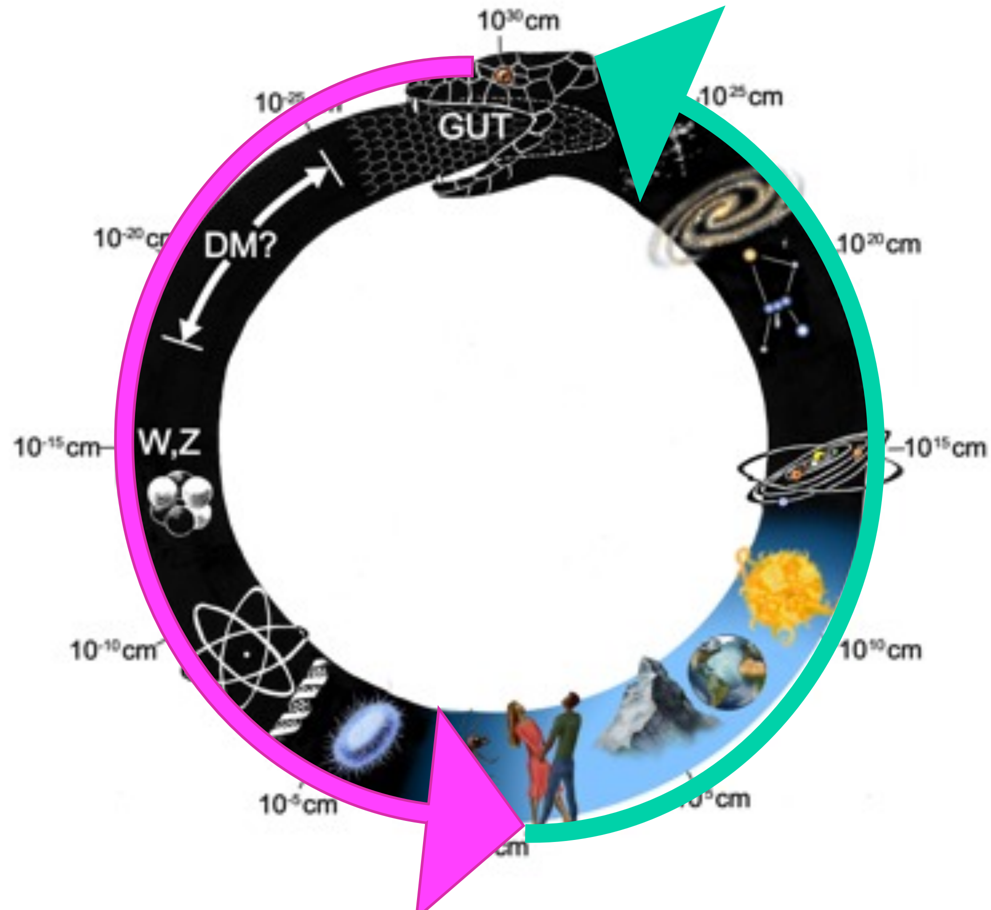
$e^{66}=$ $4\text{x}10^{28}$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\pi^2}{30}g(T)T^4 - \frac{kT^2}{(aT)^2}$$

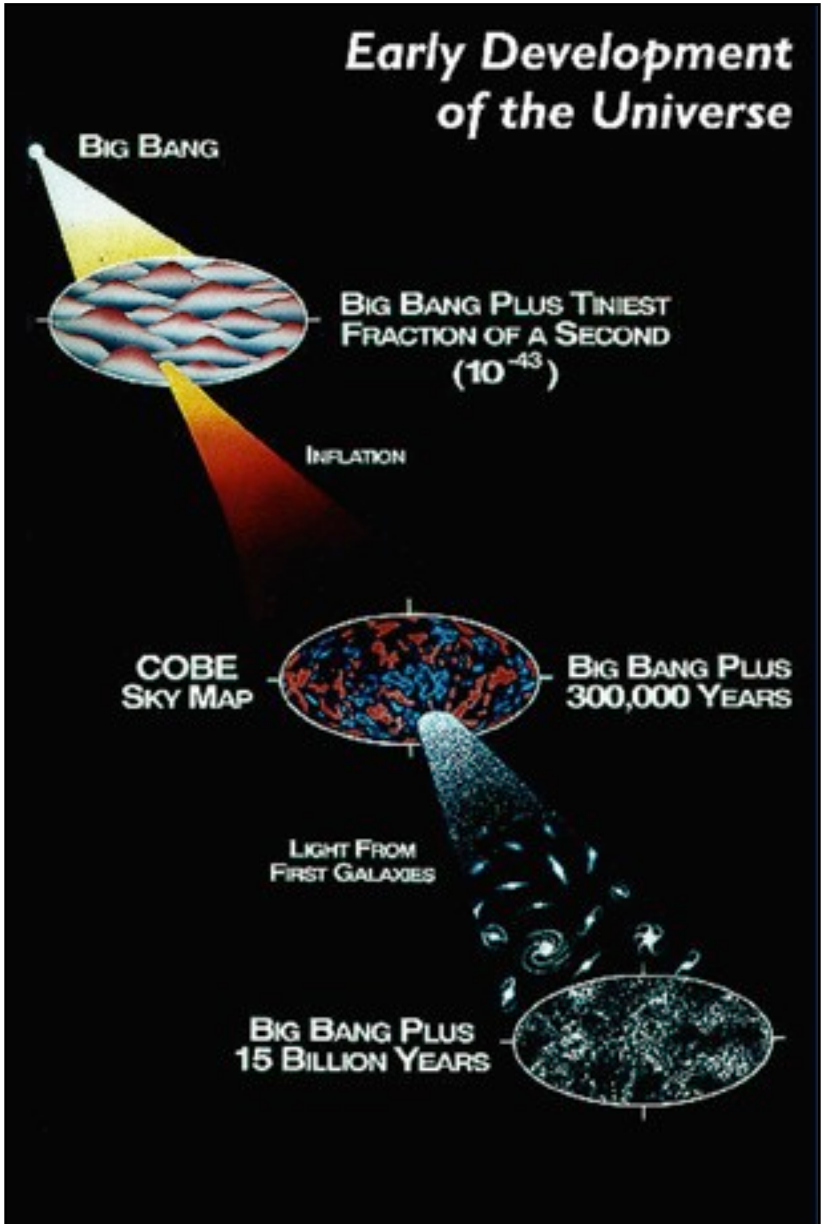
where the first term on the right hand side is the contribution of the energy density in relativistic particles and $g(T)$ is the effective number of degrees of freedom. The second term on the right hand side is the curvature term. Since $aT \approx$ constant for adiabatic expansion, it is clear that as the temperature T drops, the curvature term becomes increasingly important. The quantity $K \equiv k/(aT)^2$ is a dimensionless measure of the curvature. Today, $|K| = |\Omega - 1|\, H_o^2/T_o^2 \leq 2 \times 10^{-58}$. Unless the curvature exactly vanishes, the most "natural" value for K is perhaps $K \sim 1$. Since inflation increases a by a tremendous factor $e^{H\tau}$ at essentially constant T (after reheating), it increases aT by the same tremendous factor and thereby decreases the curvature by that factor squared. Setting $e^{-2H\tau} \lesssim 2\times 10^{-58}$ gives the needed amount of inflation: $H\tau \gtrsim 66$. This much inflation turns out to be enough to take care of the other cosmological problems mentioned above as well.

Cosmic Inflation

According to Cosmic Inflation theory, the entire visible universe was once about 10^{-30} cm in size. Its size then inflated by a factor of about 10^{30} so that when Cosmic Inflation ended (after about 10^{-32} second) it had reached the size of a baby. During its entire subsequent evolution, the size of the visible universe has increased by only about another factor of 10^{28}.



Generating the Primordial Density Fluctuations



Early phase of exponential expansion (Inflationary epoch)

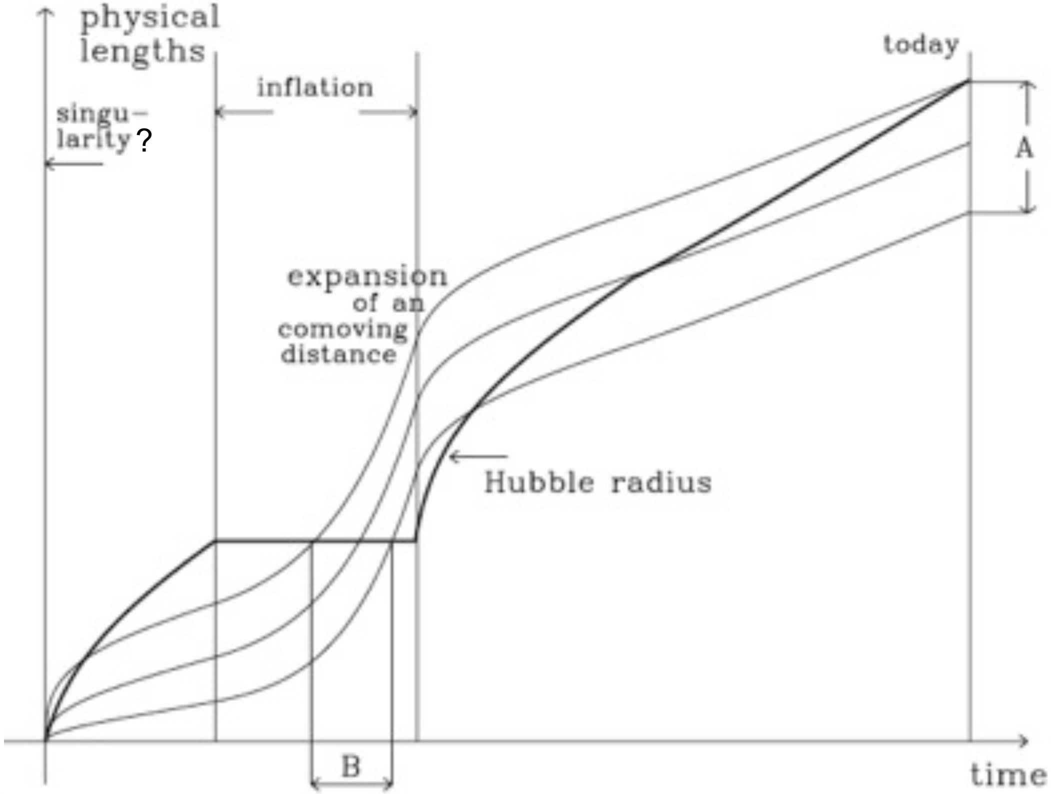
↓

Zero-point fluctuations of quantum fields are stretched and frozen

↓

Cosmic density fluctuations are frozen quantum fluctuations

Inflationary Fluctuations



Thus far, it has been sketched how inflation stretches, flattens, and smooths out the universe, thus greatly increasing the domain of initial conditions that could correspond to the universe that we observe today. But inflation also can explain the origin of the fluctuations necessary in the gravitional instability picture of galaxy and cluster formation. Recall that the very existence of these fluctuations is a problem in the standard Big Bang picture, since these fluctuations are much larger than the horizon at early times. How could they have arisen?

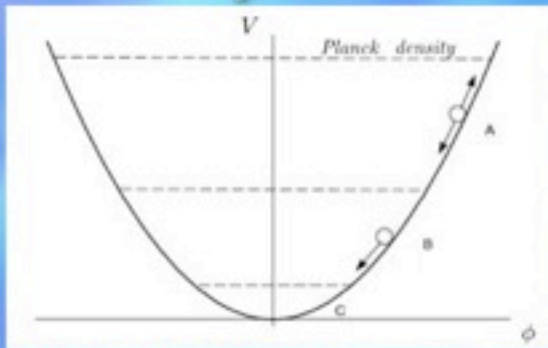
The answer in the inflationary universe scenario is that they arise from quantum fluctuations in the inflaton field ϕ whose vacuum energy drives inflation. The scalar fluctuations $\delta\phi$ during the de Sitter phase are of the order of the Hawking temperature $H/2\pi$. Because of these fluctuations, there is a time spread $\Delta t \approx \delta\phi/\dot{\phi}$ during which different regions of the same size complete the transition to the Friedmann phase. The result is that the density fluctuations when a region of a particular size re-enters the horizon are equal to (Guth & Pi 1982; see Linde 1990 for alternative approaches) $\delta_H \equiv (\delta\rho/\rho)_H \sim \Delta t/t_H = H\Delta t$. The time spread Δt can be estimated from the equation of motion of ϕ (the free Klein-Gordon equation in an expanding universe): $\ddot{\phi} + 3H\dot{\phi} = -(\partial V/\partial\phi)$. Neglecting the $\ddot{\phi}$ term, since the scalar potential V must be very flat in order for enough inflation to occur (this is called the "slow roll" approximation), $\dot{\phi} \approx -V'/(3H)$, so $\delta_H \sim H^3/V' \sim V^{3/2}/V'$. Unless there is a special feature in the potential $V(\phi)$ as ϕ rolls through the scales of importance in cosmology (producing such "designer inflation" features generally requires fine tuning — see e.g. Hodges et al. 1990), V and V' will hardly vary there and hence δ_H will be essentially constant. These are fluctuations of all the contents of the universe, so they are adiabatic fluctuations.

Thus *inflationary models typically predict a nearly constant curvature spectrum* $\delta_H =$ constant *of adiabatic fluctuations.* Some time ago Harrison (1970), Zel'dovich (1972), and others had emphasized that this is the only scale-invariant (i.e., power-law) fluctuation spectrum that avoids trouble at both large and small scales. If $\delta_H \propto M_H^{-\alpha}$, where M_H is the mass inside the horizon, then if $-\alpha$ is too large the universe will be less homogeneous on large than small scales, contrary to observation; and if α is too large, fluctuations on sufficiently small scales will enter the horizon with $\delta_H \gg 1$ and collapse to black holes (see e.g. Carr, Gilbert, & Lidsey 1995, Bullock & Primack 1996); thus $\alpha \approx 0$. The $\alpha = 0$ case has come to be known as the Zel'dovich spectrum.

Inflation predicts more: it allows the calculation of the value of the constant δ_H in terms of the properties of the scalar potential $V(\phi)$. Indeed, this proved to be embarrassing, at least initially, since the Coleman-Weinberg potential, the first potential studied in the context of the new inflation scenario, results in $\delta_H \sim 10^2$ (Guth & Pi 1982) some six orders of magnitude too large. But this does not seem to be an insurmountable difficulty; as was mentioned above, chaotic inflation works, with a sufficiently small self-coupling. Thus inflation at present appears to be a plausible solution to the problem of providing reasonable cosmological initial conditions (although it sheds no light at all on the fundamental question why the cosmological constant is so small now). Many variations of the basic idea of inflation have been worked out

Comparing different inflationary models:

- **Chaotic inflation** can start in the smallest domain of size 10^{-33} cm with total mass ~ M_p (less than a milligram) and entropy O(1)

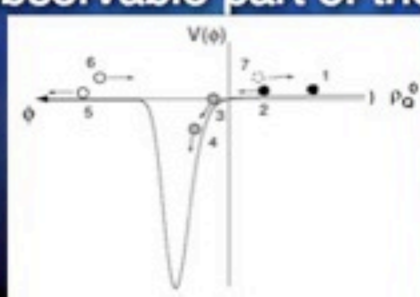


Solves flatness, mass and entropy problem

- **New inflation** can start only in a domain with mass 6 orders of magnitude greater than M_p and entropy greater than 10^9

Not very good with solving flatness, mass and entropy problem

- **Cyclic inflation** can occur only in the domain of size greater than the size of the observable part of the universe, with mass > 10^{55} g and entropy > 10^{87}



Does not solve flatness, mass and entropy problem

Andrei Linde

Is the simplest chaotic inflation natural?

Andrei Linde

- Often repeated (but incorrect) argument:

$$V = V_0 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \sum C_n \frac{\phi^n}{M_p^n}$$

Thus one could expect that the theory is ill-defined at $\phi > M_p$

However, quantum corrections are in fact proportional to

$$\left(\frac{V}{M_p^4}\right)^n \quad \text{and to} \quad \left(\frac{m^2(\phi)}{M_p^2}\right)^n$$

These terms are harmless for sub-Planckian masses and densities, even if the scalar field itself is very large.

Many Inflation Models

following Andrei Linde's classification

How Inflation Begins

Old Inflation — T_{initial} high, $\phi_{\text{in}} \approx 0$ is false vacuum until phase transition. Ends by bubble creation; Reheat by bubble collisions

New Inflation — Slow roll down $V(\phi)$, no phase transition

Chaotic Inflation — Similar to New Inflation, but ϕ_{in} essentially arbitrary: any region with $\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\partial_i\phi)^2 \lesssim V(\phi)$ inflates

Extended Inflation — Like Old Inflation, but slower (e.g., power $a \propto t^p$), so phase transition can finish

Potential $V(\phi)$ During Inflation

Chaotic typically $V(\phi) = \Lambda\phi^n$, can also use $V = V_0 e^{\alpha\phi}$, etc.

$$\Rightarrow a \propto t^p, \quad p = 16\pi/\alpha^2 \gg 1$$

How Inflation Ends

First-order phase transition — e.g., Old or Extended inflation
Faster rolling $\rightarrow$ oscillation — e.g., Chaotic $V(\phi)^2\Lambda\phi^n$
"Waterfall" — rapid roll of σ triggered by slow roll of ϕ

(Re)heating

Decay of inflatons
"Preheating" by parametric resonance, then decay

Before Inflation?

Eternal Inflation? Can be caused by

- Quantum $\delta\phi \sim H/2\pi >$ rolling $\Delta\phi = \phi\Delta t = \phi H^{-1} \approx V'/V$
- Monopoles or other topological defects