Astronomy 233 Spring 2011
Physical Cosmology

Week 9 Galaxies & Inflation Part 2. Cosmic Inflation

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Astro 233 - Spring 2011 Physical Cosmology

Week

Topics

- 1 Introduction
- 2 General Relativistic Cosmology
- 3 Big Bang Nucleosynthesis
- 4 Recombination, Dark Matter (DM)
- 5 DM Detection, Cosmic Microwave Background
- 6 Structure Formation
- 7 (Reminder: no lectures May 10 & 12)
- 8 Galaxy Formation
- 9 Galaxies; Cosmic Inflation and Before
- 10 After Inflation: Baryogenesis, Strings, ...
- 11 Student Presentations of Term Projects

Outline

Grand Unification of Forces Phase Transitions in the Early Universe Topological Defects: Strings, Monopoles Cosmic Inflation Motivation: Horizon, Flatness, Dragons, Structure How much inflation is needed?

Grand Unification

The basic premise of grand unification is that the known symmetries of the elementary particles result from a larger (and so far unknown) symmetry group G. Whenever a phase transition occurs, part of this symmetry is lost, so the symmetry group changes. This can be represented mathematically as

 $G \rightarrow H \rightarrow ... \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1).$

Here, each arrow represents a symmetry breaking phase transition where matter changes form and the groups - G, H, SU(3), etc. - represent the different types of matter, specifically the symmetries that the matter exhibits and they are associated with the different fundamental forces of nature.

The liquid phase of water is rotationally symmetric, that is, it looks the same around each point regardless of the direction in which we look. We could represent this large three-dimensional symmetry by the group G (actually SO(3)). The solid form of frozen water, however, is not uniform in all directions; the ice crystal has preferential lattice directions along which the water molecules align. The group describing these different discrete directions H, say, will be smaller than G. Through the process of freezing, therefore, the original symmetry G is broken down to H.

Grand Unified Theory

GUT refers to a theory in physics that unifies the strong interaction and electroweak interaction. Several such theories have been proposed, but none is currently universally accepted. The (future) theory that will also include gravity is termed theory of everything. Some common GUT models are:

- Georgi-Glashow (1974) model -- SU(5)
- SO(10)
- Flipped SU(5) -- SU(5)×U(1)
- Pati-Salam model -- SU(4)×SU(2)×SU(2)
- E₆

GUT models generically predict the existence of topological defects such as monopoles, cosmic strings, domain walls, and others. None have been observed and their absence is known as the monopole problem in cosmology.

There is still no hard evidence nature is described by a GUT theory. In fact, since the Higgs particle hasn't been discovered yet, it's not even certain if the Standard Model is fully accurate.

Topological Defects

These arise when some n-component scalar field $\phi_i(x) = 0$ because of topological trapping that occurs as a result of a phase transition in the early universe (as I will explain shortly).

If the ϕ field is complex then n=2, and $\phi_i(x) = 0$ occurs along a linear locus of points, a string, in three dimensional space. This corresponds to a 2-dimensional world-sheet in the 3+1 dimensions of spacetime.

If the ϕ field has three components, then $\phi_i(x) = 0$ occurs at a point in three dimensional space, a monopole. This corresponds to a 1-dimensional world-line in the 3+1 dimensions of spacetime.

If the ϕ field has four components, then $\phi_i(x) = 0$ occurs at a point in space-time, an instanton. A related concept is texture.

Phase transitions

The cosmological significance of symmetry breaking is due to the fact that symmetries are restored at high temperature (just as it is for liquid water when ice melts). For extremely high temperatures in the early universe, we will even achieve a grand unified state G. Viewed from the moment of creation forward, the universe will pass through a succession of phase transitions at which the strong nuclear force will become differentiated and then the weak nuclear force and electromagnetism.

Phase transitions can have a wide variety of important implications including the formation of topological defects -cosmic strings, domain walls, monopoles and textures, or it may even trigger a period of exponential expansion (inflation).

Phase transitions can be either dramatic - first order - or smooth second order.

During a first-order phase transition, the matter fields get trapped in a `false vacuum' state from which they can only escape by nucleating bubbles of the new phase, that is, the `true vacuum' state.



First-order phase transitions (illustrated below) occur through the formation of bubbles of the new phase in the middle of the old phase; these bubbles then expand and collide until the old phase disappears completely and the phase transition is complete.



First-order phase transitions proceed by bubble nucleation. A bubble of the new phase (the true vacuum) forms and then expands until the old phase (the false vacuum) disappears. A useful analogue is boiling water in which bubbles of steam form and expand as they rise to the surface.

Second-order phase transitions, on the other hand, proceed smoothly. The old phase transforms itself into the new phase in a continuous manner. There is energy (specific heat of vaporization, for example) associated with a first order phase transition.

Either type of phase transition can produce stable configurations called "topological defects."

Cosmic Strings & Other Topological Defects

Topological defects are stable configurations that are in the original, symmetric or old phase, but nevertheless for topological reasons they persist after a phase transition to the asymmetric or new phase is completed - because to unwind them would require a great deal of energy. There are a number of possible types of defects, such as domain walls, cosmic strings, monopoles, and textures. The type of defect is determined by the symmetry properties of the matter and the nature of the phase transition.

Domain walls: These are two-dimensional objects that form when a discrete symmetry is broken at a phase transition. A network of domain walls effectively partitions the universe into various `cells'. Domain walls have some rather peculiar properties. For example, the gravitational field of a domain wall is repulsive rather than attractive.



Cosmic strings: These are one-dimensional (that is, line-like) objects which form when an axial or cylindrical symmetry is broken. Strings can be associated with grand unified particle physics models, or they can form at the electroweak scale. They are very thin and may stretch across the visible universe. A typical GUT string has a thickness that is less then a trillion times smaller that the radius of a hydrogen atom, but a 10 km length of one such string would weigh as much as the earth itself!



Cosmic strings are associated with models in which the set of minima are not simply-connected, that is, the vacuum manifold has `holes' in it. The minimum energy states on the left form a circle and the string corresponds to a non-trivial winding around this.

Monopoles: These are zero-dimensional (point-like) objects which form when a spherical symmetry is broken. Monopoles are predicted to be supermassive and carry magnetic charge. The existence of monopoles is an inevitable prediction of grand unified theories (GUTs - more on this shortly); why the universe isn't filled with them is one of the puzzles of the standard cosmology.



Textures: These form when larger, more complicated symmetry groups are completely broken. Textures are delocalized topological defects which are unstable to collapse. A speculation that the largest "cold spot" in the WMAP CMB data was caused by cosmic textures was published by Cruz et al. (2007, Science 318, 1612).



Examples of delocalized texture configurations in one and two dimensions.

By 2000, it was clear that **cosmic defects are not the main source of the CMB anisotropies**.



Figure 3: Current data (as complied by Knox[22]) with two defect models (dashed) and an inflation-based model (solid). The upper defect model has a standard ionization history and the lower model has an ionization history specifically designed to produce a sharper, shifted peak.

Andreas Albrecht, Defect models of cosmic structure in light of the new CMB data, XXXVth Rencontres de Moriond ``Energy Densities in the Universe'' (2000).

A Cosmic Microwave Background Feature Consistent with a Cosmic Texture

M. Cruz, 1,2* N. Turok, 3 P. Vielva, 1 E. Martínez-González, 1 M. Hobson⁴ SCIENCE VOL 318 7 DECEMBER 2007

The Cosmic Microwave Background provides our most ancient image of the universe and our best tool for studying its early evolution. Theories of high-energy physics predict the formation of various types of topological defects in the very early universe, including cosmic texture, which would generate hot and cold spots in the Cosmic Microwave Background. We show through a Bayesian statistical analysis that the most prominent 5°-radius cold spot observed in all-sky images, which is otherwise hard to explain, is compatible with having being caused by a texture. From this model, we constrain the fundamental symmetry-breaking energy scale to be $\phi_0 \approx 8.7 \times 10^{15}$ gigaelectron volts. If confirmed, this detection of a cosmic defect will probe physics at energies exceeding any conceivable terrestrial experiment.

The Axis of Evil revisited

Kate Land, Joao Magueijo, 2007 MNRAS, 378, 153 Abstract: In light of the three-year data release from WMAP we reexamine the evidence for the ``Axis of Evil" (AOE) [anomalous alignment of CMB multipoles in the direction $I \approx -100$, b = 60]. We discover that previous statistics are not robust with respect to the data-sets available and different treatments of the galactic plane. We identify the cause of the instability and implement an alternative ``model selection" approach. A comparison to Gaussian isotropic simulations find the features significant at the 94-98% level, depending on the particular AOE model. The Bayesian evidence finds lower significance, ranging from ``substantial" to no evidence for the most general AOE model.



The zone of the CS has been placed at the center of the black circle.

CMB Lensing and the WMAP Cold Spot

Sudeep Das^{1,*} and David N. Spergel^{1,2,3,†}

Cosmologists have suggested a number of intriguing hypotheses for the origin of the "WMAP cold spot", the coldest extended region seen in the CMB sky, including a very large void and a collapsing texture. Either hypothesis predicts a distinctive CMB lensing signal. We show that the upcoming generation of high resolution CMB experiments such as ACT and SPT should be able to detect the signatures of either textures or large voids. If either signal is detected, it would have profound implications for cosmology.

Some theorists have speculated that the Cold Spot is a secondary effect, generated at some intermediate distance between us and the last scattering surface. One such model proposes that the Cold Spot may have been caused by the Rees-Sciama effect due to an underdense void of comoving radius ~200h⁻¹Mpc and fractional density contrast $\delta \sim -0.3$ at redshift of z < 1 [8, 9]. Interestingly, [10] reported a detection of an underdense region with similar characteristics in the distribution of extragalactic radio sources in the NRAO VLA Sky Survey in the direction of the Cold Spot, a claim which has recently been challenged [11]. An alternative view [12] proposes that the spot was caused by the interaction of the CMB photons with a cosmic texture, a type of topological defect that can give rise to hot and cold spots in the CMB [13]. Bayesian analysis by [14] claims that the texture hypothesis seems to be favored over the void explanation, mainly because such large voids as required by the latter is highly unlikely to form in a CDM structure formation scenario.

[8] K. T. Inoue and J. Silk, ApJ 648, 23 (2006), arXiv:astroph/ 0602478.

[9] K. T. Inoue and J. Silk, ApJ 664, 650 (2007), arXiv:astro-ph/0612347.

[10] L. Rudnick, S. Brown, and L. R. Williams, ApJ 671, 40 (2007).

[11] K. M. Smith and D. Huterer, ArXiv:0805.2751.

[12] M. Cruz, N. Turok, P. Vielva, E. Mart'inez-Gonz'alez, and M. Hobson, Science 318, 1612 (2007).

[13] N. Turok and D. Spergel, Physical Review Letters 64, 2736 (1990).

[14] M. Cruz, E. Martinez-Gonzalez, P. Vielva, J. M. Diego, M. Hobson, and N. Turok, ArXiv:0804.2904.

Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Are There Cosmic Microwave Background Anomalies? C. Bennett et al. WMAP7 Jan 2010

arXiv:1001.4758v1

A simple six-parameter LCDM model provides a successful fit to WMAP data, both when the data are analyzed alone and in combination with other cosmological data. Even so, it is appropriate to search for any hints of deviations from the now standard model of cosmology, which includes inflation, dark energy, dark matter, baryons, and neutrinos. The cosmological community has subjected the WMAP data to extensive and varied analyses. While there is widespread agreement as to the overall success of the six-parameter LCDM model, various "anomalies" have been reported relative to that model. In this paper we examine potential anomalies and present analyses and assessments of their significance. In most cases we find that claimed anomalies depend on posterior selection of some aspect or subset of the data. Compared with sky simulations based on the best fit model, one can select for low probability features of the WMAP data. Low probability features are expected, but it is not usually straightforward to determine whether any particular low probability feature is the result of the a posteriori selection or of non-standard cosmology. We examine in detail the properties of the power spectrum with respect to the LCDM model. We examine several potential or previously claimed anomalies in the sky maps and power spectra, including cold spots, low guadrupole power, guadropole-octupole alignment, hemispherical or dipole power asymmetry, and quadrupole power asymmetry. We conclude that there is no compelling evidence for deviations from the LCDM model, which is generally an acceptable statistical fit to WMAP and other cosmological data.

Why do cosmic topological defects form?

If cosmic strings or other topological defects *can* form at a cosmological phase transition, then they *will* form. This was first pointed out by Tom Kibble and, in a cosmological context, the defect formation process is known as the Kibble mechanism.

The simple fact is that causal effects in the early universe can only propagate (as at any time) at the speed of light c. This means that at a time t, regions of the universe separated by more than a distance d=ct can know nothing about each other. In a symmetry breaking phase transition, different regions of the universe will choose to fall into different minima in the set of possible states (this set is known to mathematicians as the vacuum manifold). Topological defects are precisely the "boundaries" between these regions with different choices of minima, and their formation is therefore an inevitable consequence of the fact that different regions cannot agree on their choices.

For example, in a theory with two minima, plus + and minus -, then neighboring regions separated by more than ct will tend to fall randomly into the different states (as shown below). Interpolating between these different minima will be a domain wall.



Cosmic strings will arise in slightly more complicated theories in which the minimum energy states possess `holes'. The strings will simply correspond to non-trivial `windings' around these holes (as illustrated at right).

The Kibble mechanism for the formation of cosmic strings.



Topological defects can provide a unique link to the physics of the very early universe. Furthermore, they can crucially affect the evolution of the universe, so their study is an unavoidable part of any serious attempt to understand the early universe. The cosmological consequences vary with the type of defect considered. Domain walls and monopoles are cosmologically catastrophic. Any cosmological model in which they form will evolve in a way that contradicts the basic observational facts that we know about the universe. Such models must therefore be ruled out! Cosmic inflation was invented to solve this problem.

Cosmic strings and textures are (possibly) much more benign. Among other things, they were until recently thought to be a possible source of the fluctuations that led to the formation of the large-scale structures we observe today, as well as the anisotropies in the Cosmic Microwave Background. However, the CMB anisotropies have turned out not to agree with the predictions of this theory.

GUT Monopoles

A simple SO(3) GUT illustrates how nonsingular monopoles arise. The Lagragian is $\mathcal{L} = \frac{1}{2} D_{\mu} \Phi^{a} D^{\mu} \Phi^{a} - \frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} - \frac{1}{8} \lambda (\Phi^{a} \Phi^{a} - \sigma^{2})^{2},$ $F^{a}_{\mu\nu} = \partial_{\mu} A^{a}_{\nu} - \partial_{\nu} A^{a}_{\mu} - e \varepsilon_{abc} A^{b}_{\mu} A^{c}_{\nu},$ $D_{\mu} \Phi^{a} = \partial_{\mu} \Phi^{a} - e \varepsilon_{abc} A^{b}_{\mu} \Phi^{c}.$

The masses of the resulting charged vector and Higgs bosons after spontaneous symmetry breaking are $M_V^2 = e^2 \sigma^2$,

 $M_S^2 = \lambda \sigma^2.$

If the Higgs field Φ^a happens to rotate about a sphere in SO(3) space as one moves around a sphere about any particular point in x-space, then it must vanish at that point. Remarkably, if we identify the massless vector field as the photon, this configuration corresponds to a nonsingular magnetic monopole, as was independently discovered by 'tHooft and Polyakov. The monopole has magnetic charge twice the minimum Dirac value, g = $2\pi/e = (4\pi/e^2)(e/2) \approx 67.5 e$.

The singular magnetic field is cut off at scale σ , and as a result the GUT monopole has mass $M_{monopole} \approx M_V / \alpha \approx M_{GUT} / \alpha \approx 10^{18}$ GeV.

The first accurate calculation of the mass of the 't Hooft - Polyakov nonsingular monopole was Bais & Primack (Phys. Rev. D13:819,1976).

GUT Monopole Problem

The Kibble mechanism produces ~ one GUT monopole per horizon volume when the GUT phase transition occurs. These GUT monopoles have a number density over entropy

$$n_M/s \sim 10^2 (T_{GUT}/M_{PI})^3 \sim 10^{-13}$$

(compared to $n_B/s \sim 10^{-9}$ for baryons) Their annihilation is inefficient since they are so massive, and as a result they are about as abundant as gold atoms but 10^{16} times more massive, so they "overclose" the universe. This catastrophe must be avoided! This was Alan Guth's initial motivation for inventing cosmic inflation.

I will summarize the key ideas of inflation theory, following my lectures at the Jerusalem Winter School, published as the first chapter in Avishai Dekel & Jeremiah Ostriker, eds., *Formation of Structure in the Universe* (Cambridge University Press, 1999), and Dierck-Ekkehard Liebscher, *Cosmology* (Springer, 2005) (available online through the UCSC library).

Motivations for Inflation

PROBLEM SOLVED

Horizon	Homogeneity, Isotropy, Uniform T
Flatness/Age	Expansion and gravity balance
"Dragons"	Monopoles, doman walls,banished
Structure	Small fluctuations to evolve into galaxies, clusters, voids

Cosmological constant $\Lambda > 0 \Rightarrow$ space repels space, so the more space the more repulsion, \Rightarrow de Sitter exponential expansion $a \propto e^{\sqrt{\Lambda t}}$.

Inflation is exponentially accelerating expansion caused by effective cosmological constant ("false vacuum" energy) associated with hypothetical scalar field ("inflaton").

	Forces of Nature	Spin
Known	Gravity	2
Known	Strong, weak, and electromagnetic	1
Goal of LHC	Mass (Higgs Boson)	0
Early universe	Inflation (Inflaton)	0

Inflation lasting only $\sim 10^{-32}$ s suffices to solve all the problems listed above. Universe must then convert to ordinary expansion through conversion of false to true vacuum ("re-"heating).

Joel Primack, in Formation of Structure in the Universe, ed. Dekel & Ostriker (Cambridge Univ Press, 1999)

Inflation Basics

The basic idea of inflation is that before the universe entered the present adiabatically expanding Friedmann era, it underwent a period of de Sitter exponential expansion of the scale factor, termed *inflation* (Guth 1981). Actually, inflation is never precisely de Sitter, and any superluminal (faster-than-light) expansion is now called inflation. Inflation was originally invented to solve the problem of too many GUT monopoles, which, as mentioned in the previous section, would otherwise be disastrous for cosmology.

The de Sitter cosmology corresponds to the solution of Friedmann's equation in an empty universe (i.e., with $\rho = 0$) with vanishing curvature (k = 0) and positive cosmological constant $(\Lambda > 0)$. The solution is $a = a_o e^{Ht}$, with constant Hubble parameter $H = (\Lambda/3)^{1/2}$. There are analogous solutions for k = +1 and k = -1 with $a \propto \cosh Ht$ and $a \propto \sinh Ht$ respectively. The scale factor expands exponentially because the positive cosmological constant corresponds effectively to a negative pressure. de Sitter space is discussed in textbooks on general relativity (for example, Rindler 1977, Hawking & Ellis 1973) mainly for its geometrical interest. Until cosmological inflation was considered, the chief significance of the de Sitter solution in cosmology was that it is a limit to which all indefinitely expanding models with $\Lambda > 0$ must tend, since as $a \to \infty$, the cosmological constant term ultimately dominates the right hand side of the Friedmann equation. Joel Primack, in *Formation of Structure in the Universe*, (Cambridge Univ Press, 1999) As Guth (1981) emphasized, the de Sitter solution might also have been important in the very early universe because the vacuum energy that plays such an important role in spontaneously broken gauge theories also acts as an effective cosmological constant. A period of de Sitter inflation preceding ordinary radiation-dominated Friedmann expansion could explain several features of the observed universe that otherwise appear to require very special initial conditions: the horizon, flatness/age, monopole, and structure formation problems. (See Table 1.6.)

Let us illustrate how inflation can help with the horizon problem. At recombination $(p^+ + e^- \rightarrow H)$, which occurs at $a/a_o \approx 10^{-3}$, the mass encompassed by the horizon was $M_H \approx 10^{18} M_{\odot}$, compared to $M_{H,o} \approx 10^{22} M_{\odot}$ today. Equivalently, the angular size today of the causally connected regions at recombination is only $\Delta\theta \sim 3^{\circ}$. Yet the fluctuation in temperature of the cosmic background radiation from different regions is very small: $\Delta T/T \sim 10^{-5}$. How could regions far out of causal contact have come to temperatures that are so precisely equal? This is the "horizon problem". With inflation, it is no problem because the entire observable universe initially lay inside a single causally connected region that subsequently inflated to a gigantic scale. Similarly, inflation exponentially dilutes any preceeding density of monopoles or other unwanted relics (a modern version of the "dragons" that decorated the unexplored borders of old maps). Joel Primack, in Formation of Structure in the Universe, (Cambridge Univ Press, 1999)

In the first inflationary models, the dynamics of the very early universe was typically controlled by the self-energy of the Higgs field associated with the breaking of a Grand Unified Theory (GUT) into the standard 3-2-1 model: $GUT \rightarrow SU(3)_{color} \otimes [SU(2) \otimes U(1)]_{electroweak}$. This occurs when the cosmological temperature drops to the unification scale $T_{GUT} \sim 10^{14} \,\text{GeV}$ at about 10^{-35} s after the Big Bang. Guth (1981) initially considered a scheme in which inflation occurs while the universe is trapped in an unstable state (with the GUT unbroken) on the wrong side of a maximum in the Higgs potential. This turns out not to work: the transition from a de Sitter to a Friedmann universe never finishes (Guth & Weinberg 1981). The solution in the "new inflation" scheme (Linde 1982; Albrecht and Steinhardt 1982) is for inflation to occur after barrier penetration (if any). It is necessary that the potential of the scalar field controlling inflation ("inflaton") be nearly flat (i.e., decrease very slowly with increasing inflaton field) for the inflationary period to last long enough. This nearly flat part of the potential must then be followed by a very steep minimum, in order that the energy contained in the Higgs potential be rapidly shared with the other degrees of freedom ("reheating"). A more general approach, "chaotic" inflation, has been worked out by Linde (1983, 1990) and others; this works for a wide range of inflationary potentials, including simple power laws such as $\lambda \phi^4$. However, for the amplitude of the fluctuations to be small enough for consistency with observations, it is necessary that the inflaton self-coupling be very small, for example $\lambda \sim 10^{-14}$ for the ϕ^4 model. This requirement prevents a Higgs field from being the inflaton, since Higgs fields by definition have gauge couplings to the gauge field (which are expected to be of order unity), and these would generate self-couplings of similar magnitude even if none were present.

It turns out to be <u>necessary to inflate by a factor $\gtrsim e^{66}$ in order to solve e^{66} = the flatness problem, i.e., that $\Omega_0 \sim 1$. (With $H^{-1} \sim 10^{-34}$ s during the de 4×10^{28} Sitter phase, this implies that the inflationary period needs to last for only a relatively small time $\tau \gtrsim 10^{-32}$ s.) The "flatness problem" is essentially the question why the universe did not become curvature dominated long ago. Neglecting the cosmological constant on the assumption that it is unimportant after the inflationary epoch, the Friedmann equation can be written</u>

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \frac{\pi^2}{30} g(T) T^4 - \frac{kT^2}{(aT)^2}$$

where the first term on the right hand side is the contribution of the energy density in relativistic particles and g(T) is the effective number of degrees of freedom. The second term on the right hand side is the curvature term. Since $aT \approx \text{constant}$ for adiabatic expansion, it is clear that as the temperature T drops, the curvature term becomes increasingly important. The quantity $K \equiv k/(aT)^2$ is a dimensionless measure of the curvature. Today, |K| = $|\Omega - 1| H_o^2/T_o^2 \leq 2 \times 10^{-58}$. Unless the curvature exactly vanishes, the most "natural" value for K is perhaps $K \sim 1$. Since inflation increases a by a tremendous factor $e^{H\tau}$ at essentially constant T (after reheating), it increases aT by the same tremendous factor and thereby decreases the curvature by that factor squared. Setting $e^{-2H\tau} \lesssim 2 \times 10^{-58}$ gives the needed amount of inflation: $H\tau \gtrsim 66$. This much inflation turns out to be enough to take care of the other cosmological problems mentioned above as well.



COSMIC TIME



Generating the Primordial Density Fluctuations

