Lecture

Angular Momentum

Tidal-Torque Theory

Halo spin

Angular-momentum distribution within halos

Gas Condensation and Disk Formation

The AM Problem(s)

Thin disk, thick disk, bulge

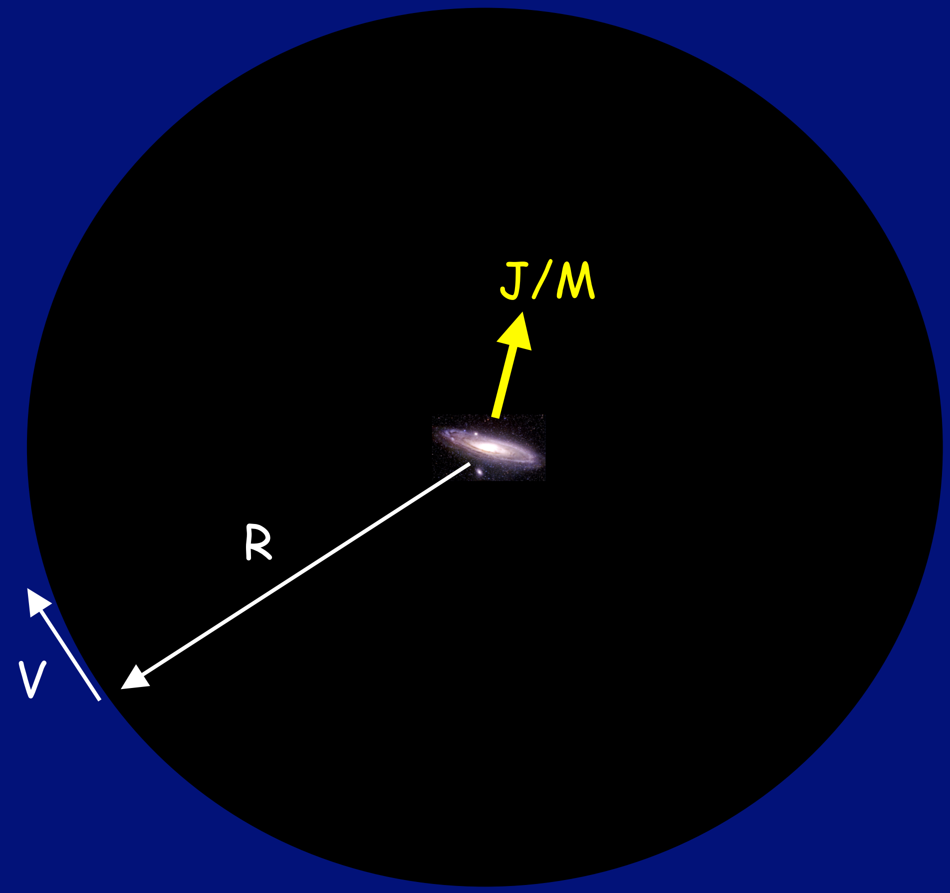

TIME

Disk Size

Spin parameter $\lambda \sim \frac{J/M}{RV}$

Conservation of specific angular momentum $const. = J/M \sim \lambda R_{\mathrm{virial}} V \sim R_{\mathrm{disk}} V$

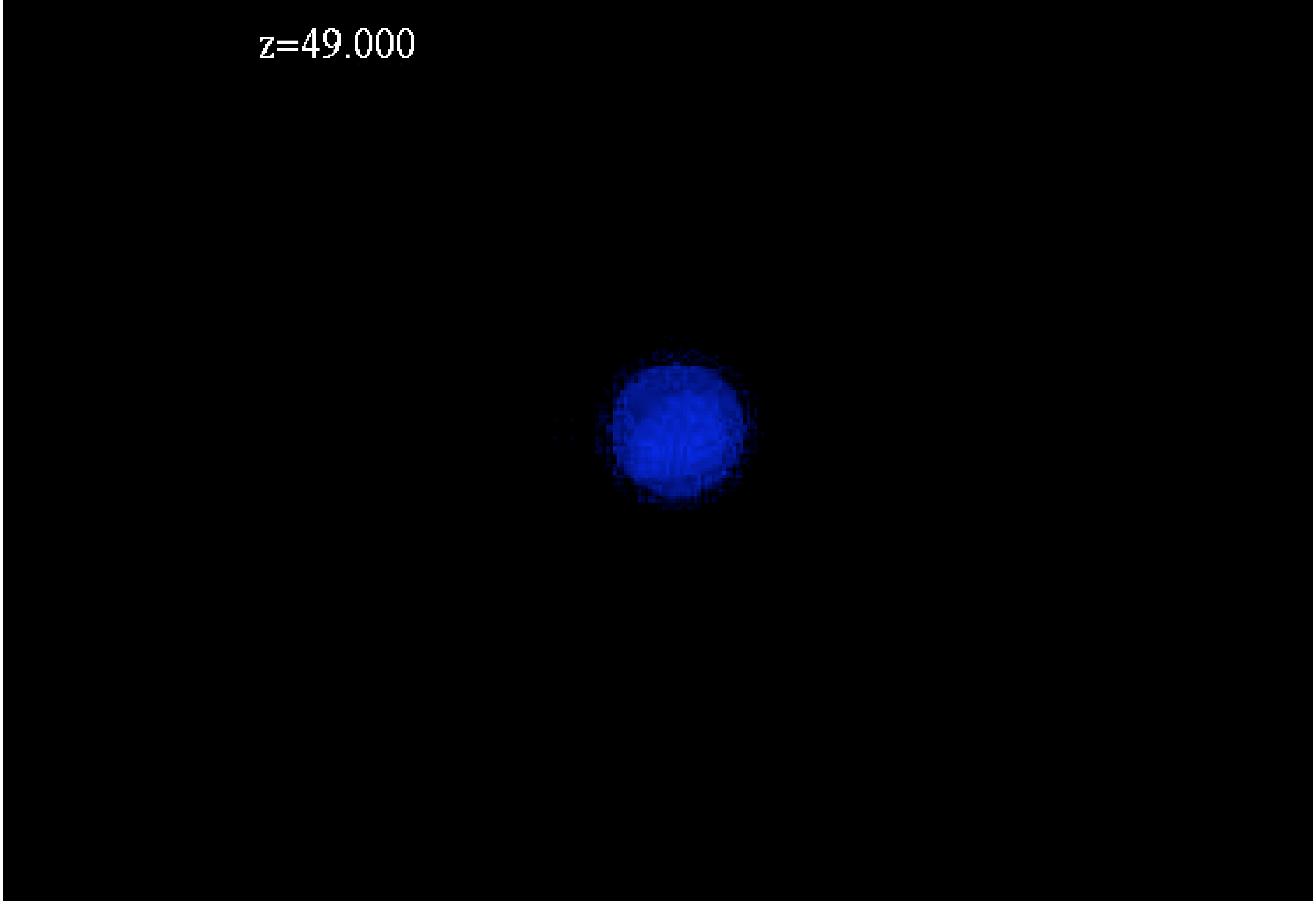
→ $\frac{R_{\mathrm{disk}}}{R_{\mathrm{virial}}} \sim \lambda$



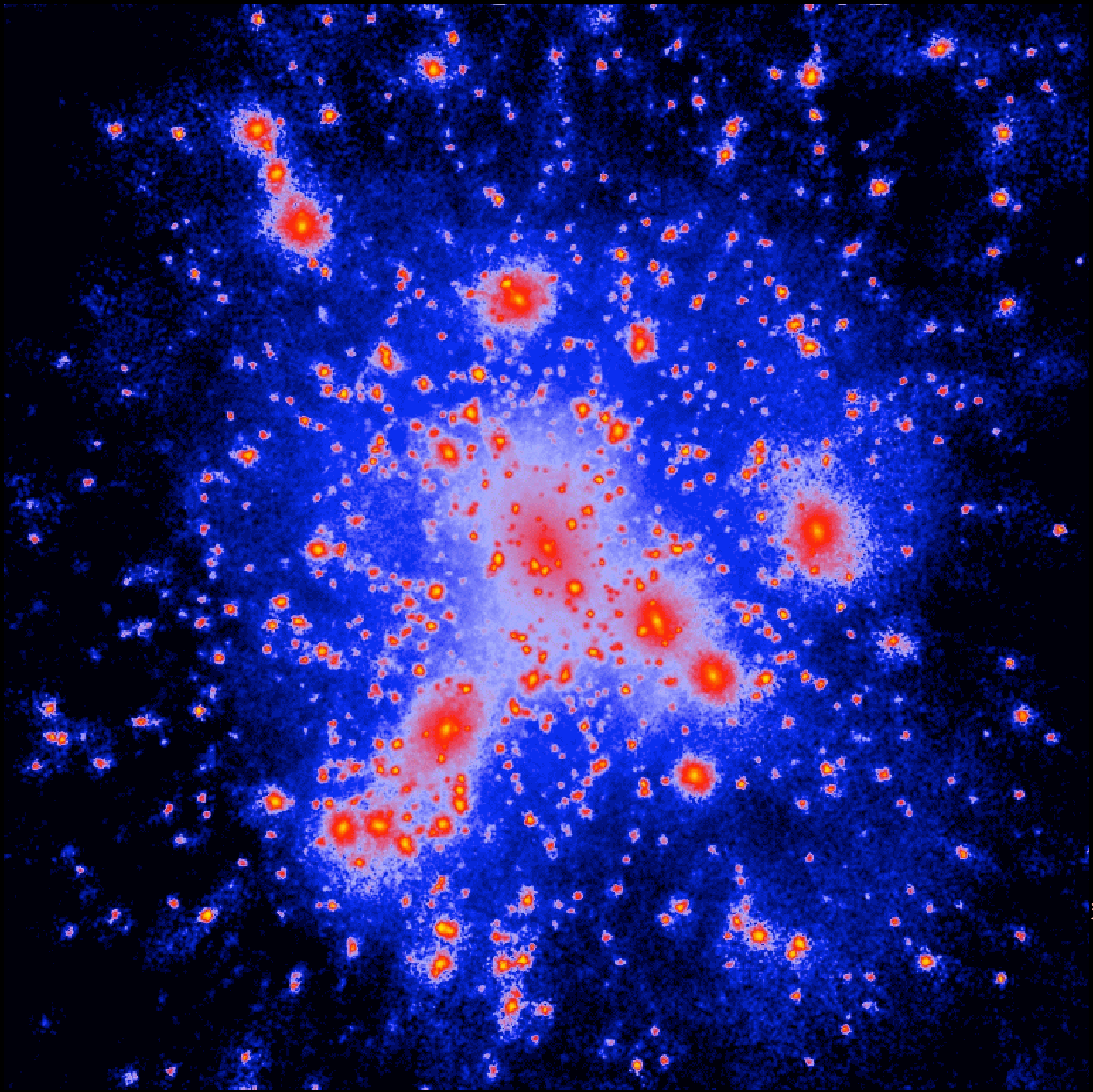
Tidal-Torque Theory (TTT)

Peebles 1976   White 1984

N-body simulation of Halo Formation



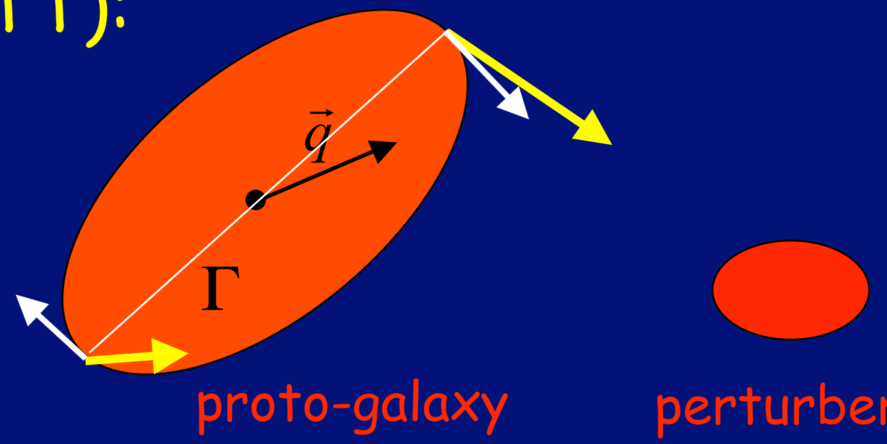
N-body simulation of Halo Formation



Origin of Angular Momentum

Tidal Torque Theory (TTT):

Peebles 1976 White 1984



Result:

$$J_i \propto t\, \varepsilon_{ijk} T_{jl} I_{lk}$$

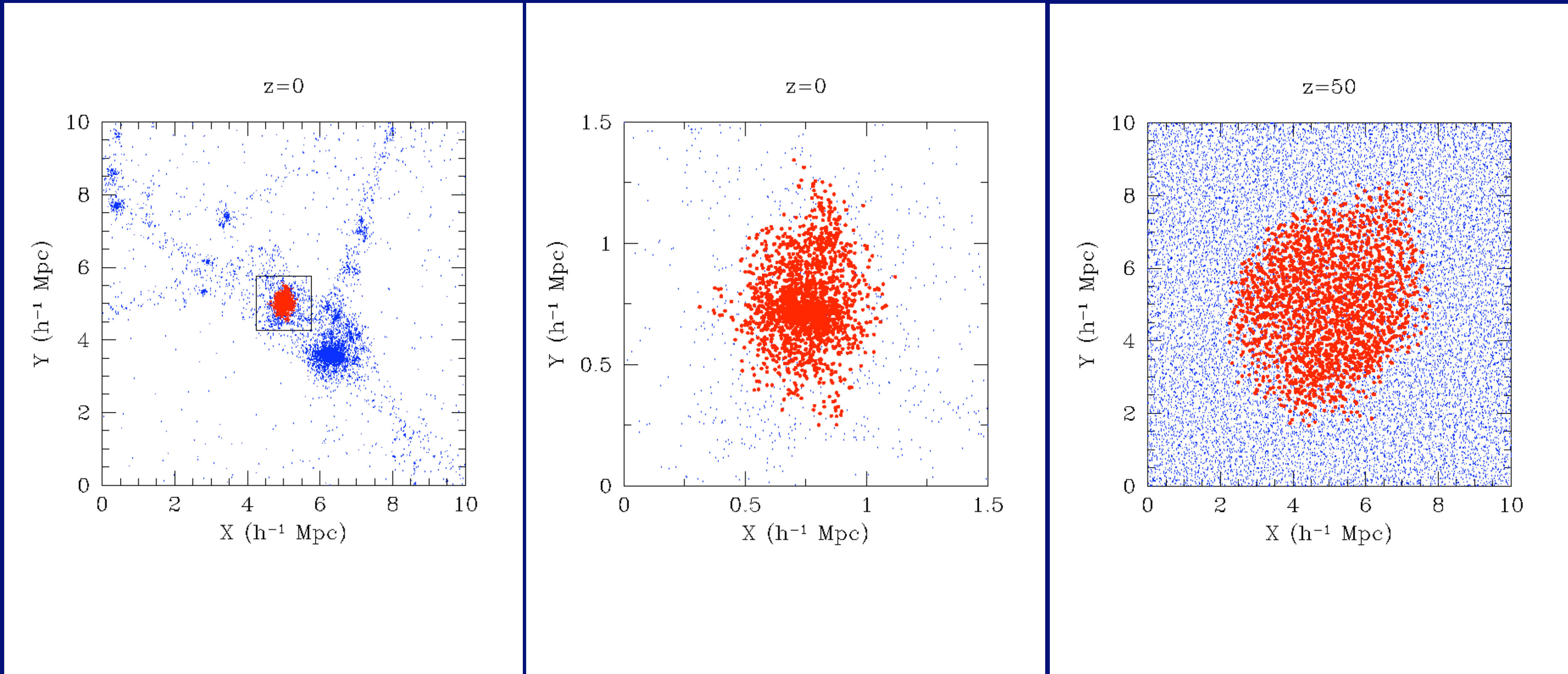
Tidal: $T_{ij} = -\dfrac{\partial^2 \phi}{\partial q_i\, \partial q_j}$ Inertia: $I_{ij} = \rho_0 a_0^{\,3} \int_\Gamma q_i\, q_j\, d^3q$

Tidal-Torque Theory

Halo

Proto-halo:

a Lagrangian patch



Tidal-Torque Theory

angular momentum in Eulerian patch

$$\vec{L}(t) = \int_{\gamma\ \text{Eulerian}} \rho(\vec{r},t)\,[\vec{r}(t) - \vec{R}_{cm}(t)] \times [\vec{v}(t) - \vec{V}_{cm}(t)]\ d^3r$$

comoving coordinates

$$\vec{x} \equiv \vec{r}/a \quad \vec{v} \equiv a\dot{\vec{x}} \quad \delta \equiv \rho/\bar{\rho}(t) - 1$$

$$\vec{L}(t) = \bar{\rho}(t)\,a^3(t) \int_{\gamma} [1 + \delta(\vec{x},t)]\,[\vec{x}(t) - \vec{X}_{cm}(t)] \times \dot{\vec{x}}\ d^3x$$

const. in m.d.

displacement from Lagrangian q to Eulerian x

$$\vec{q} \to \vec{x} \quad \vec{x}(\vec{q},t) = \vec{q} - \vec{S}(\vec{q},t)$$

laminar flow

$$1 + \delta[x(q,t)] = J^{-1}_{acobian}(q,t) \quad \to \quad (1+\delta)\,d^3x = d^3q$$

$$\vec{L}(t) = \bar{\rho}_0 a_0^3 \int_{\Gamma\ \text{Lagrangian}} [(q - \bar{q}) + (S(q,t) - \bar{S})] \times \dot{S}(q,t)\ d^3q$$

average over q in _

Zel'dovich approximation

$$S(q,t) = -D(t)\nabla\phi(q) \quad \phi(q) = \varphi_{\text{grav}}(q,t)/[4\pi G\rho(t)a^2(t)D(t)] \quad \to \quad \vec{S}\ //\ \dot{\vec{S}}$$

$$\vec{L}(t) = -a^2(t)\dot{D}(t)\ \bar{\rho}_0 a_0^3 \int_{\Gamma} (q - \bar{q}) \times \nabla\varphi(q)\ d^3q$$

in a flat universe $a^2\dot{D} \propto D^{3/2} \propto t$ in EdS

2nd-order Taylor expansion of potential about $q_{cm}=0$

$$\phi(\vec{q}) \approx \phi(0) + \left.\frac{\partial\phi}{\partial q_i}\right|_{\vec{q}=0} q_i + \frac{1}{2}\left.\frac{\partial^2\phi}{\partial q_i \partial q_j}\right|_{\vec{q}=0} q_i q_j \qquad \vec{q} \equiv \vec{q} - \bar{\vec{q}}$$

$$L_i(t) = a^2(t)\dot{D}(t)\ \varepsilon_{ijk} D_{jl} I_{lk}$$

Deformation tensor

$$D_{jl} \equiv -\left.\frac{\partial^2\phi}{\partial q_j \partial q_l}\right|_{q=q_{cm}=0}$$

Inertia tensor

$$I_{lk} \equiv \bar{\rho}_0 a_0^3 \int_{\Gamma} q_l q_k\, d^3q$$

antisymmetric tensor ε_{ijk}

Tidal-Torque Theory

$$L_i(t) = a^2(t)\dot{D}(t)\,\varepsilon_{ijk} T_{jl} I_{lk}$$

$$D_{jl} \equiv -\left.\frac{\partial^2 \phi}{\partial q_j \partial q_l}\right|_{q=q_{cm}=0}$$

Deformation tensor

$$I_{lk} \equiv \bar{\rho}_0 a_0^3 \int_\Gamma q_l q_k d^3 q$$

Inertia tensor

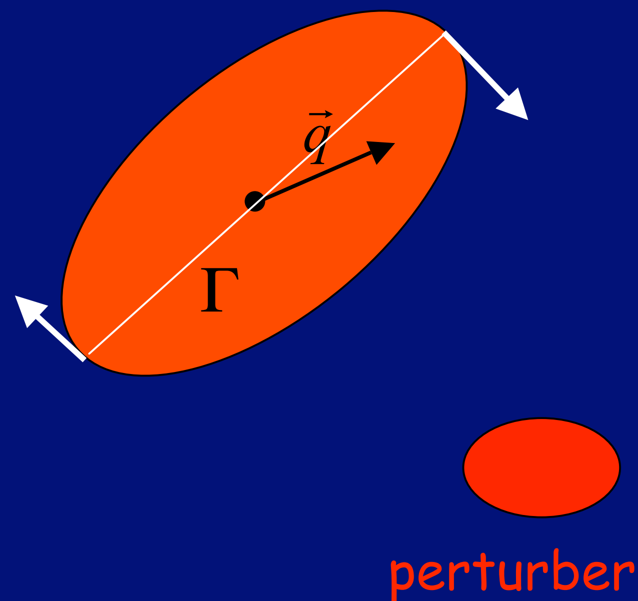
ε_{ijk}

antisymmetric

Tidal tensor = Shear tensor $T_{ij} \equiv D_{ij} - D_{ii}\delta_{ij}/3$ Only the trace-less part contributes

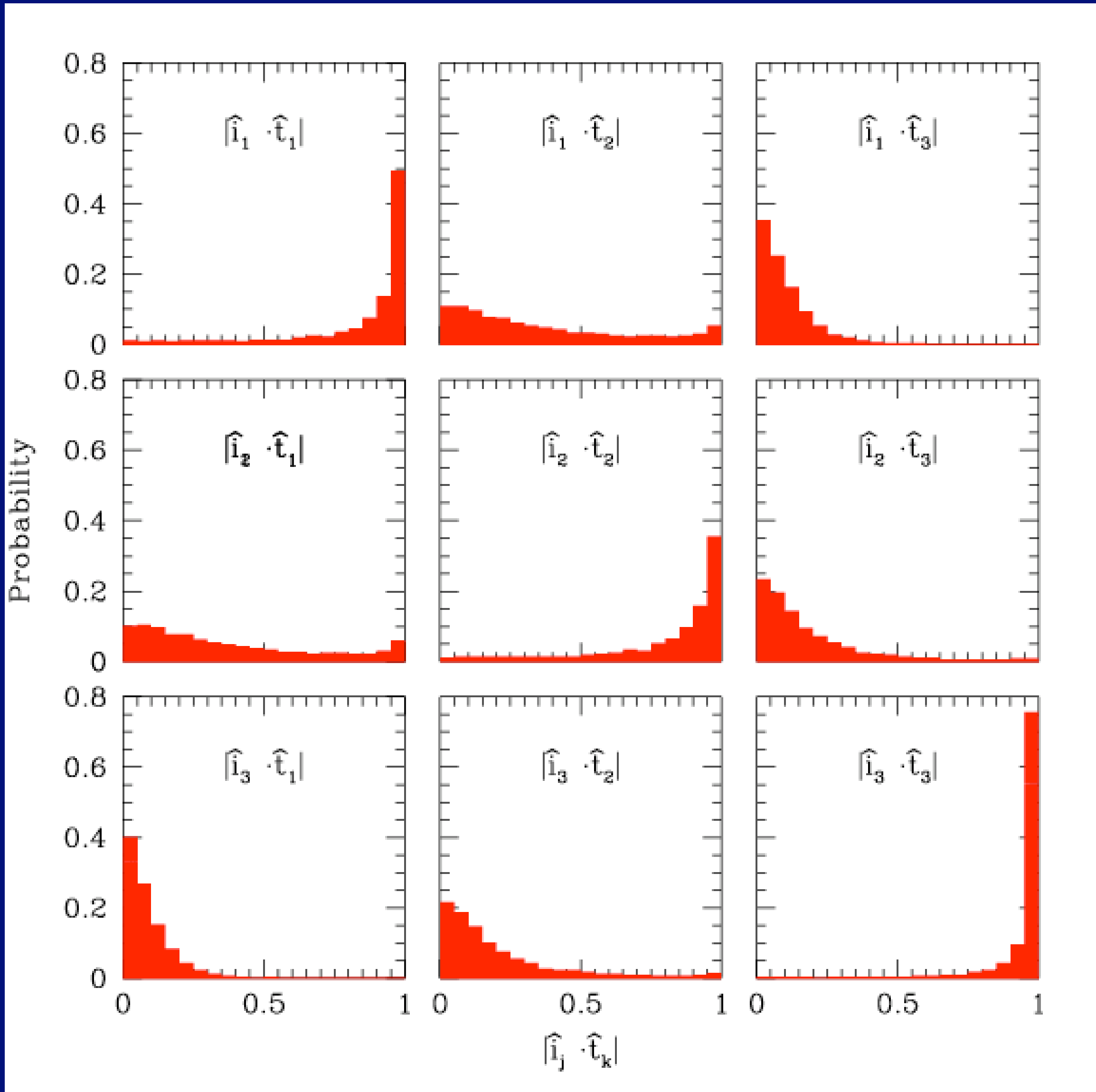
Quadrupolar Inertia $I_{ij} - I_{ii}\delta_{ij}/3$

L by gravitational coupling of
Quadrupole moment of _ with
Tidal field from neighboring fluctuations
°T and I must be misaligned.



L∝t till
~turnaround

TTT vs Simulations (Porciani, Dekel & Hoffman 2002)



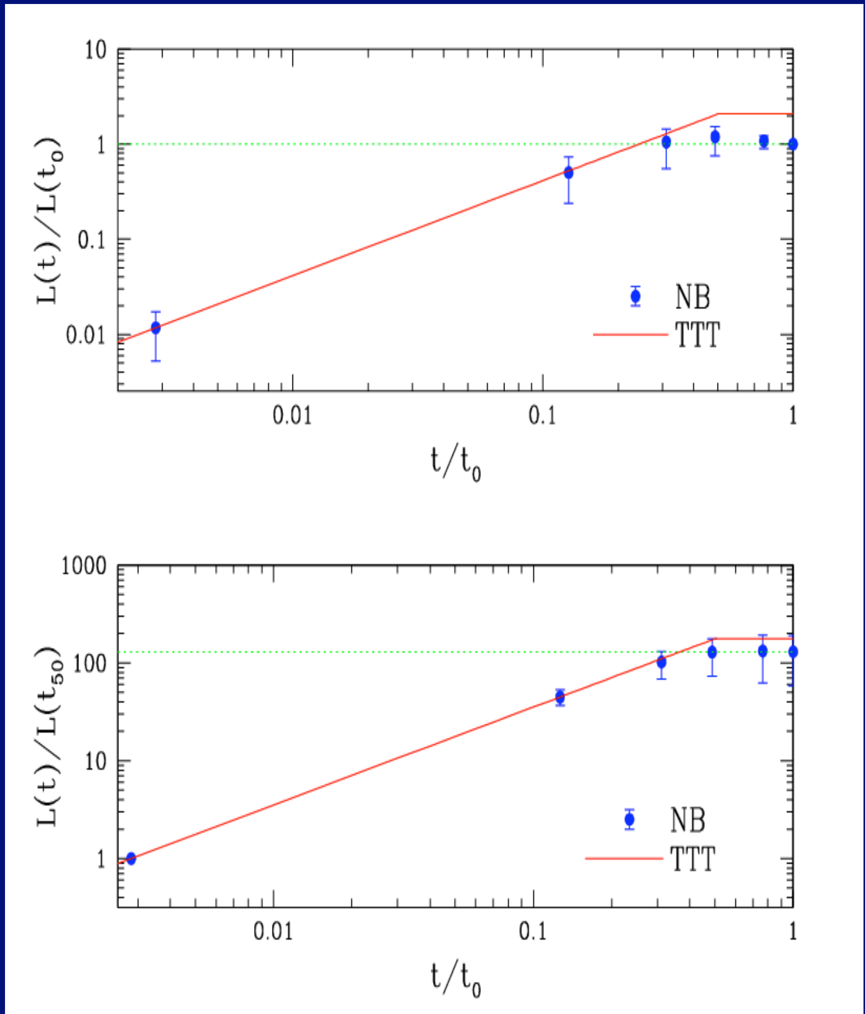
Alignment of T and I:
Spin originates from the residual misalignment.

°˙Small spin !

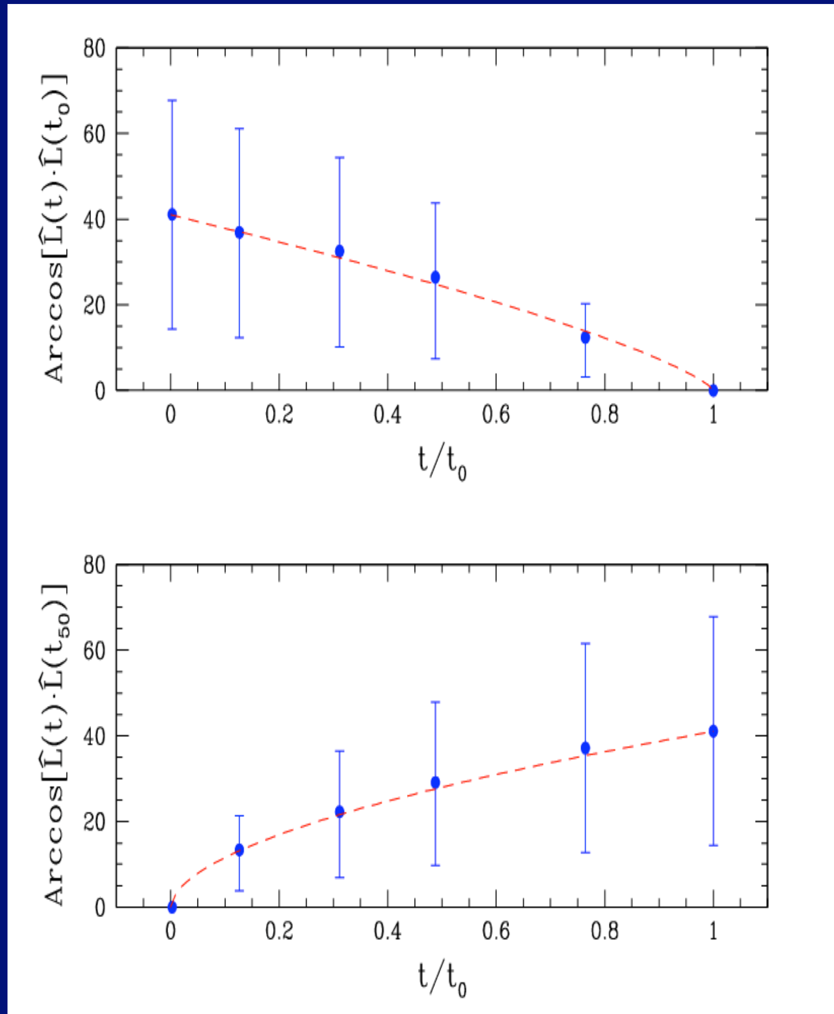
TTT vs. Simulations: Amplitude Growth Rate

Porciani, Dekel & Hoffman 02

Amplitude

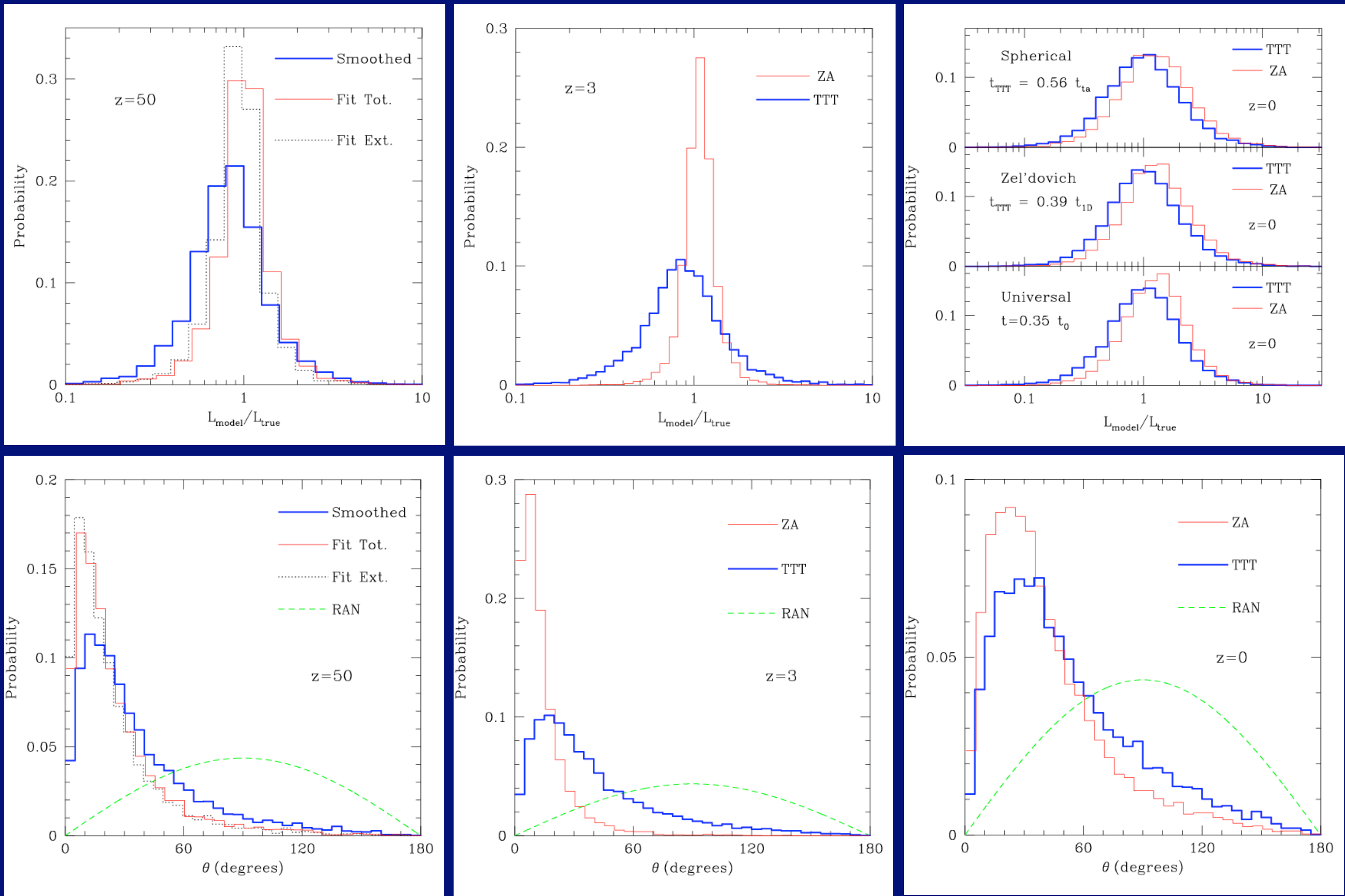


Direction



TTT vs Simulations: Scatter

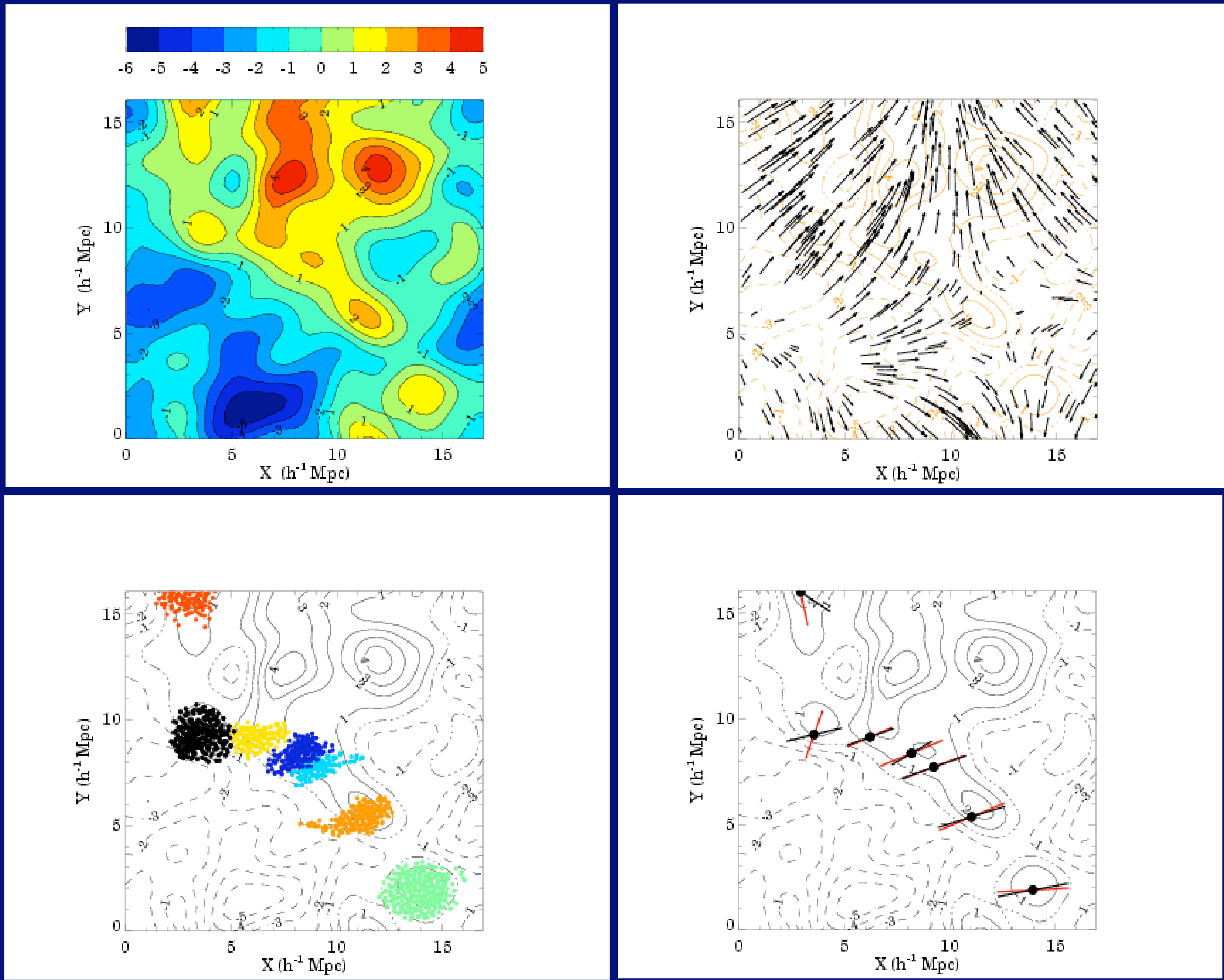
(Porciani, Dekel & Hoffman 2002)



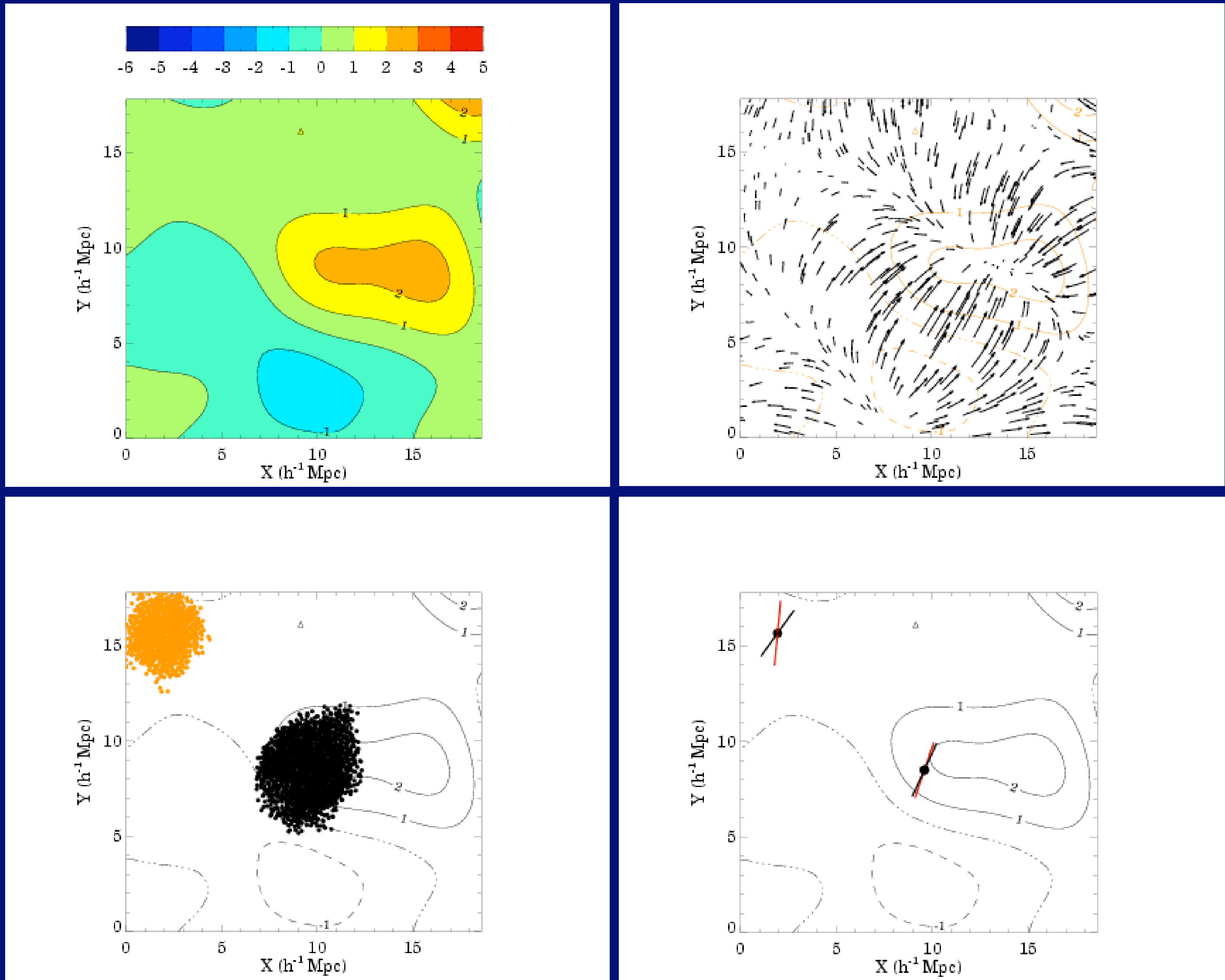
TTT predicts the spin amplitude to within a factor of ~2,

but it is not a very reliable predictor of spin direction.

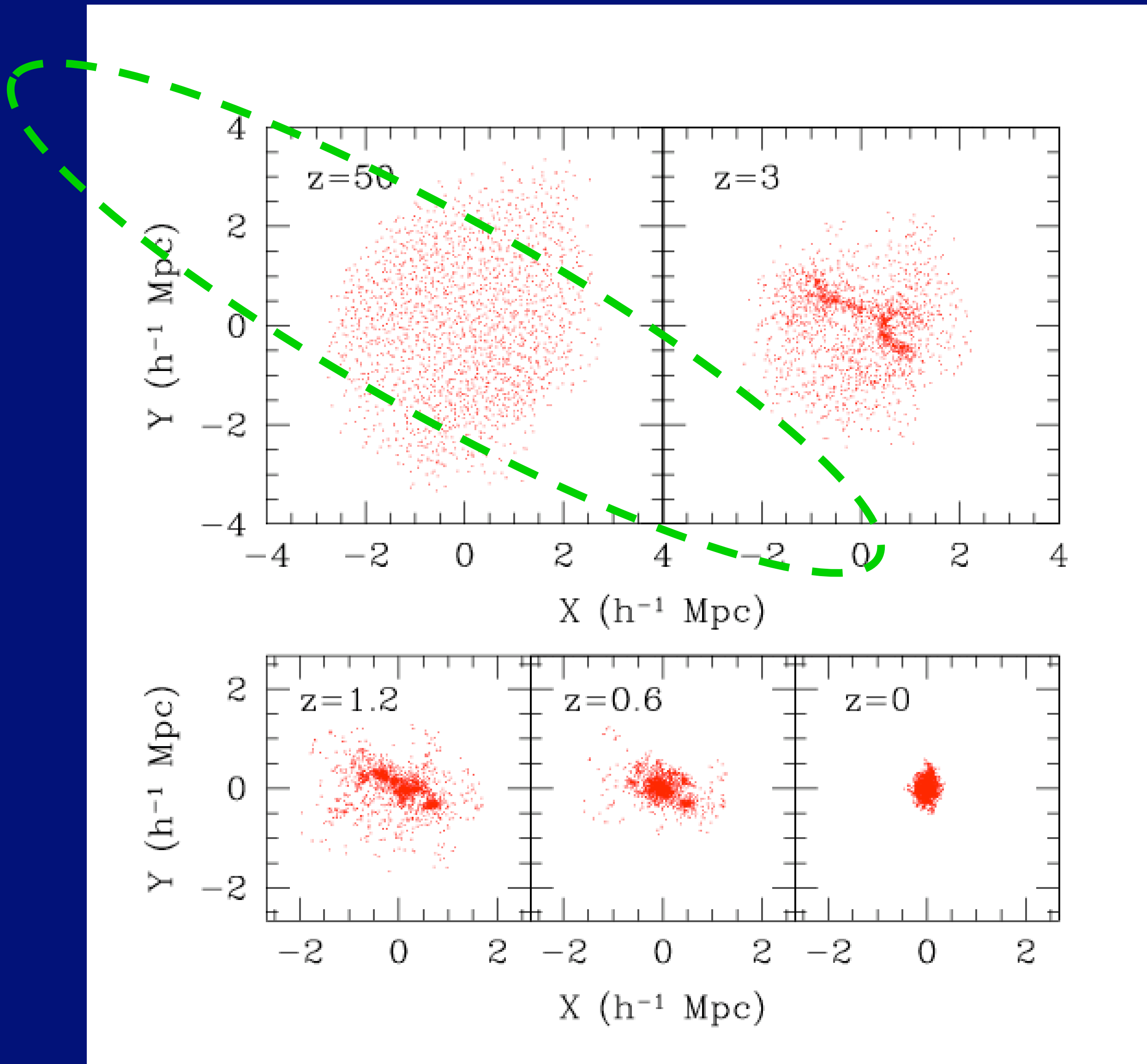
Alignment of I and T: Protohalos and Filaments



Alignment of I and T: Protohalos and Filaments



Stages in Halo Formation



Spin axis and Large-Scale Structure

TTT:

$$J_x = \frac{\partial^2 \phi}{\partial y \partial z}(I_{yy} - I_{zz})$$

$$J_y = \frac{\partial^2 \phi}{\partial x \partial z}(I_{xx} - I_{zz})$$

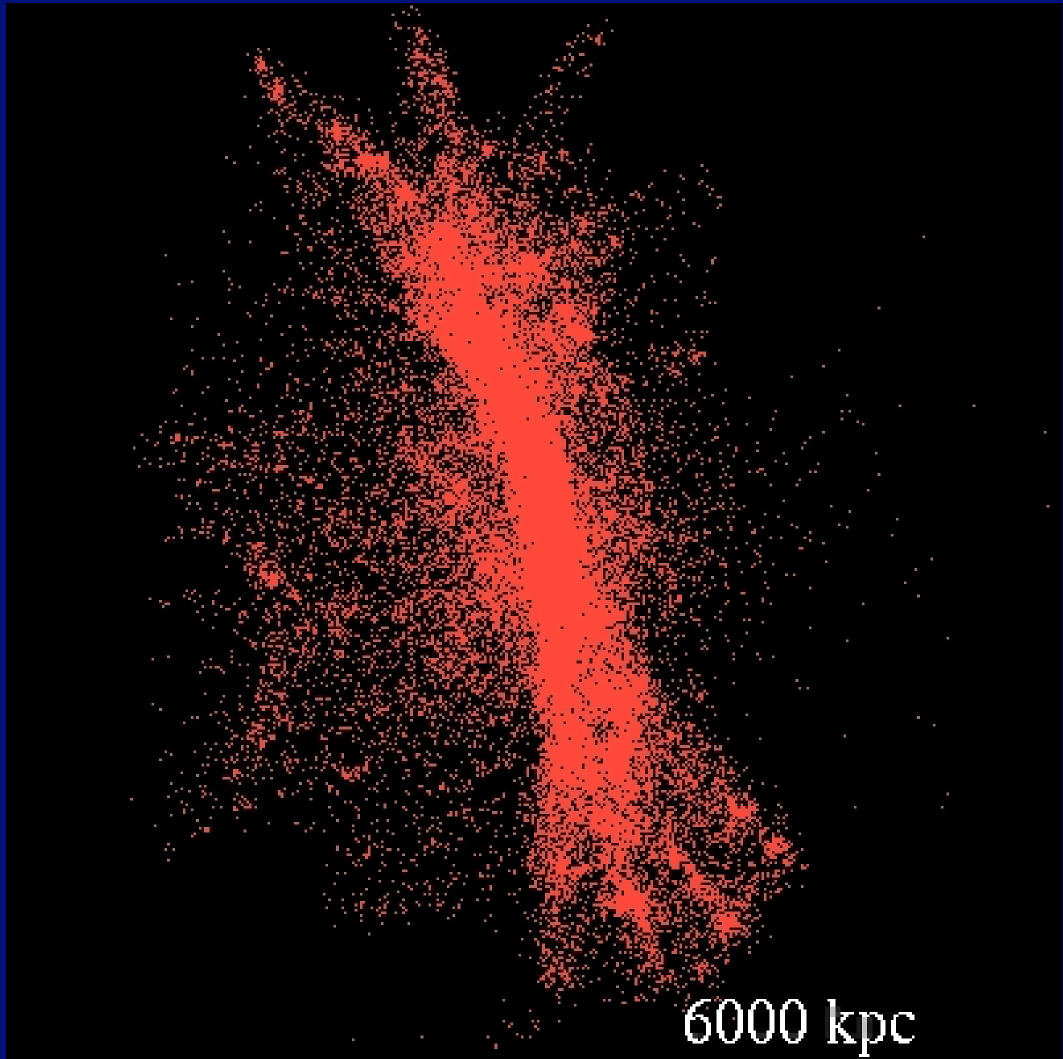
$$J_z = \frac{\partial^2 \phi}{\partial x \partial y}(I_{xx} - I_{yy})$$

$$I_{xx} > I_{yy} > I_{zz}$$

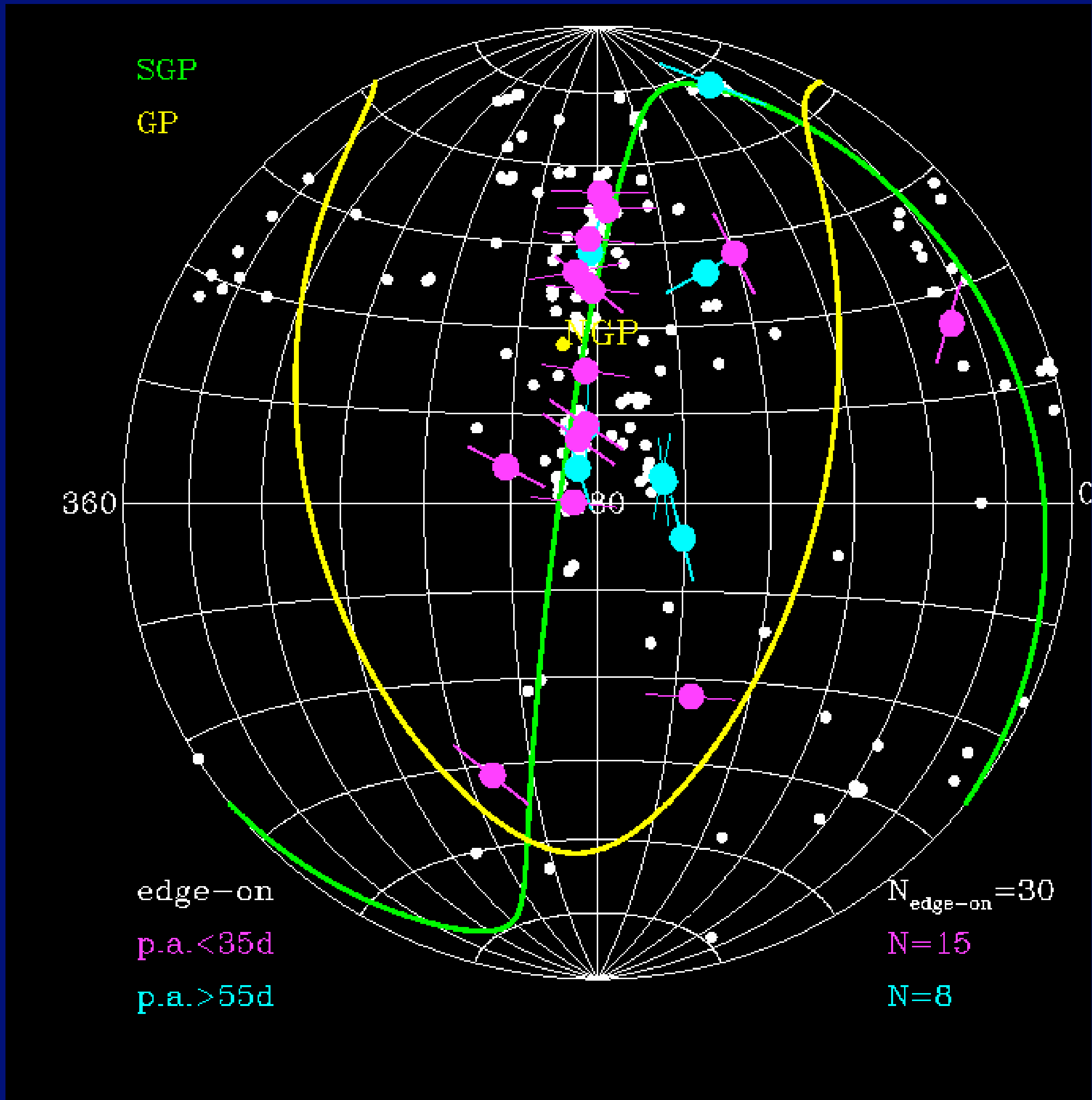
The spin direction is correlated with the intermediate principal axis of the Iij tensor at turnaround.

In a large-scale pancake: the spin axis should tend to lie in the plane.

Spin axis and Large-Scale Structure



Disk-Pancake Alignment in the Local Supercluster



Halo Spin Parameter

Fall & Efstathiou 1980

Barnes & Efstathiou 1984

Steinmetz et al. 1994-…

Bullock et al. 2001b

Halo Spin Parameter

Peebles 76: dimensionless $\lambda \equiv \frac{J|E|^{1/2}}{GM^{5/2}}$

Bullock et al. 2001 $\lambda \equiv \sqrt{\frac{3}{4}}\frac{J/M}{RV}$

same for isothermal sphere

$$|E| = \frac{3}{2}M\sigma^2 \quad \sigma^2 = \frac{1}{2}\frac{GM}{R} \quad V^2 = 2\sigma^2$$

TTT:

$$J \sim a^2\dot{D}\ \nabla^2\phi_0\ MR_0^2 \sim a^{1/2}M^{5/3}$$

$$a^2\dot{D} \sim t \sim a^{3/2}$$

J determined at turnaround

$$\delta \sim D\nabla^2\phi \quad \rightarrow \quad \text{when } \delta \sim 1\text{:}\ \nabla^2\phi_0 \sim D^{-1} \sim a^{-1}$$

$$\text{comoving } R_0^3 \sim M/\bar{\rho}_0 \sim M$$

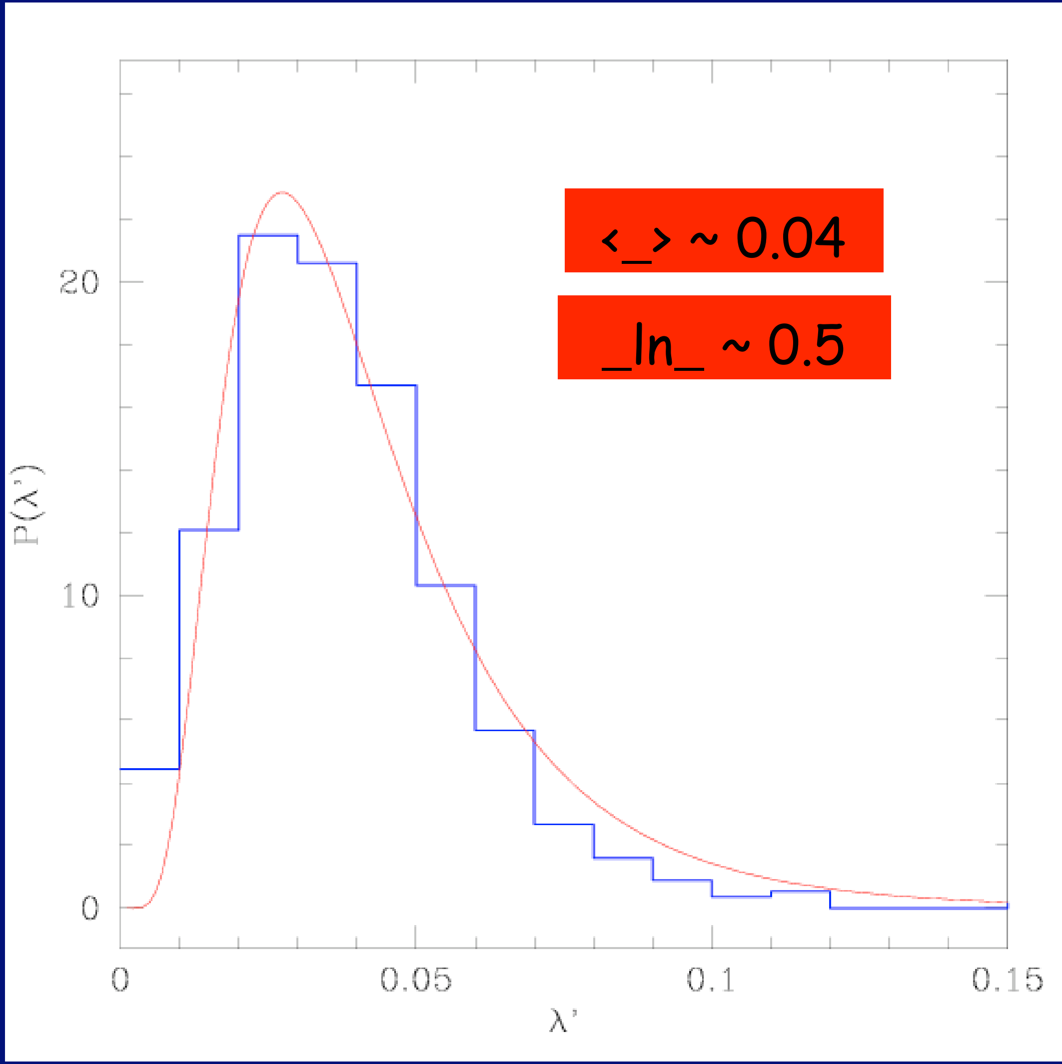
$$E \sim M^2/R \sim a^{-1}M^{5/3}$$

$$\text{Physical } R^3 \sim \rho^{-1}M \sim a^3M$$

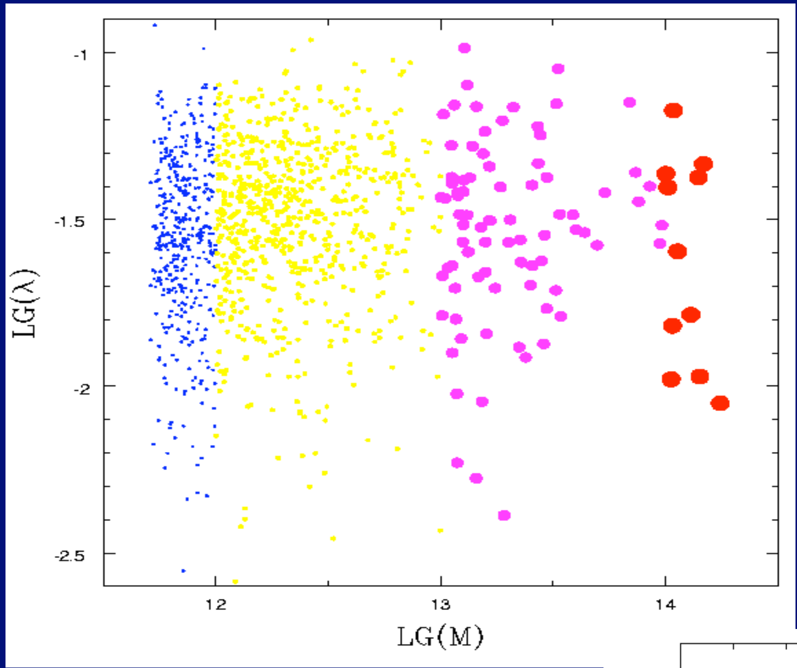
→ _ is constant, independent of a or M

simulations: _~0.05

Distribution of Halo Spins

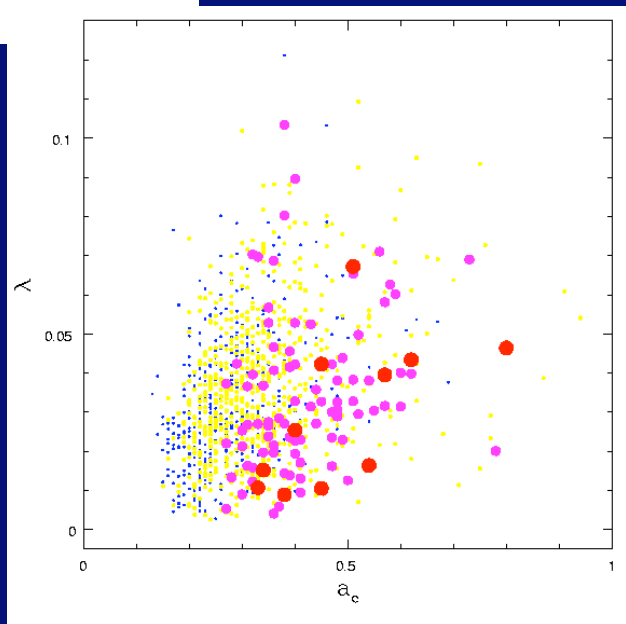


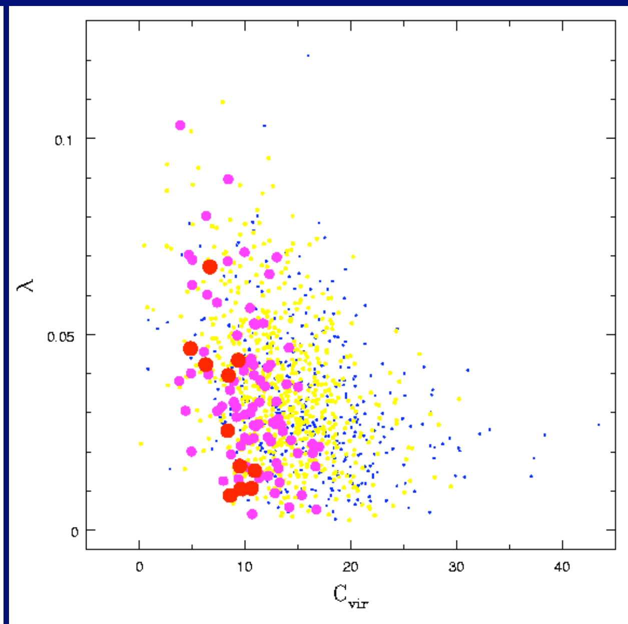
Spin vs Mass, Concentration, History



_ distribution is universal

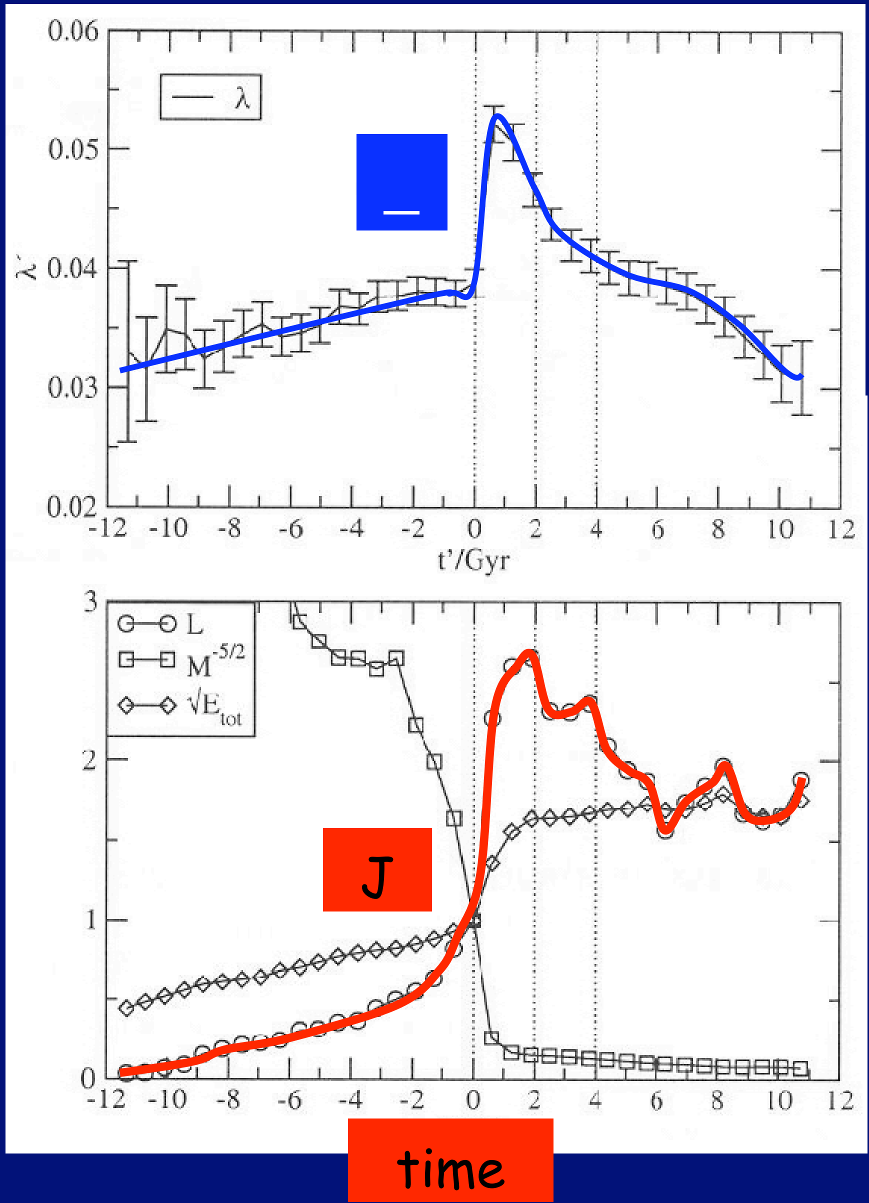
_ correlated with a_c,
anti-correlated with C



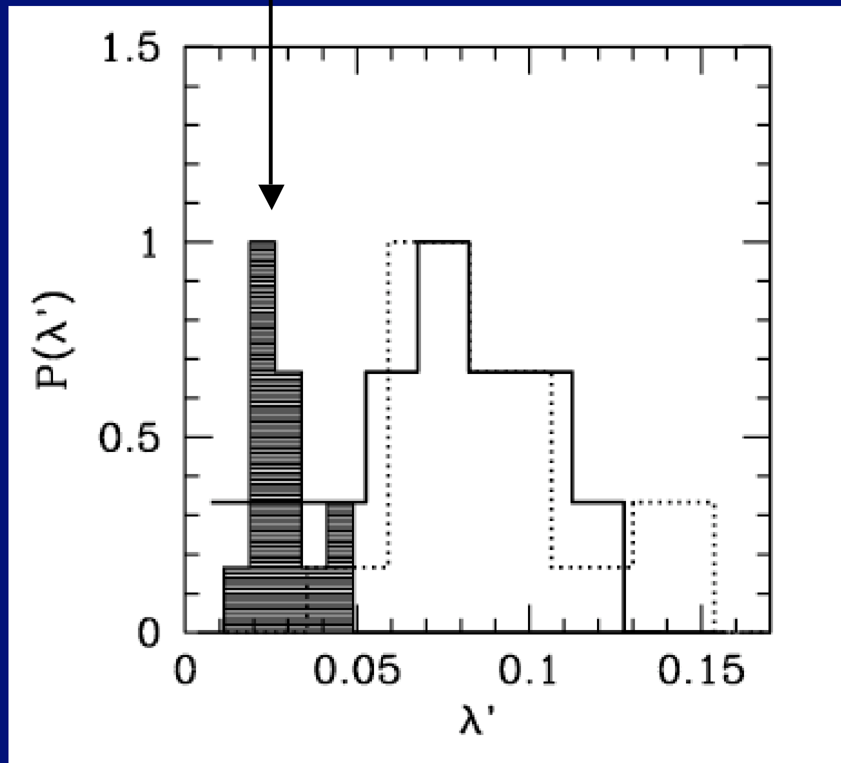


Spin Jump in a Major Merger

Burkert & D'onghia 04



quiet halos with no recent major merger



J Distribution inside Halos

Bullock et al. 2001b

Universal Distribution of J inside Halos

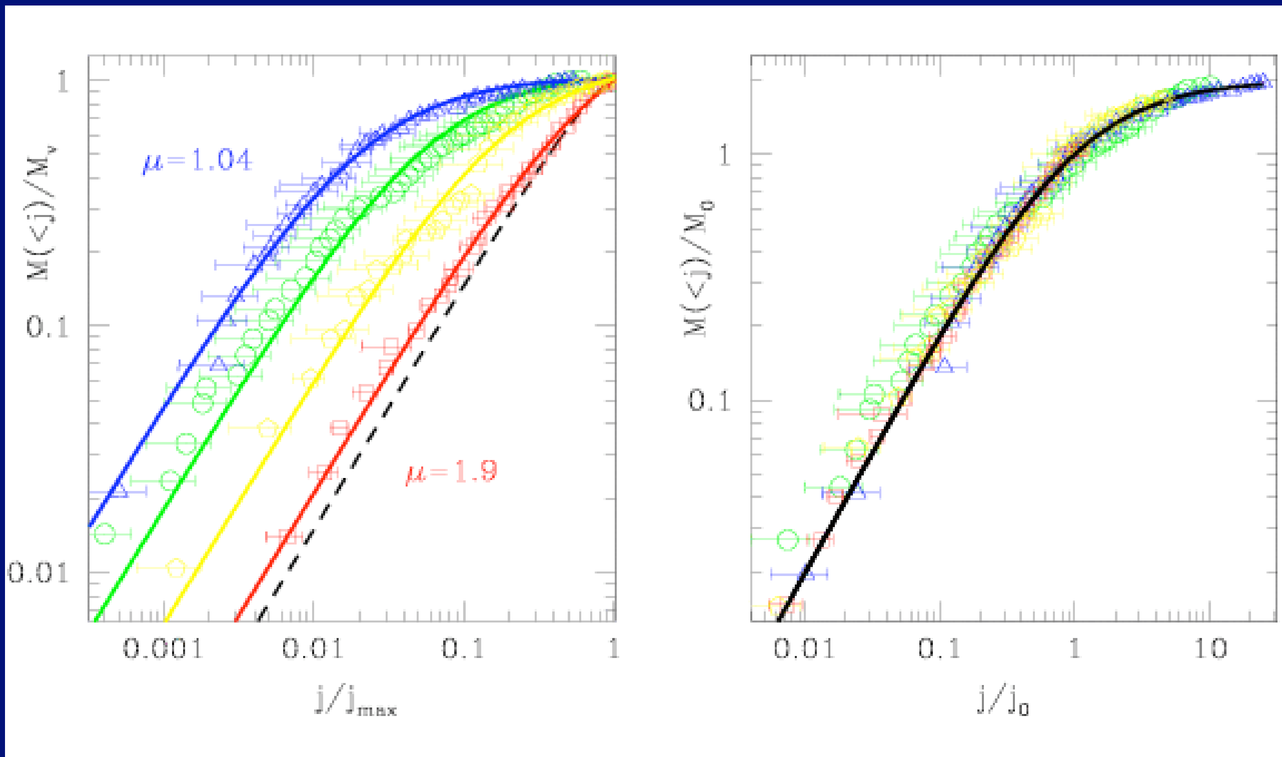
Bullock et al. 2001b

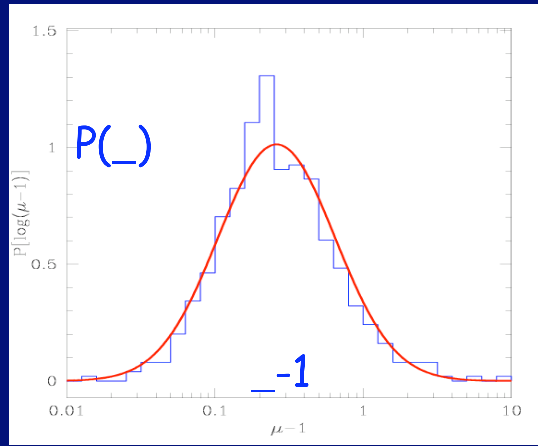
$$M(<j) = M_{vir} \frac{\mu j}{j_0 + j} \qquad \mu > 1$$

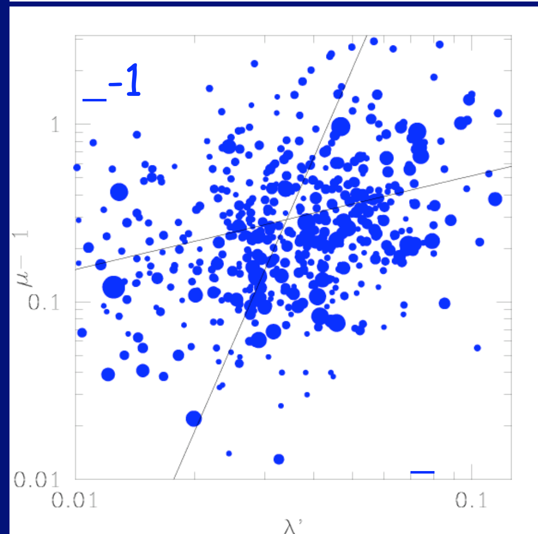
$$j_{max} = \frac{j_0}{\mu - 1} \qquad J/M = j_0 b(\mu) = \sqrt{2} V R \lambda' \qquad b(\mu) \equiv -\mu \ln(1 - \mu^{-1}) - 1$$

Two parameter family:

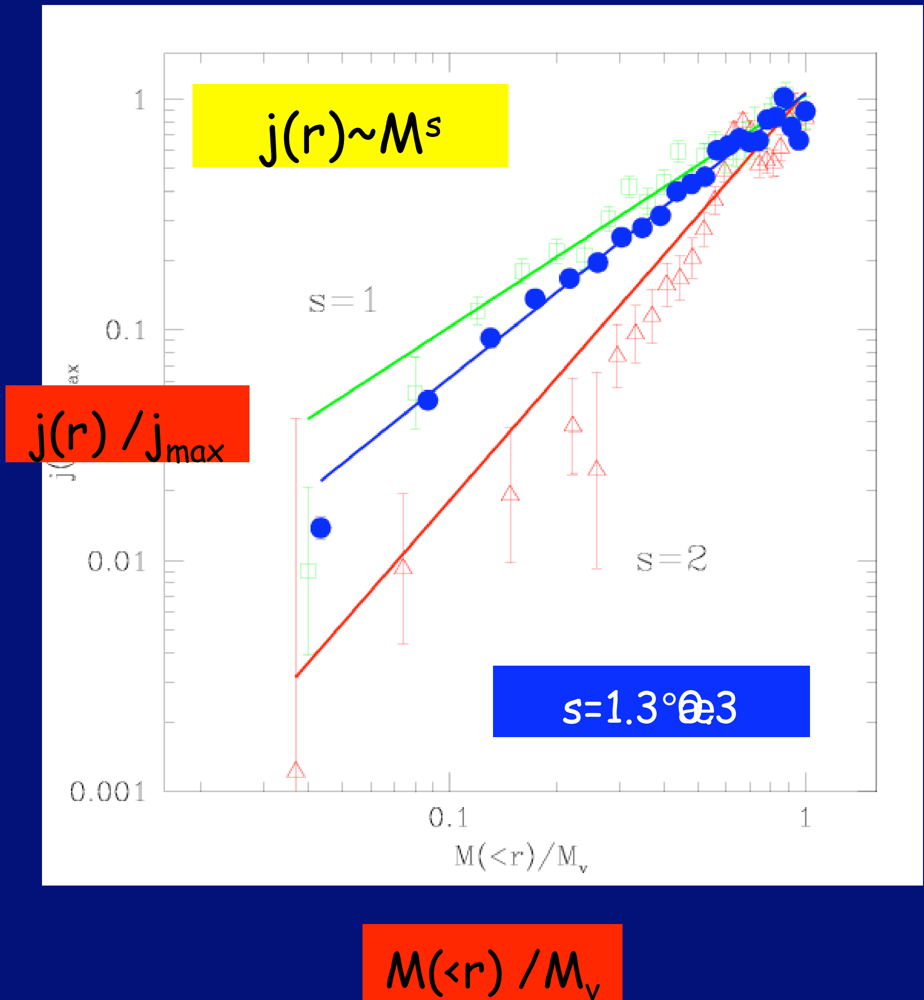
spin parameter _ and shape parameter _

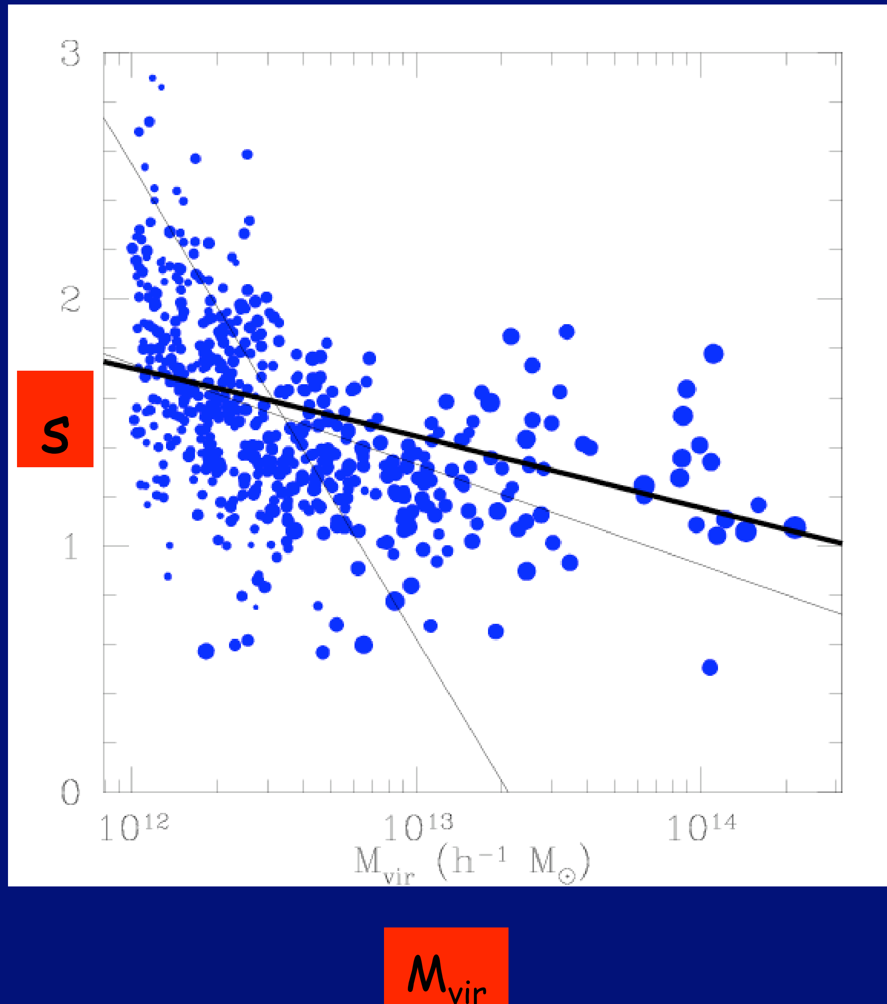






Distribution of J with radius:
a power-law profile





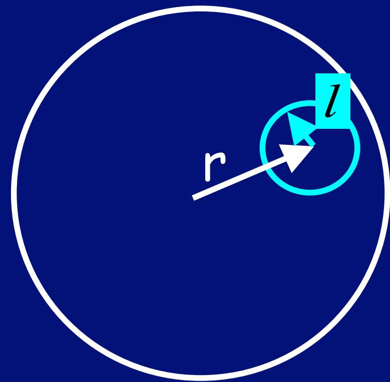
Distribution of J in space

Toy model: J by minor mergers

Tidal radius

$$\frac{m(l_t)}{l_t^2} = \frac{l_t}{r^3}\left(2M(r) - r\frac{dM}{dr}\right)$$

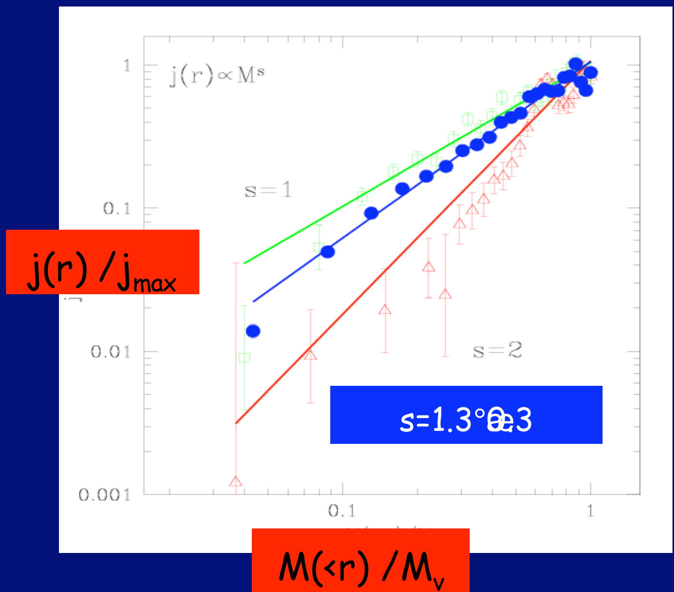
$$M \propto r^{\alpha} \quad \rightarrow \quad m[l_t(r)] \propto M(r)$$

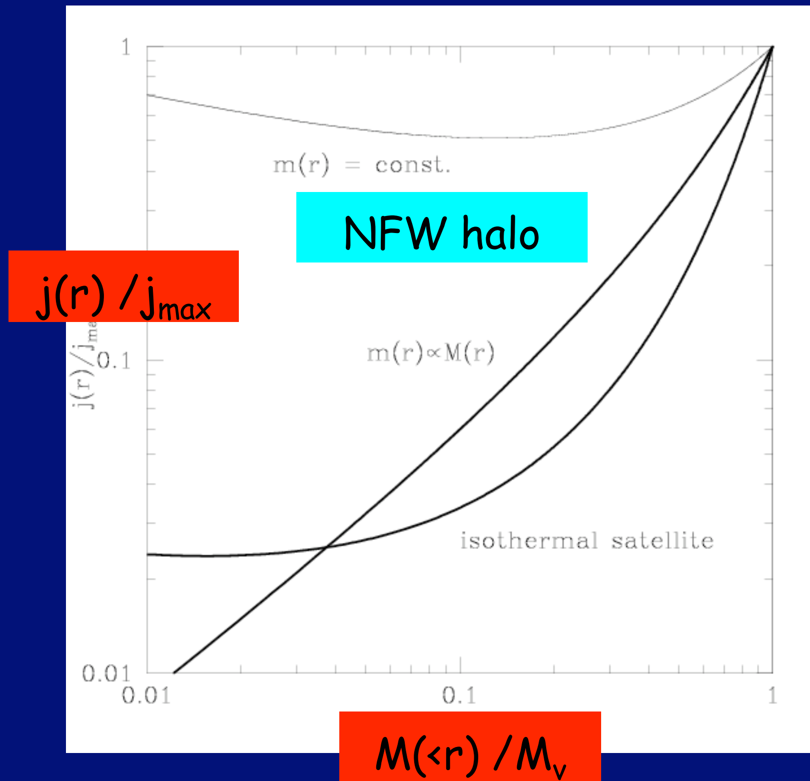


Assume m and j are deposited locally in a shell r

$$4\pi r^2 \rho(r)\, j(r) = m(r)\frac{d[rV(r)]}{dr} + \frac{dm}{dr} rV(r)$$

$$M \propto r, \quad m \propto l \quad \rightarrow \quad j(M) \propto M \propto r$$



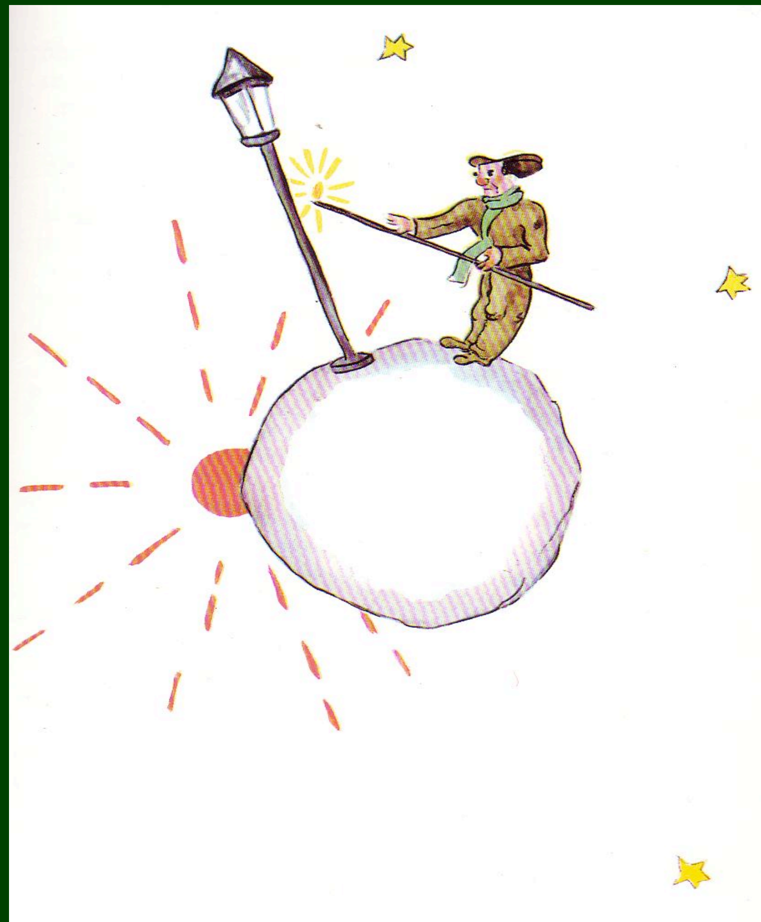


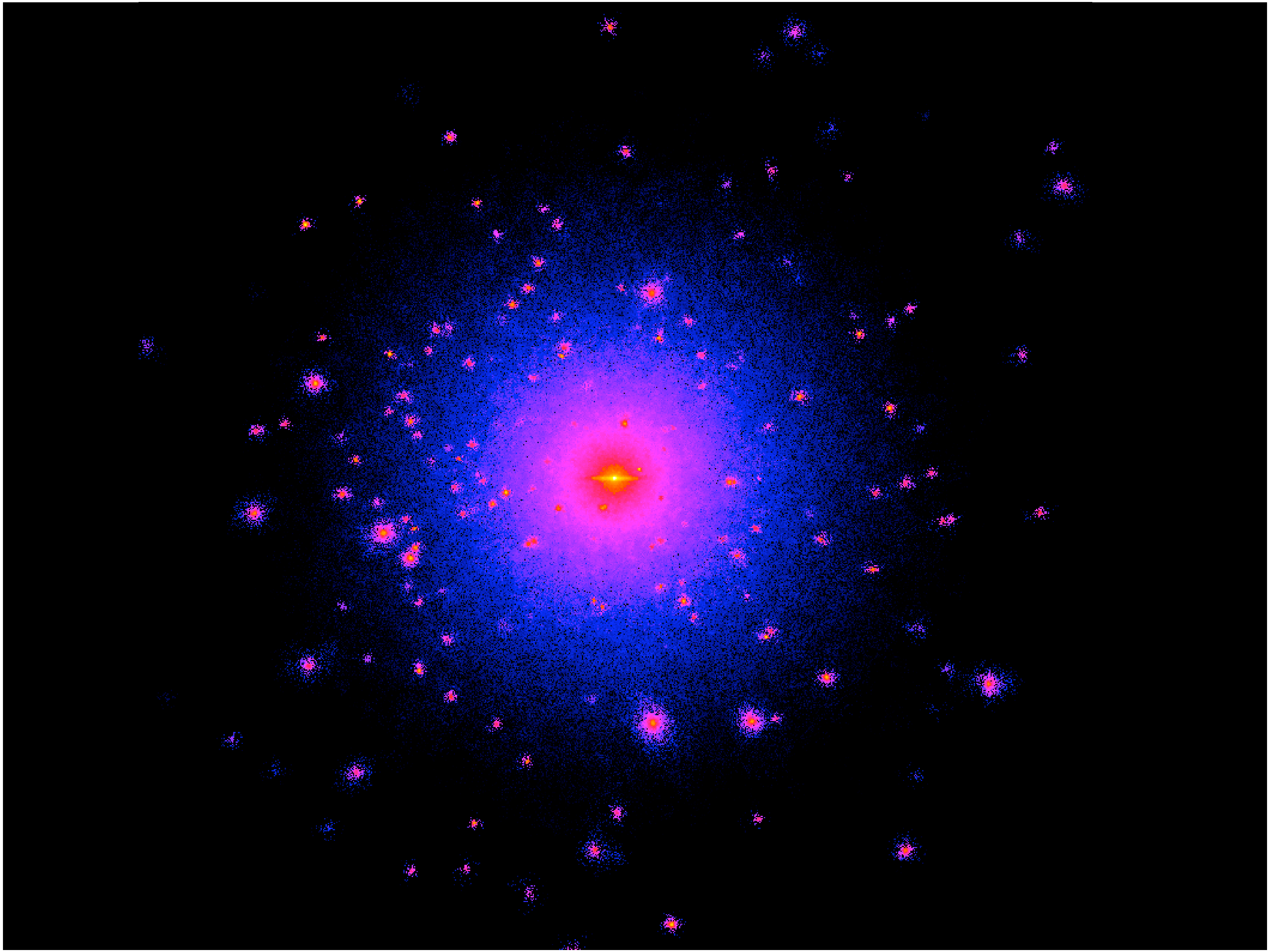
Formation of Stellar Disks and Spheroids inside DM Halos

White & Rees 1978

Fall & Efstathiou 1980

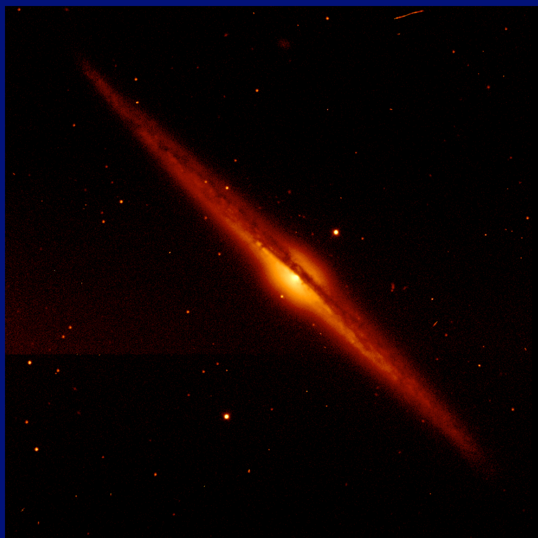
Mo, Mao & White

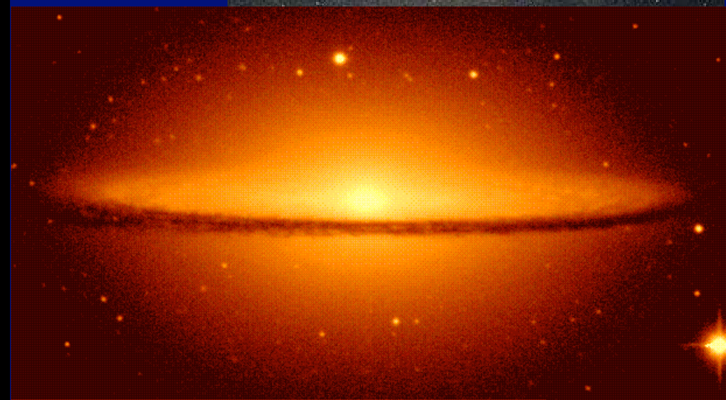




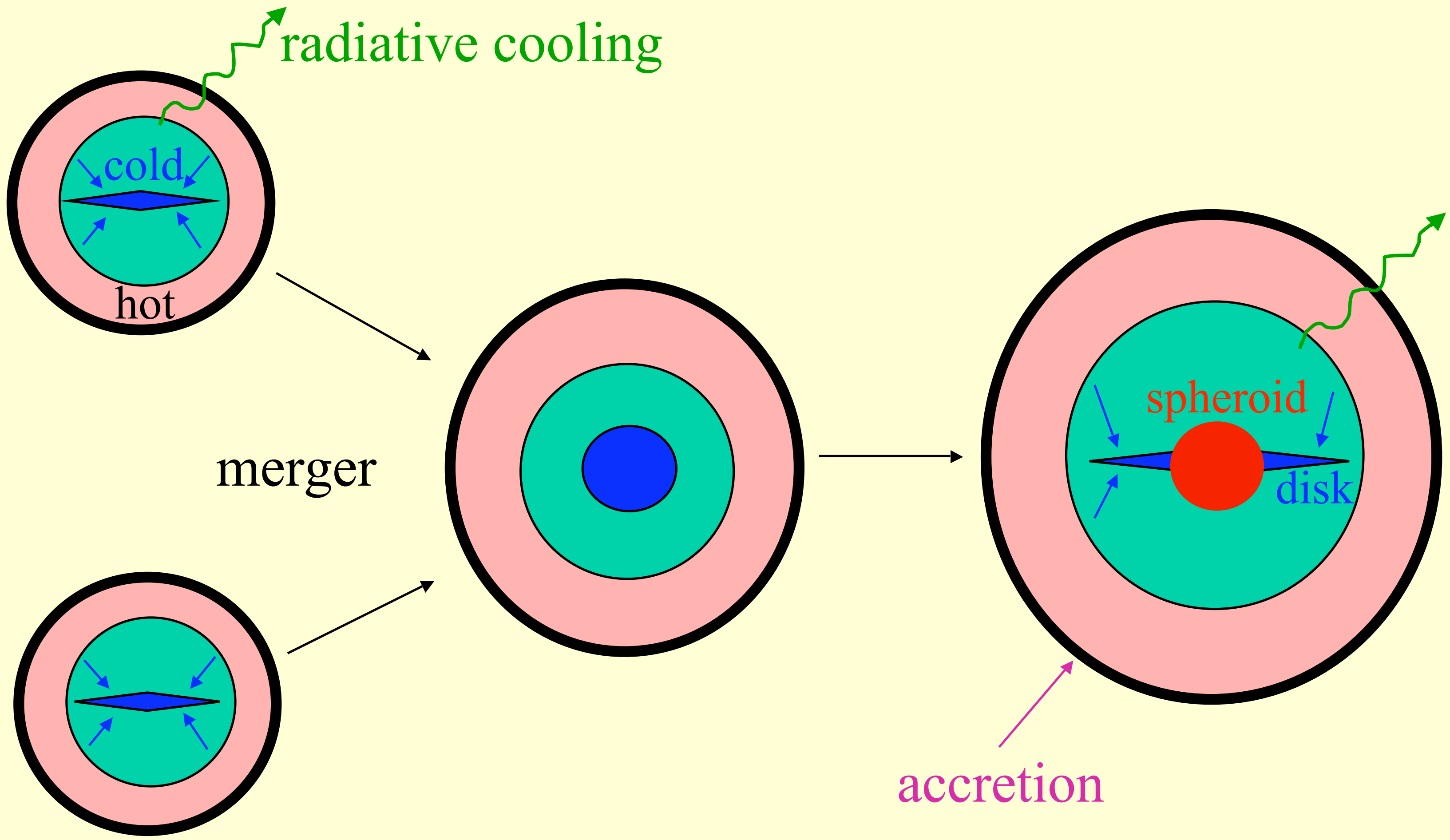
Galaxy Types: Disks and Spheroids

- The morphology of a galaxy is a transient feature dictated by the mass accretion history of its dark matter halo
  - most stars form in disks; spheroids result from subsequent mergers
  - disks result from smooth gas accretion; oldest disk stars are often used to date the last major merger event





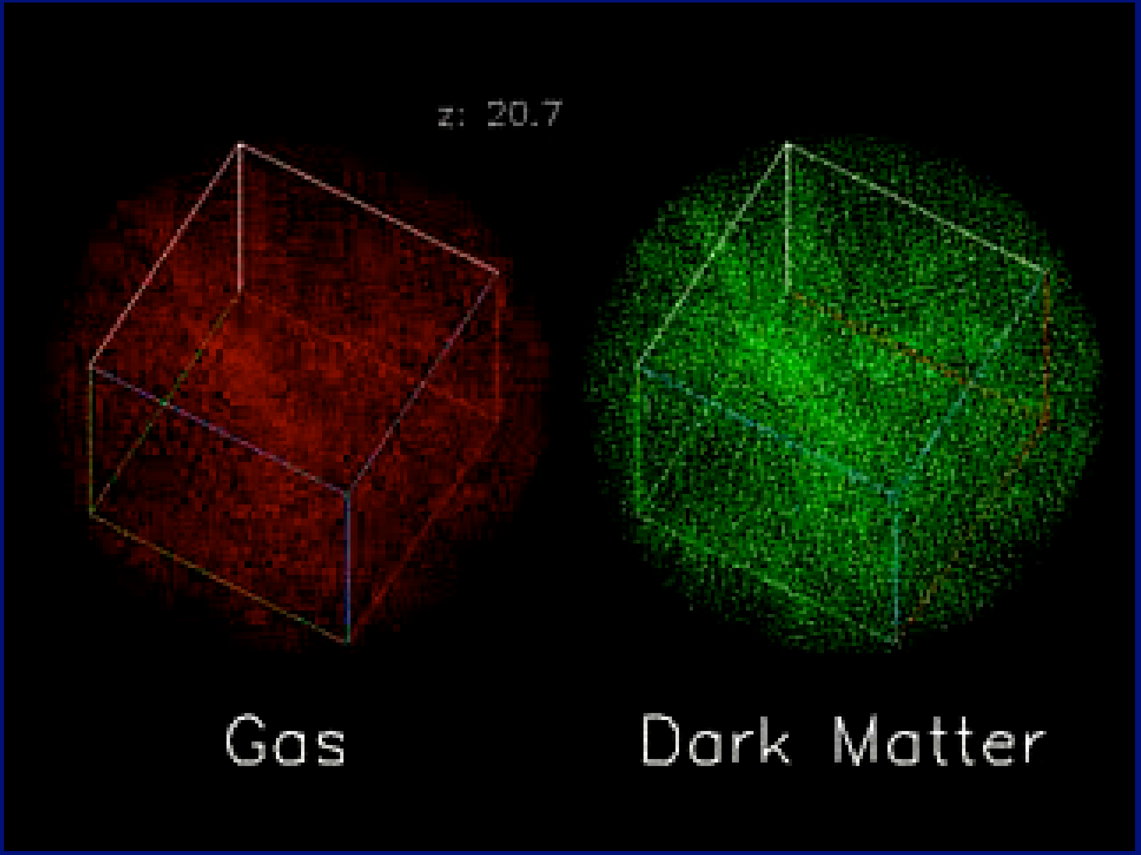
Galaxy Formation in halos



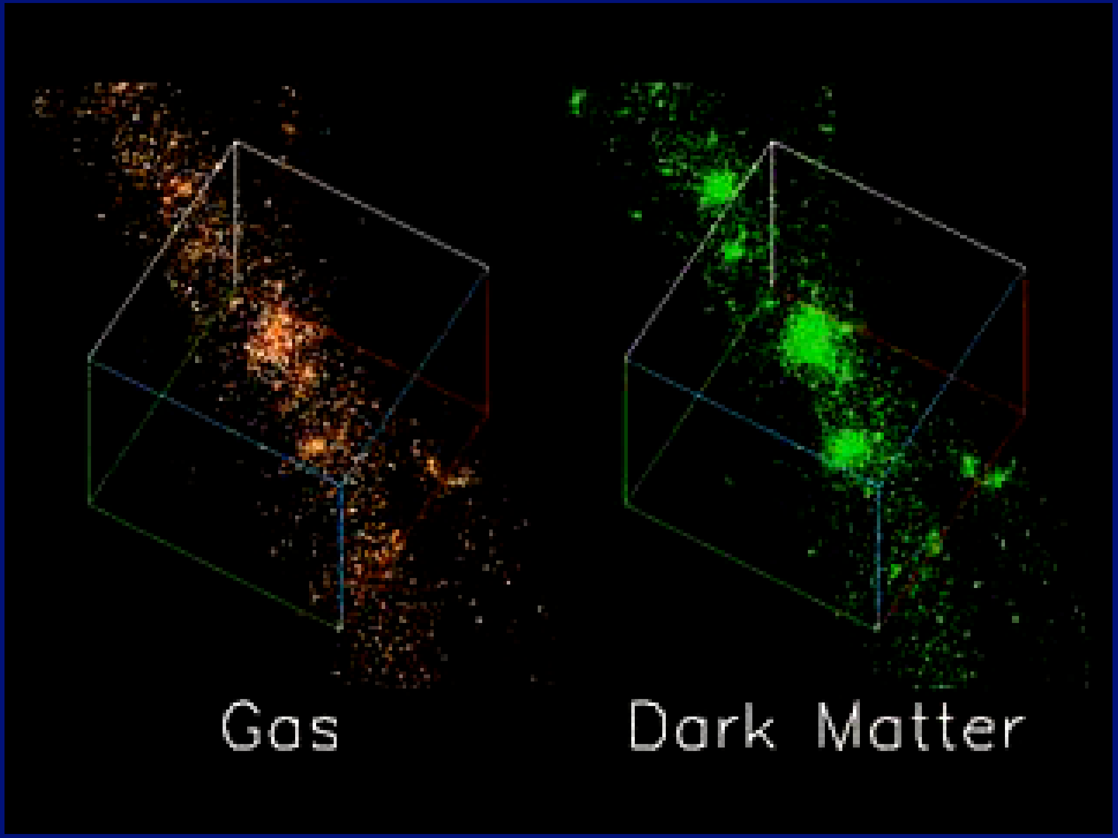
halos cold gas → young stars → old stars

Gas versus Dark Matter

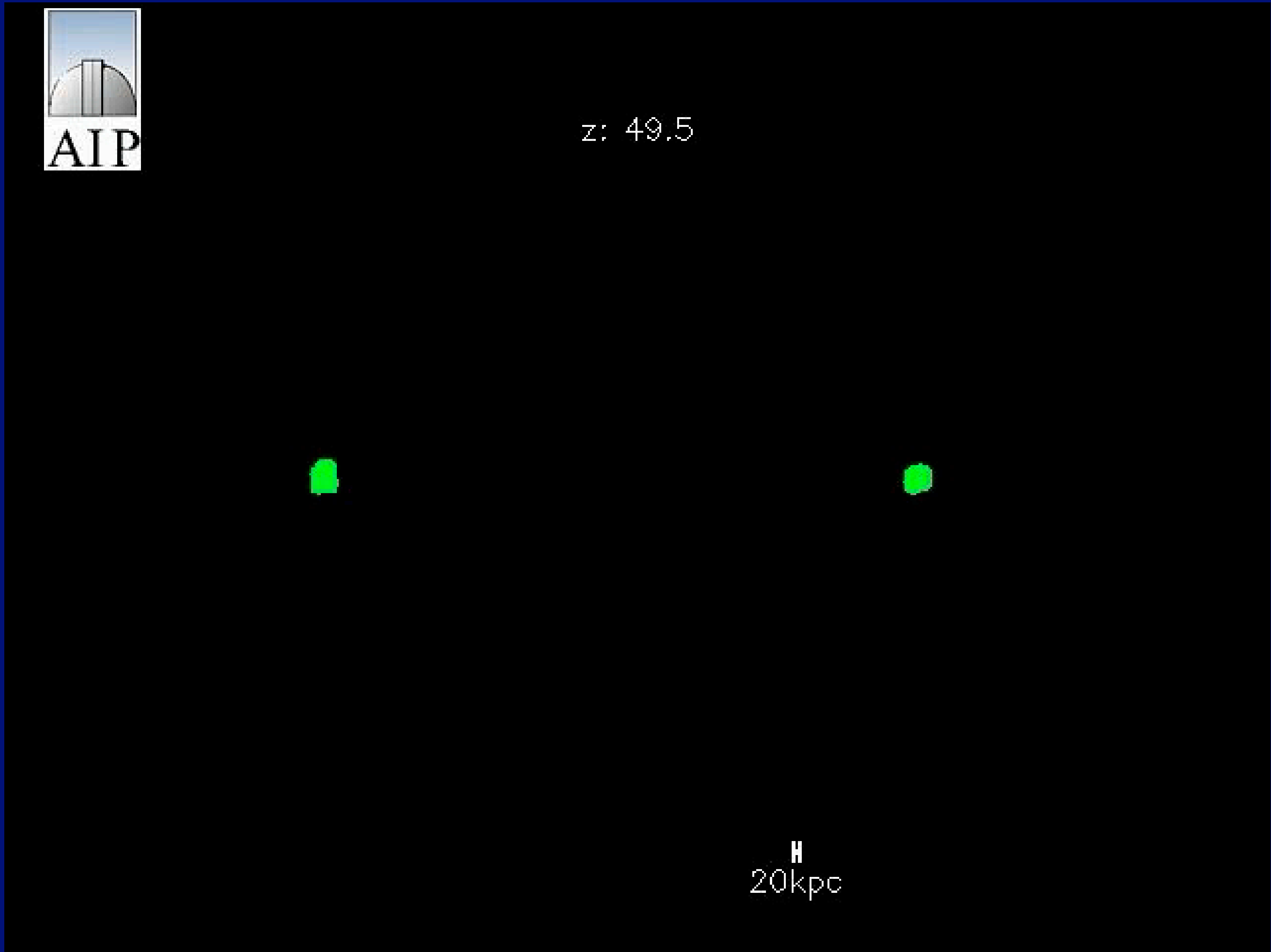
Navarro, Steinmetz



Flat gaseous disk vs spheroidal DM halo



Disk/Bulge Formation (gas only) (Navarro, Steinmetz)

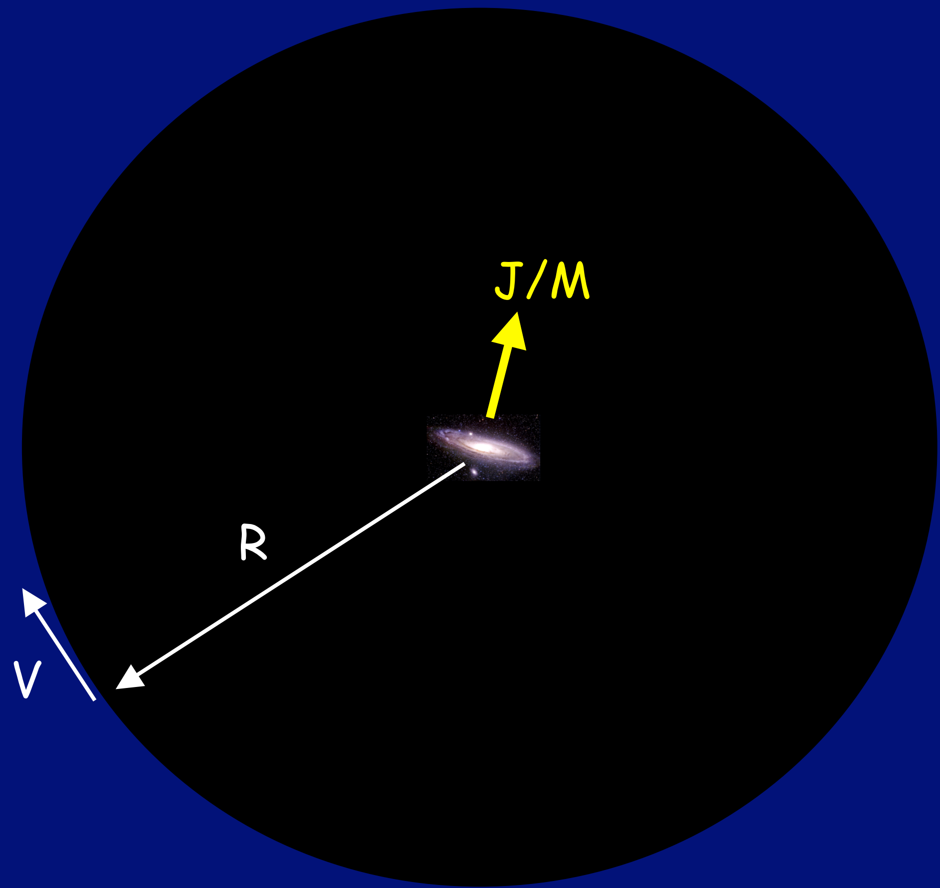


Disk Size

Spin parameter $\lambda \sim \dfrac{J/M}{RV}$

Conservation of specific angular momentum $const. = J/M \sim \lambda R_{\text{virial}} V \sim R_{\text{disk}} V$

$\rightarrow$ $\dfrac{R_{\text{disk}}}{R_{\text{virial}}} \sim \lambda$



Disk Profile from the Halo J Distribution

Assume the gas follows the halo j distribution $M_{gas}(<j) = f\,M(<j)$

Assume conservation of j
during infall from halo to disk.

In disk: $j(r) = Vr = [GM(r)r]^{1/2}$

In disk: lower j at lower r

$M_{halo}(<j) \rightarrow m_{disk}(r)$

$$M_{halo}(<j) = M_{vir}\frac{\mu j}{j_0 + j} \qquad \mu > 1 \quad \rightarrow \quad m_d(r) = f\mu M_v \frac{j(r)}{j_0 + j(r)} \qquad j(r) < j_{\max}$$

Assume isothermal sphere
No adiabatic contraction

$$M \propto r \quad \rightarrow \quad j(r) = rV(r) = rV_{vir}$$

$$\rightarrow \quad m_d(r) = f\mu M_v \frac{r}{r_d + r} \qquad r < r_{\max}$$

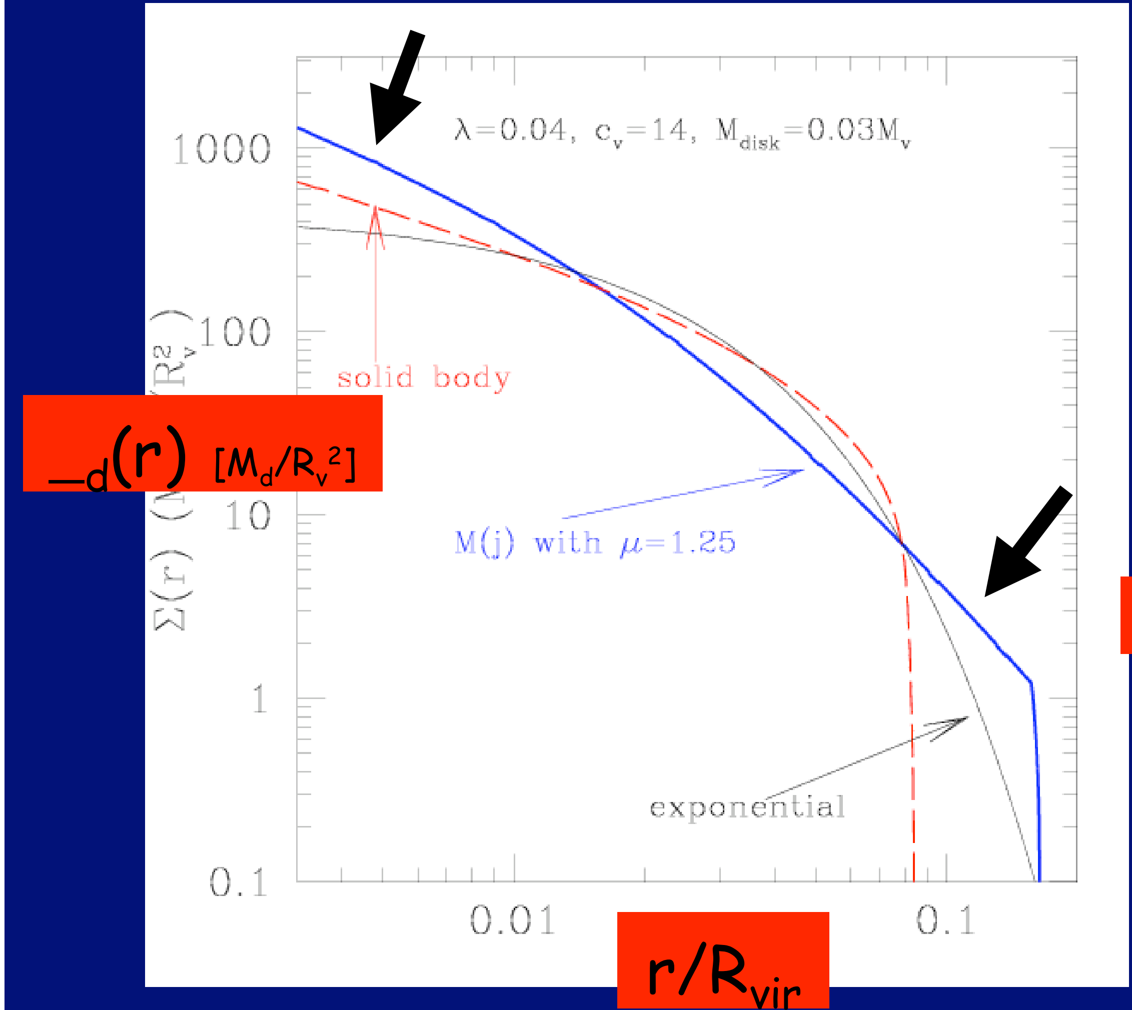
$$r_d = \sqrt{2}\lambda' R_v b^{-1}(\mu)$$

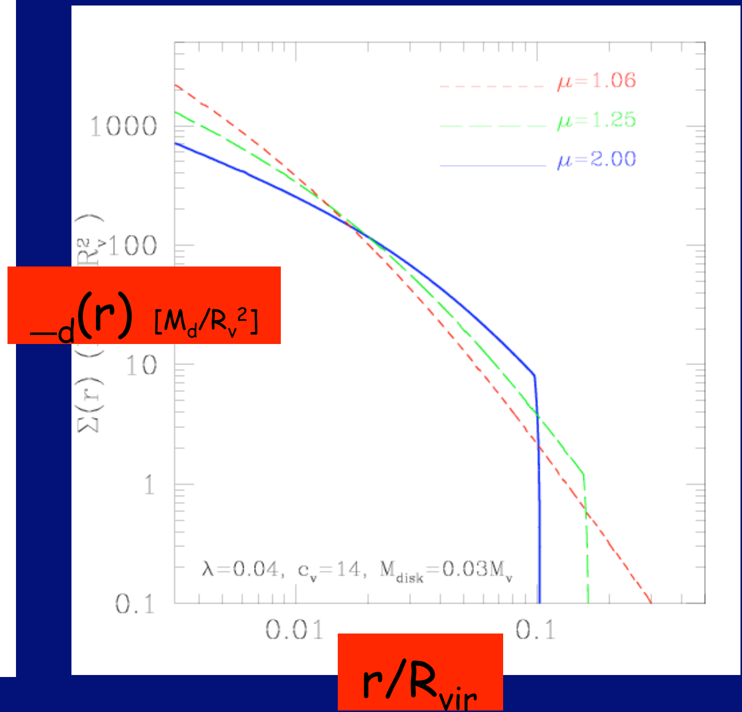
$$r_{\max} = r_d/(\mu - 1)$$

$$\Sigma_d(r) = \frac{f\mu M_v}{2\pi}\frac{r_d}{r(r_d + r)^2}$$

Disk Profile: **Shape Problem**

Bullock et al. 2001b





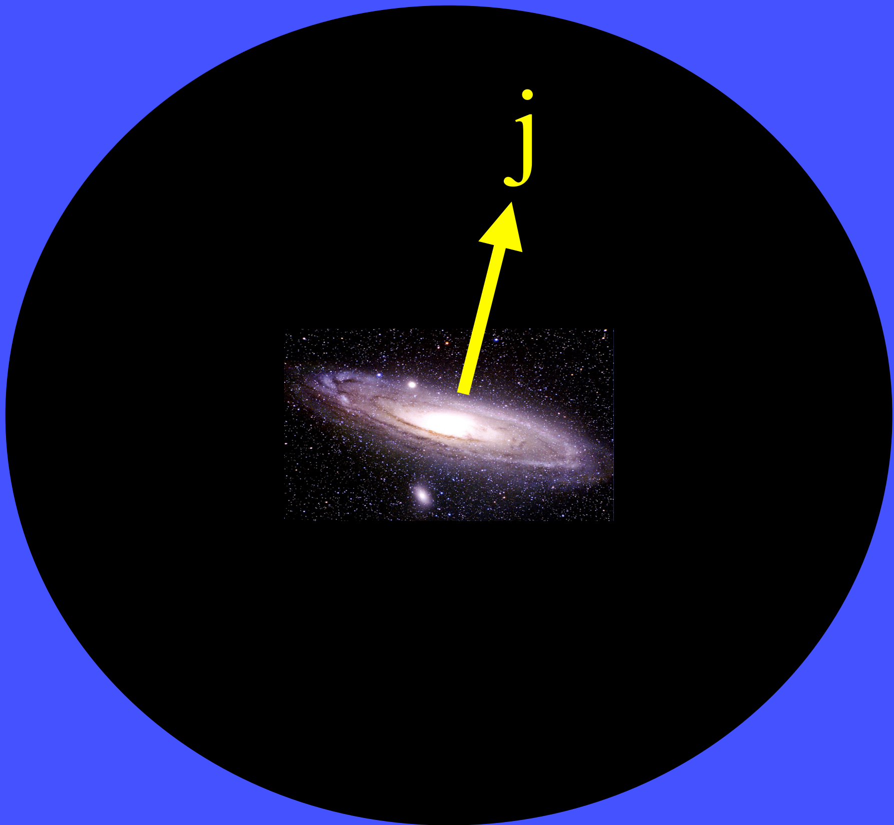
The Angular-Momentum Problem

Navarro & Steinmetz

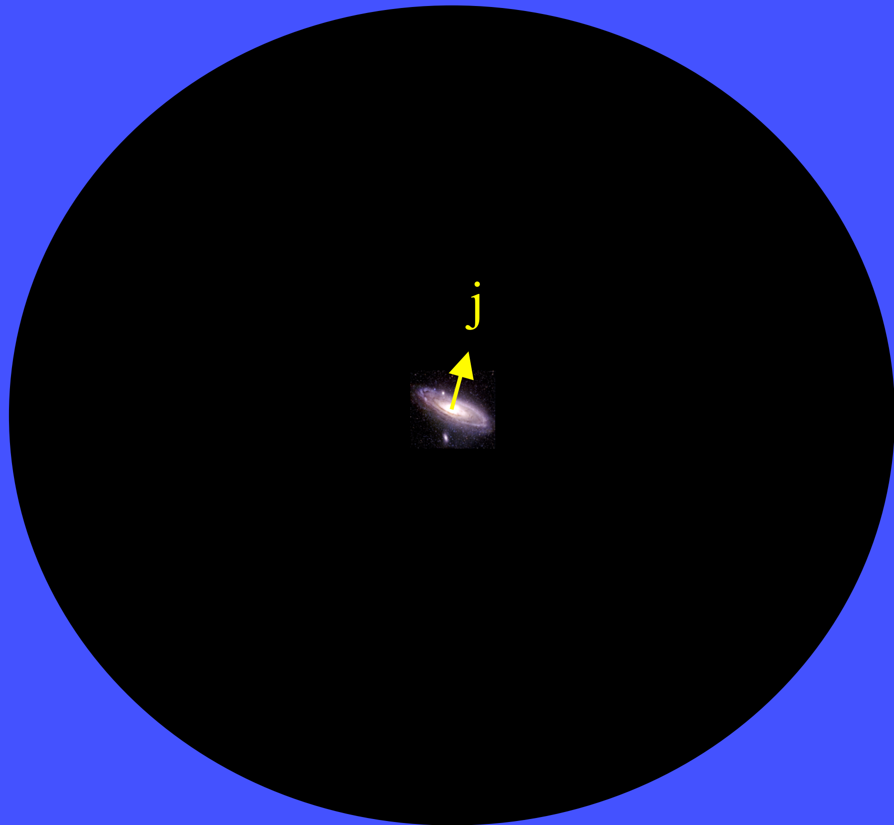
The Spin Catastrophe

Navarro & Steinmetz et al.

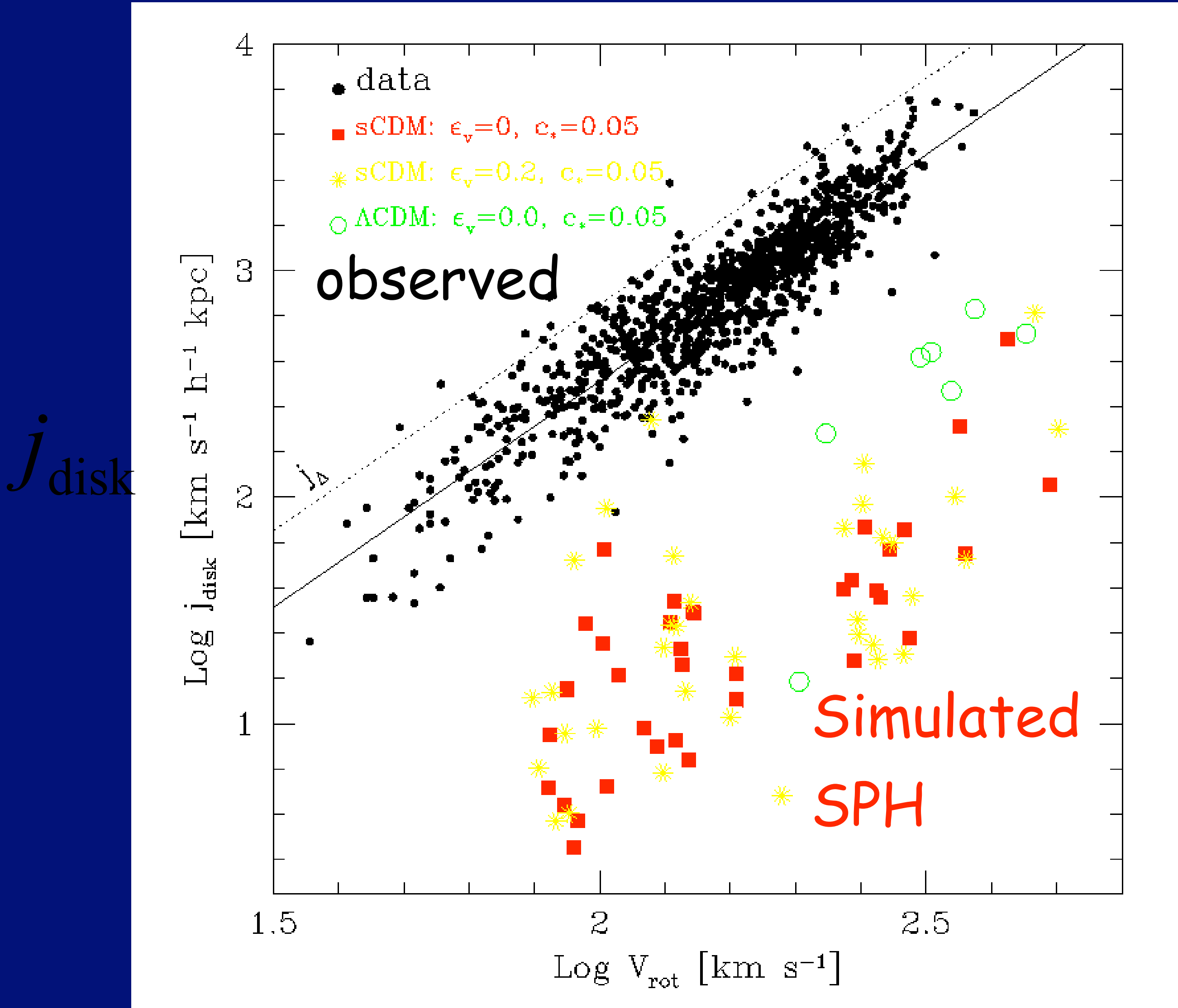
observations



simulations

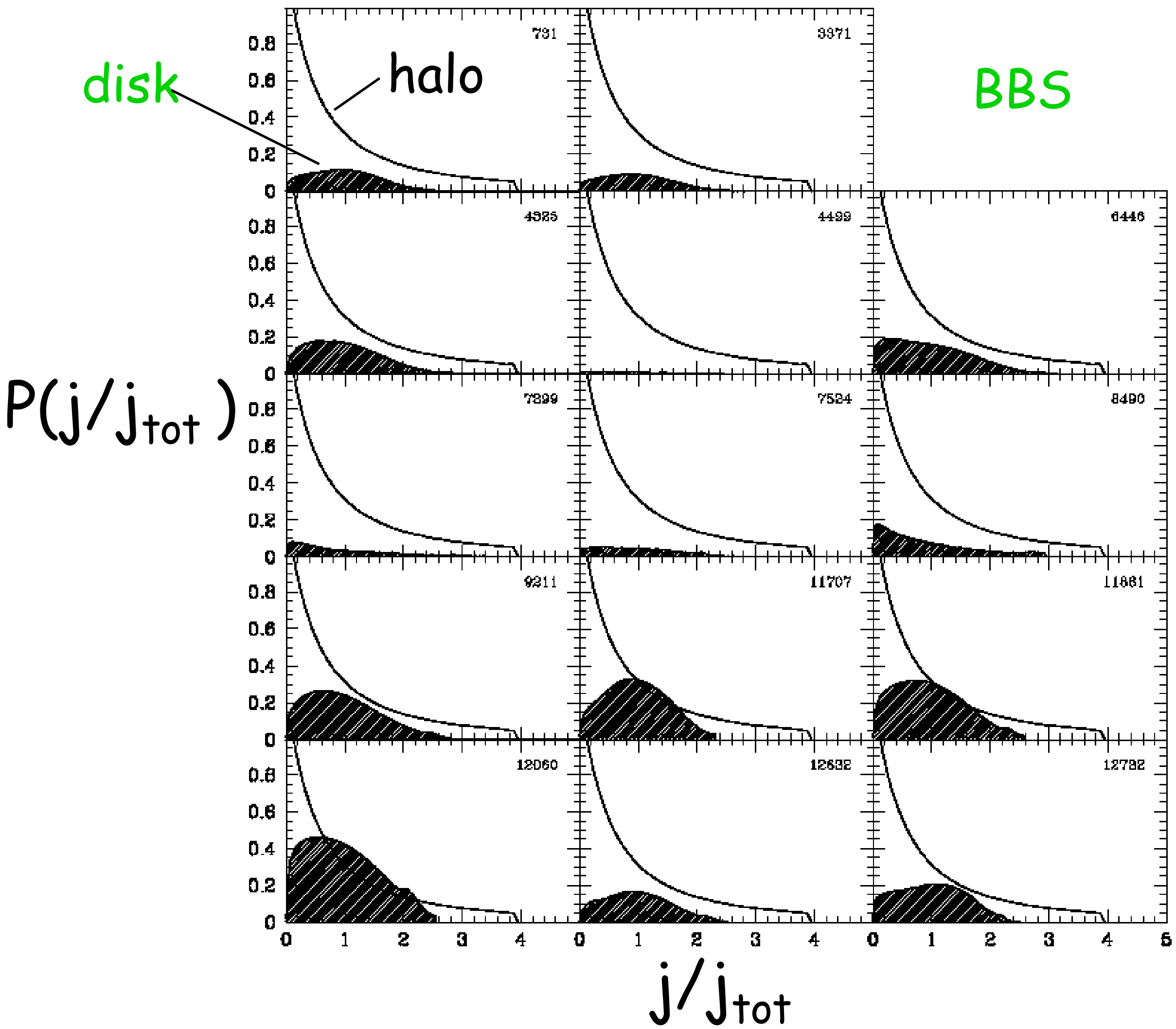


The spin catastrophe



Steinmetz, Navarro, et al.

Observed j distribution in dwarfs



Low $f_{baryons} \approx 0.03$

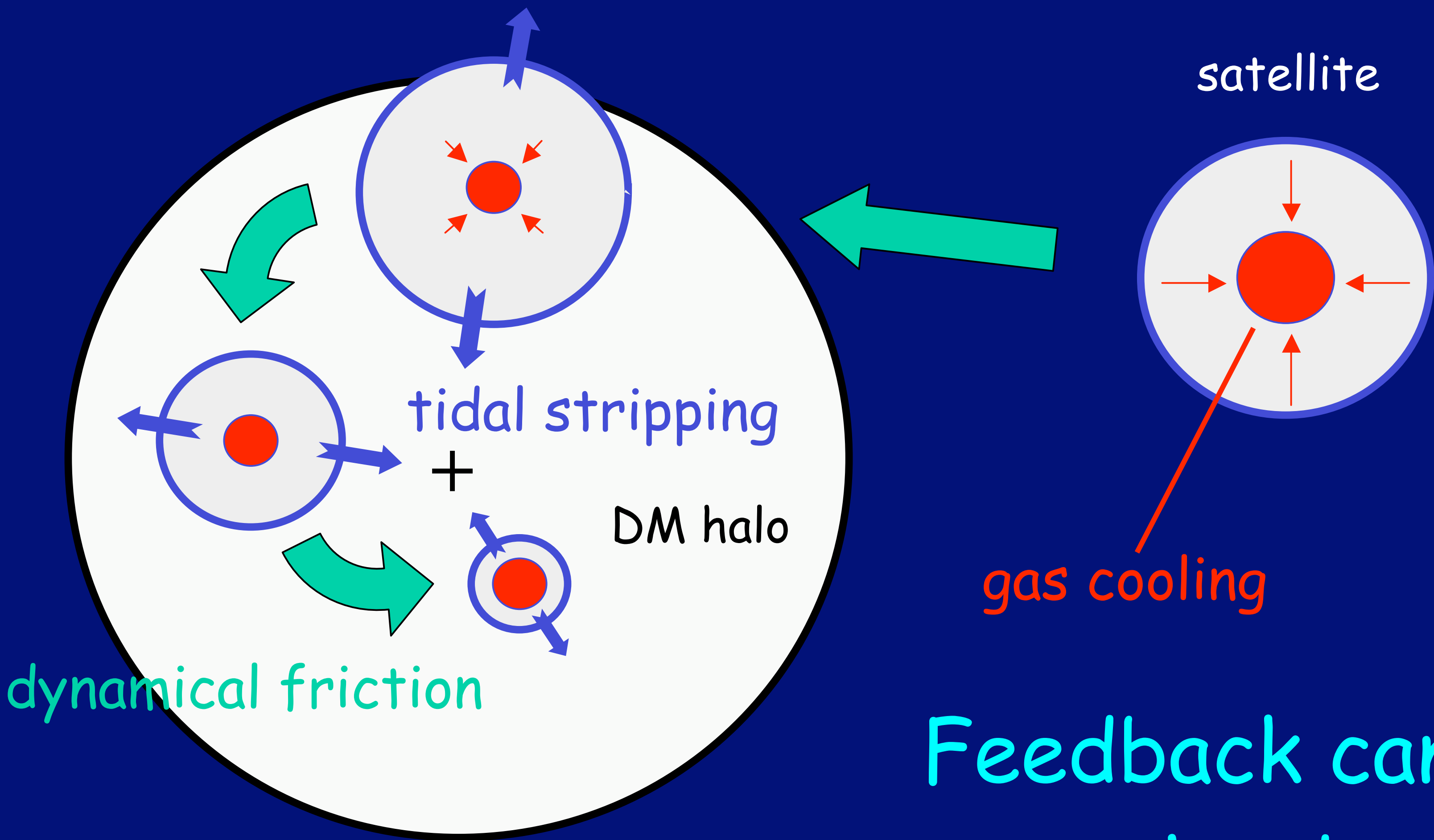
Missing low j

High $\lambda_{baryons} \approx 0.07$

van den Bosch, Burkert & Swaters 2002

Over-cooling → spin catastrophe

Maller & Dekel 02

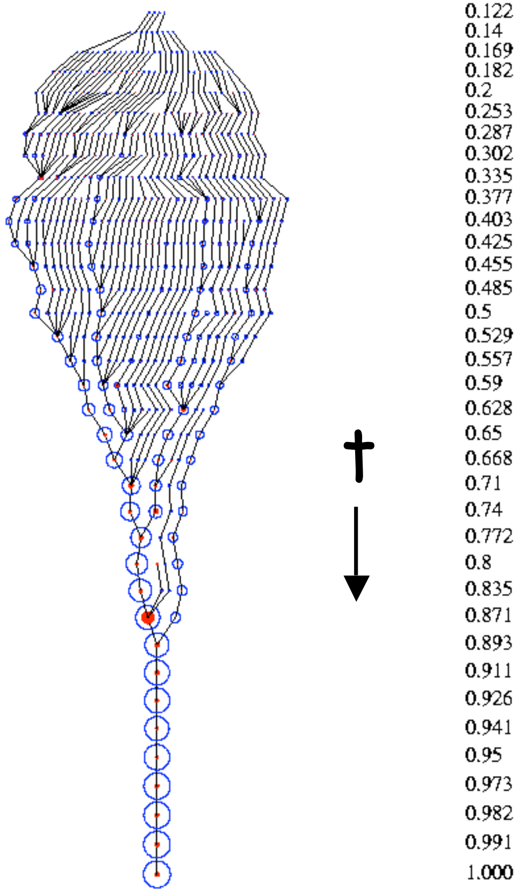


Feedback can save the day

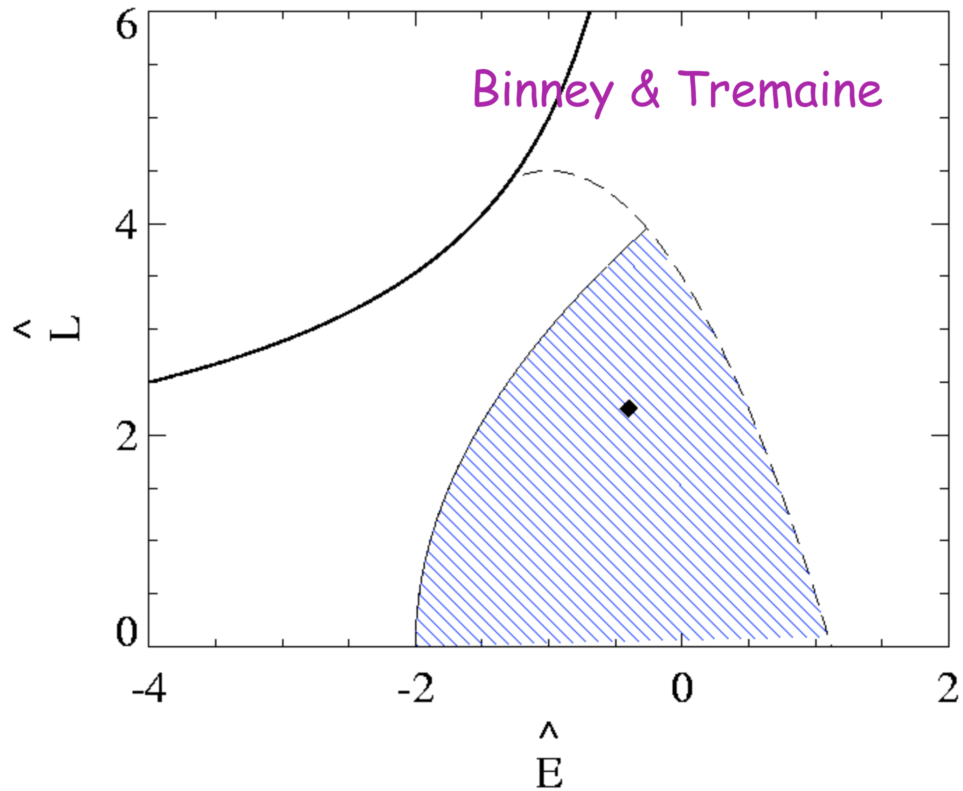
Orbital-merger model:

Add orbital angular momentum in merger history

Merger history

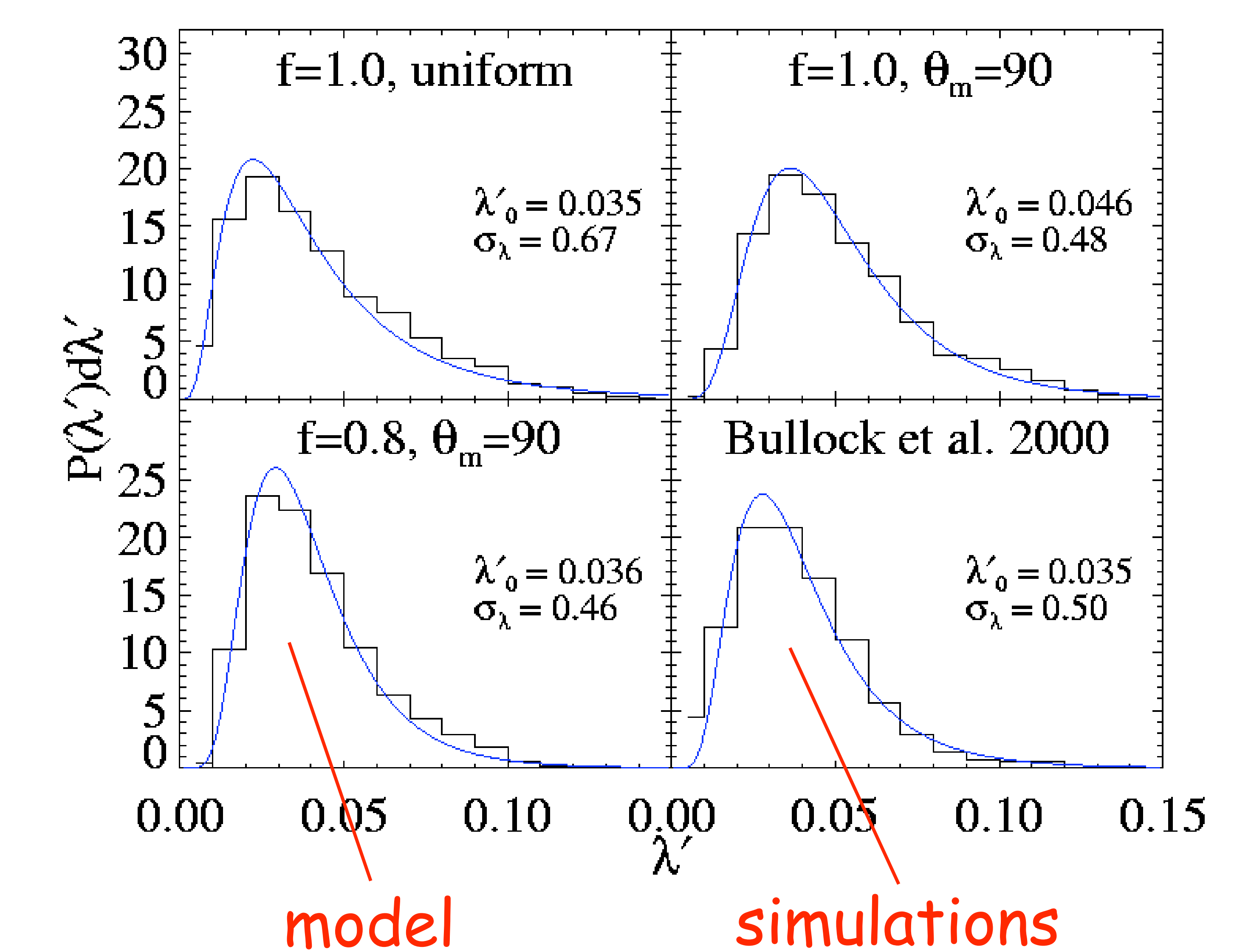


Orbit parameters



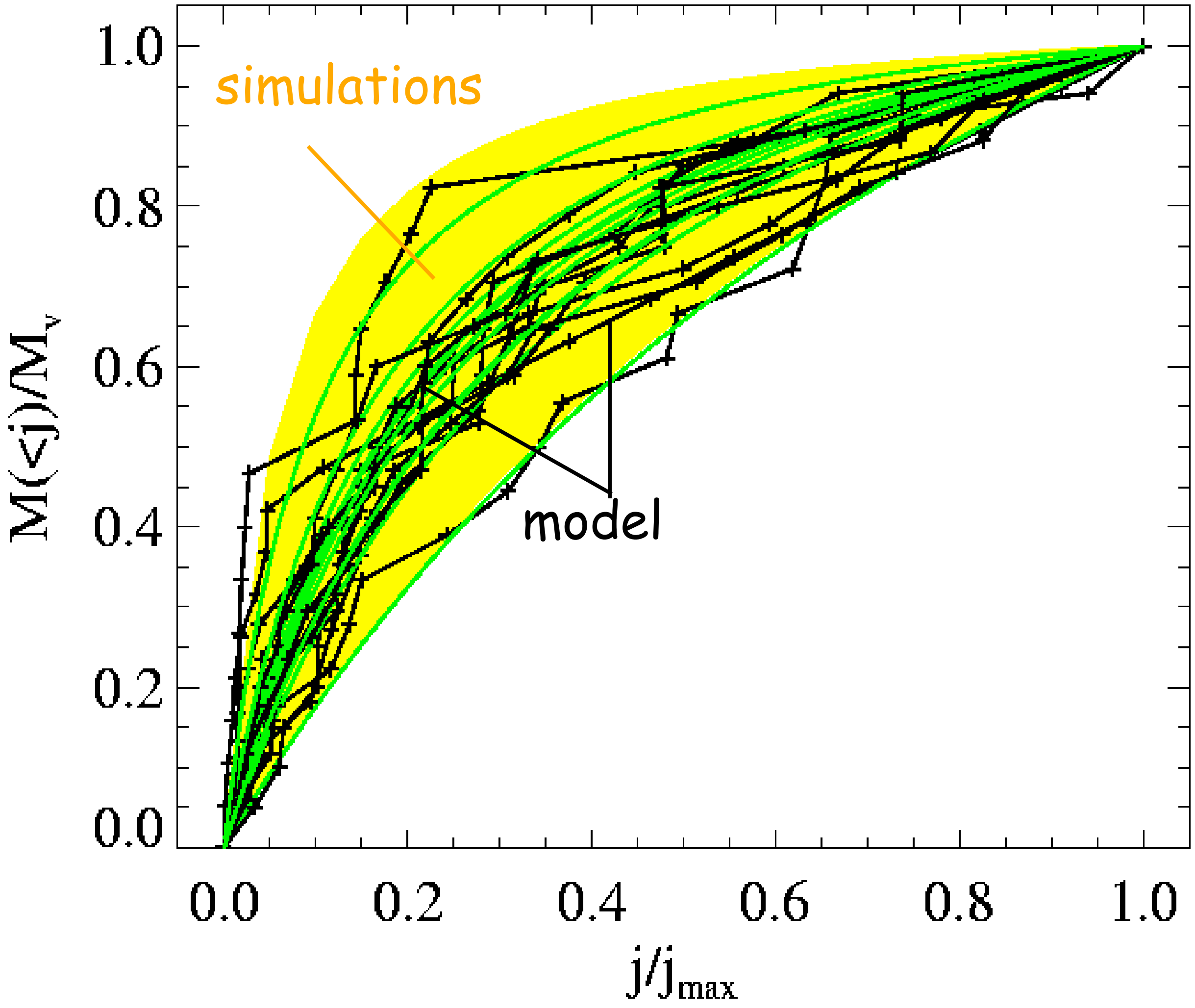
and random orientation

Succes of orbital-merger model



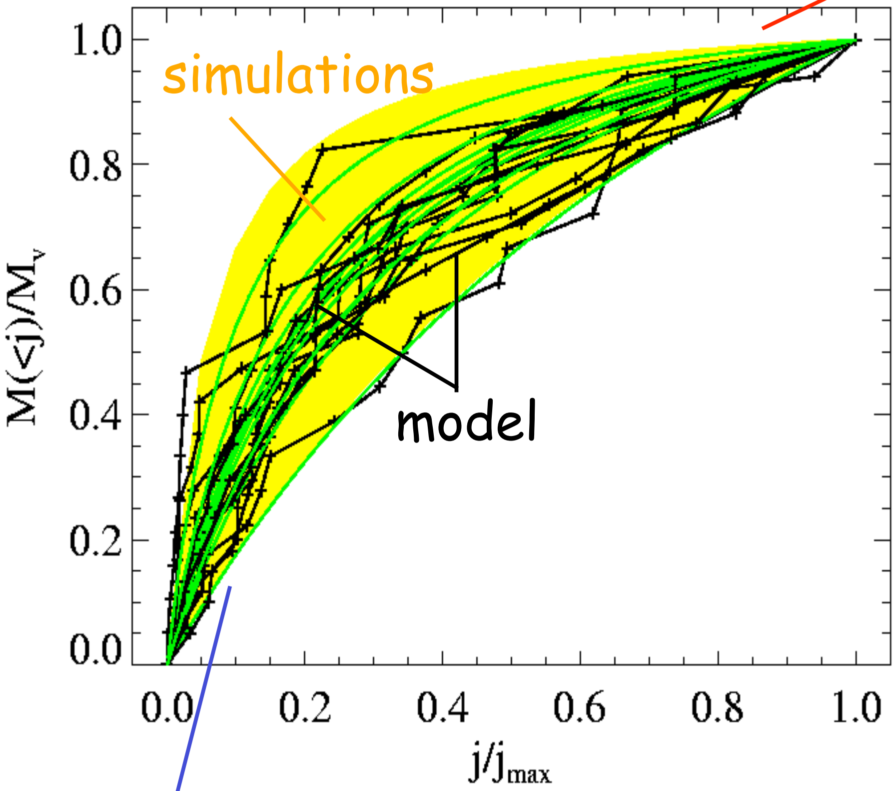
Maller, Dekel & Somerville 2002

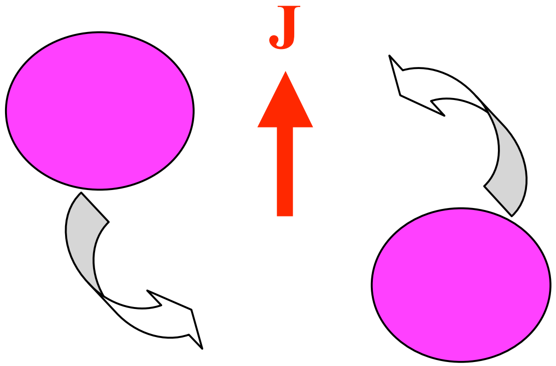
Model success: j distribution in halos

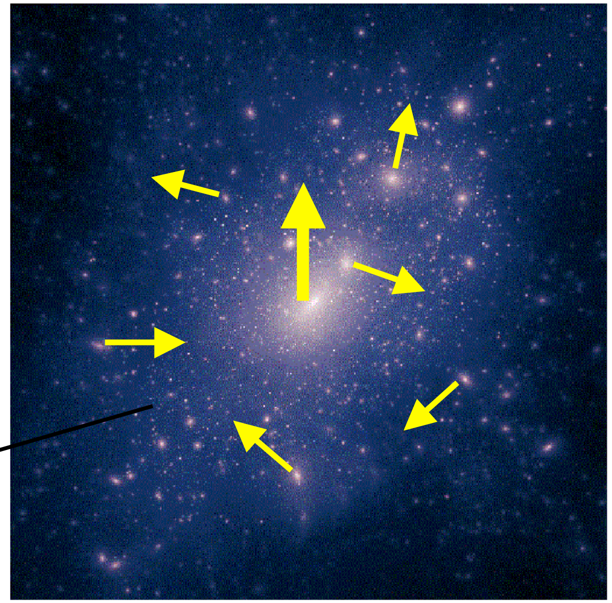


Low/high-j from minor/major mergers

High-j from major mergers







Low-j from minor mergers

Supernova Feedback: V_{SN} (Dekel & Silk 86; Dekel & Woo 03)

Energy fed to the ISM during the "adiabatic" phase:

$$E_{SN} \approx \nu\varepsilon \dot{M}_* t_{rad} \quad \propto M_* (t_{rad}/t_{ff})$$

$$\dot{M}_* \approx M_*/t_{ff}$$

$$\approx 0.01$$

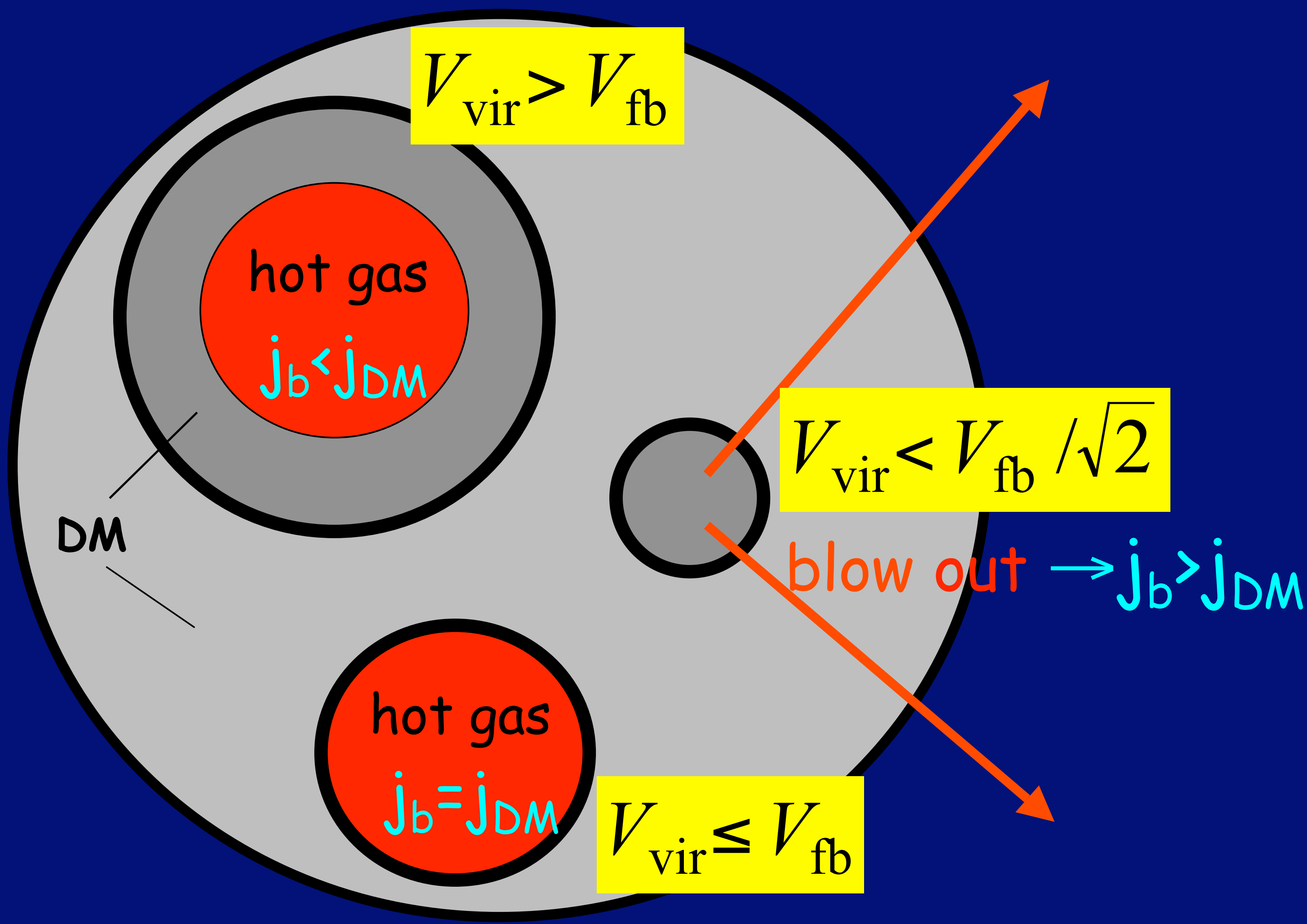
for $\Lambda \propto T^{-1}$ at $T \sim 10^5 K$

Energy required for blowout:

$$E_{SN} \approx M_{gas} V^2$$

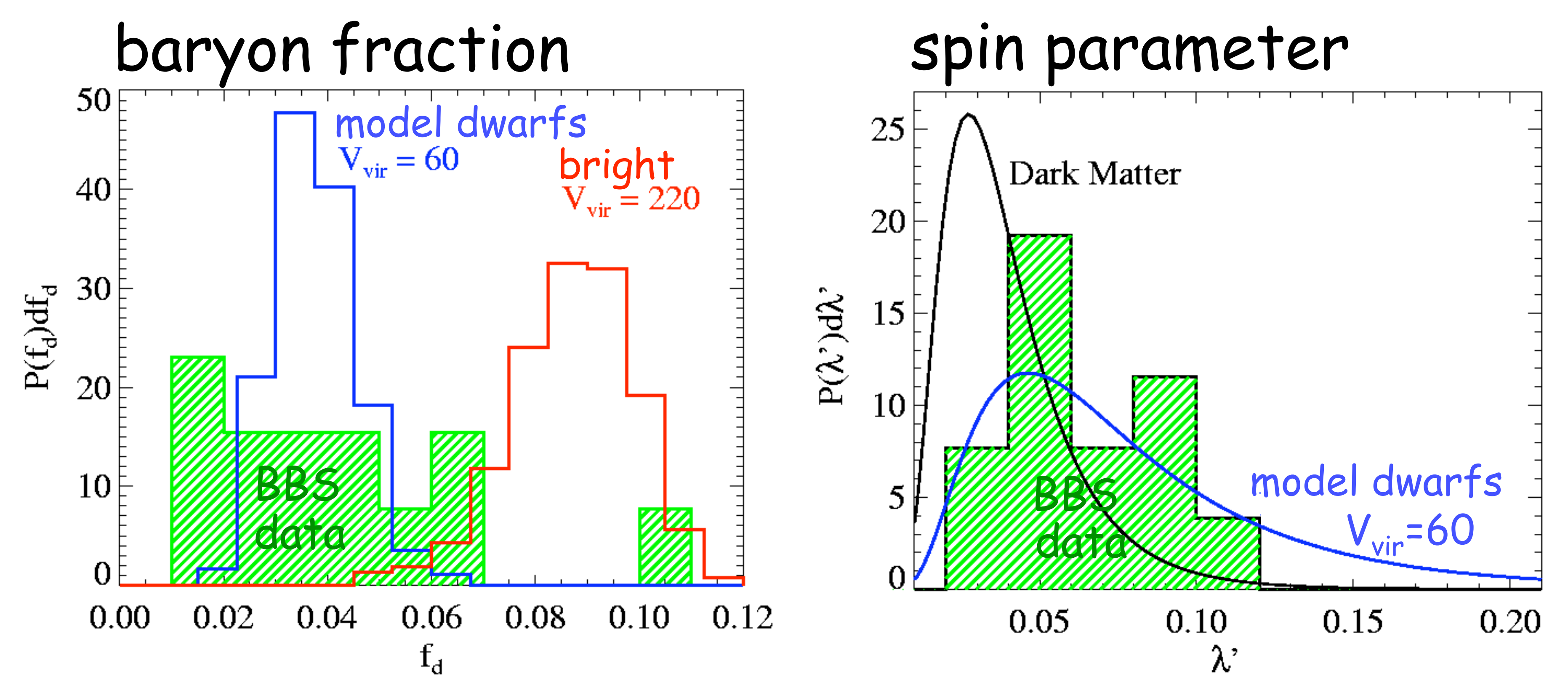
$$\rightarrow \quad V_{crit} \approx 100 \text{ km/s} \quad \rightarrow \quad M_{*crit} \approx 3\times 10^{10} M_\odot$$

Feedback in satellite halos



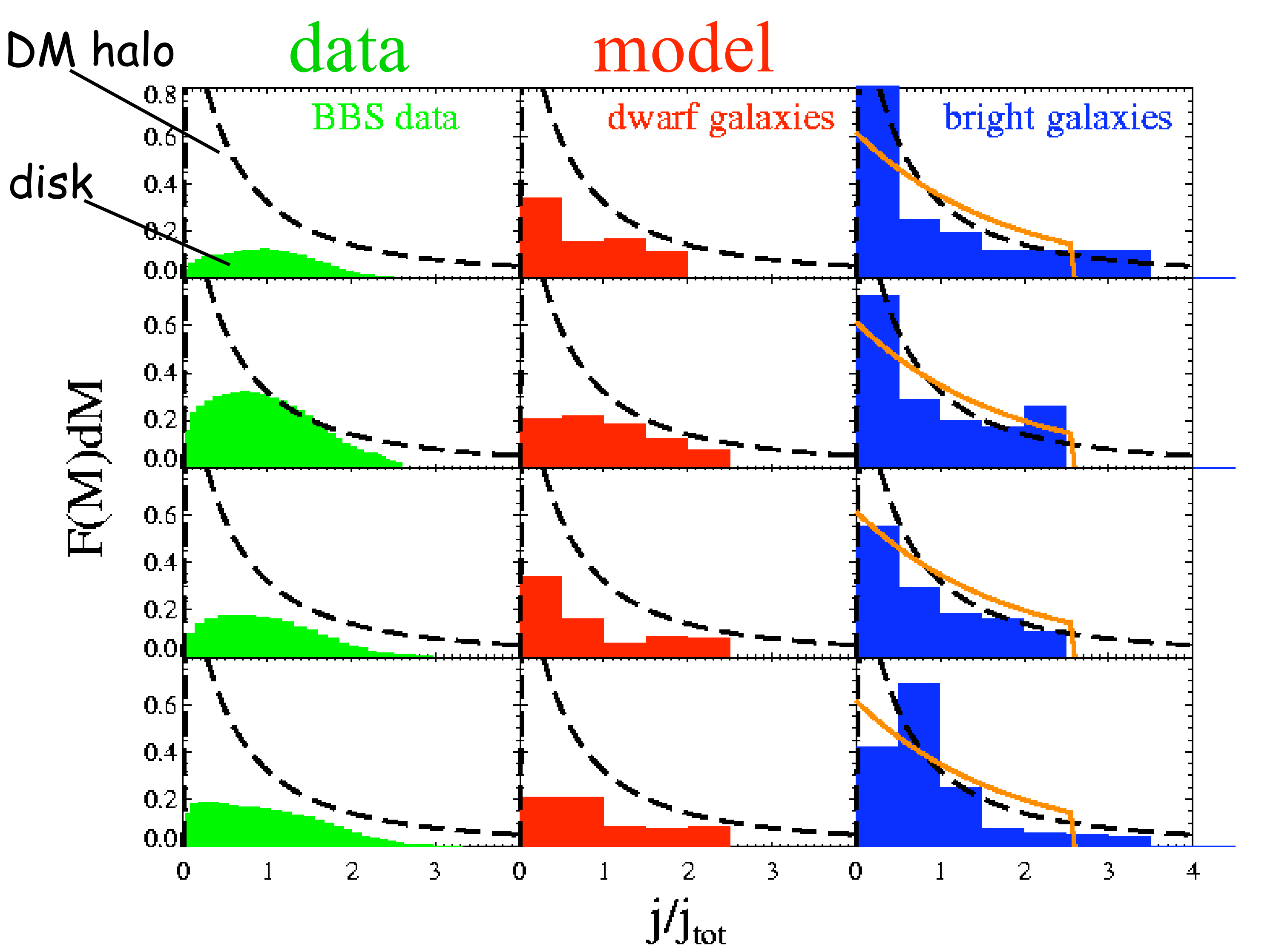
Model vs Data (Maller & Dekel 02)

BBS data: 14 dwarfs, van den Bosch, Burkert & Swaters 02



One free parameter in model: $V_{feedback} \cong 90$ km s^{-1}

J-distribution within galaxies



BBS: van den Bosch, Burkert & Swaters 2002

Summary: feedback effect on spin

In big satellites (merging to big galaxies)

heating → gas expansion $R_b \sim R_{DM}$

→ tidal stripping together → $\lambda_{bar} \sim \lambda_{DM}$

In small satellites (merging to dwarfs)

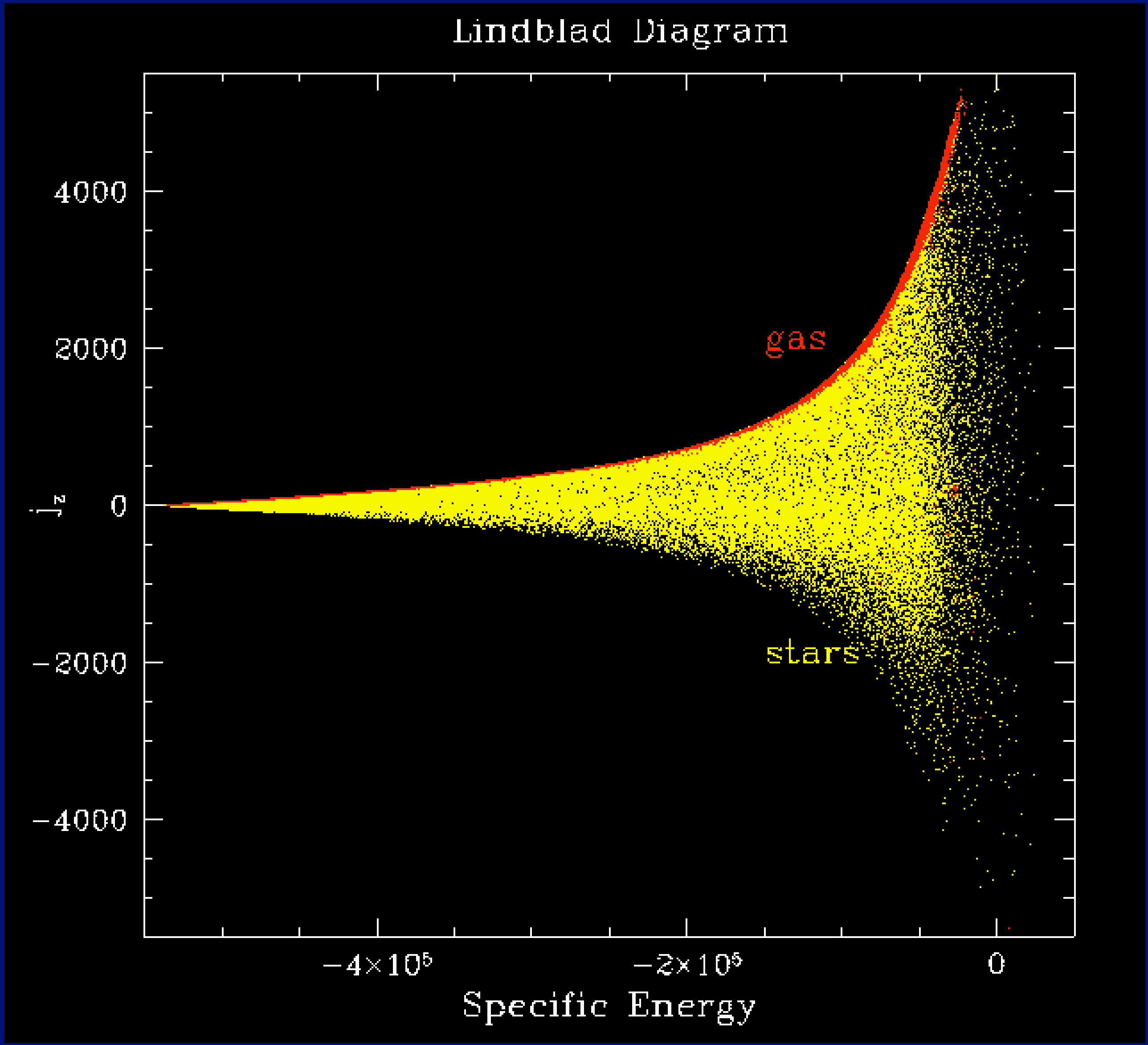
gas blowout → f_{bar} down

blowout of low j gas → $\lambda_{bar} > \lambda_{DM}$

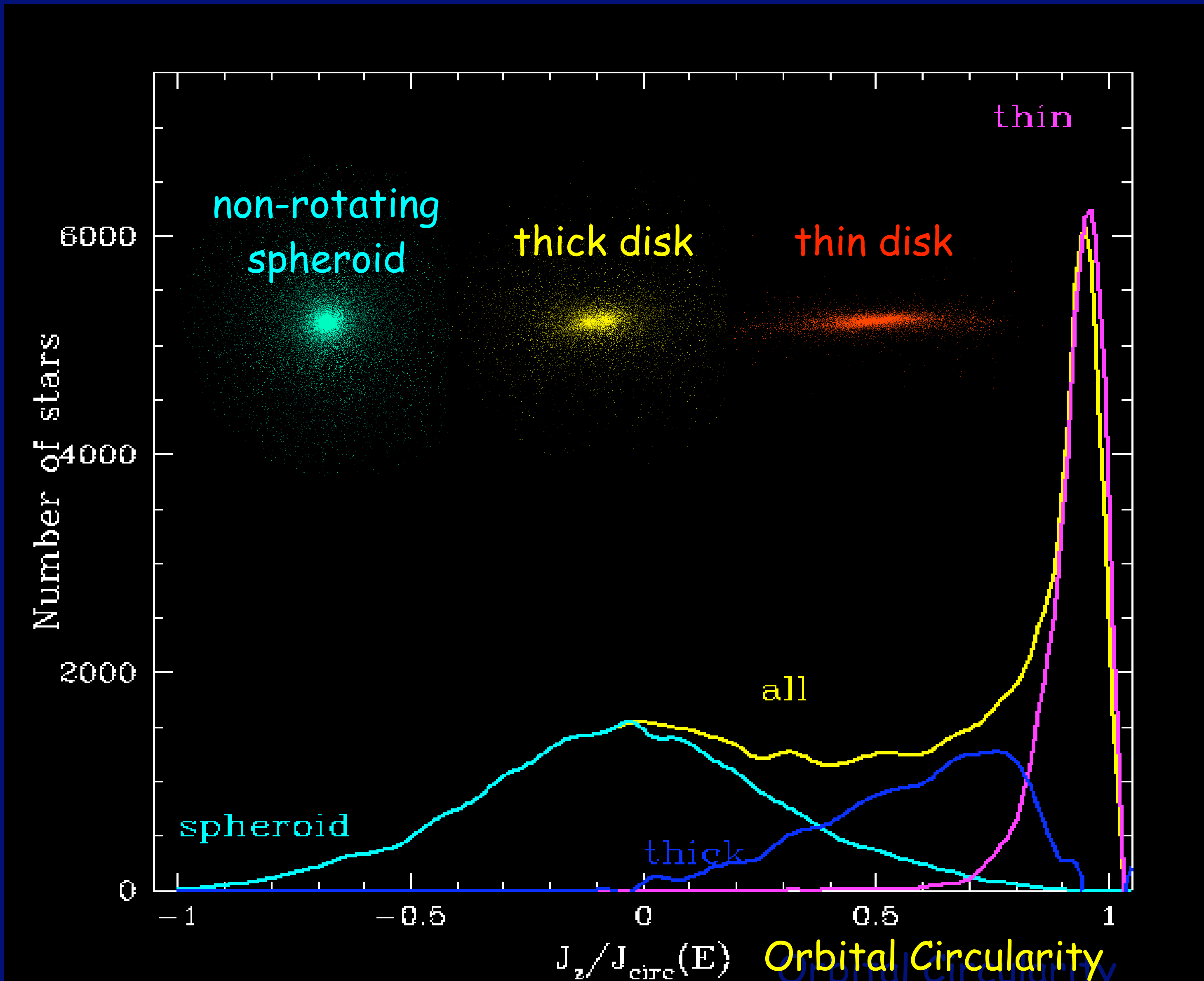
Thin Disk and Thick Disk

Navarro & Steinmetz

Dynamical Components of a Simulated galaxy



Dynamical components of a simulated galaxy



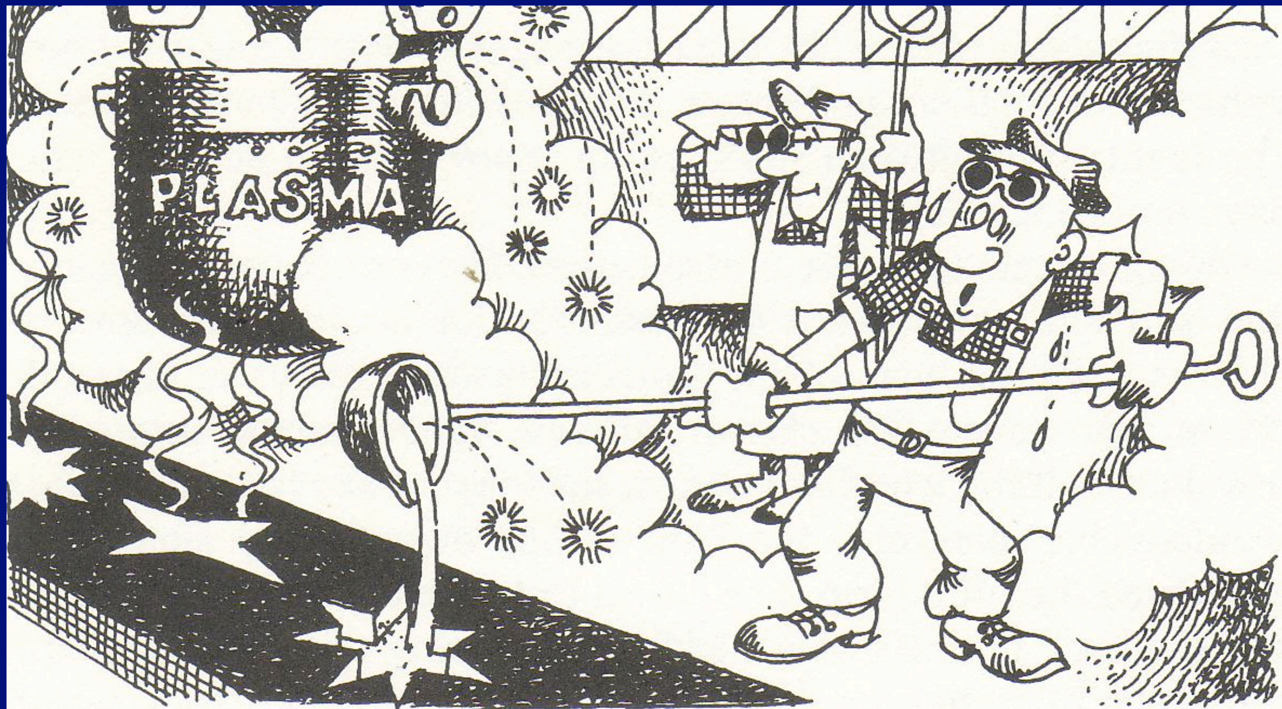
Abadi et al 03

Formation of Thick Disk

Stellar satellite merging with disk: edge-on

Formation of Thick Disk

Stellar satellite merging with disk: face-on


PLASMA

"Great PowerPoint, Kevin, but the answer is no."