Homework Set 2

DUE: Friday February 8

1. The "tired light" hypothesis states that the universe is not expanding, but that photons lose energy per unit distance

$$\frac{dE}{dr} = -KE\,.$$

Show that this hypothesis gives a distance-redshift relation that is linear in the limit $z \ll 1$. What value of K gives a Hubble constant $h = 0.7$? What are some arguments against the "tired light" hypothesis?

2. Short problems:

(a) If a neutrino has mass m_ν and decouples at $T_{\nu d} \sim 1$ MeV, show that the contribution of this neutrino and its antiparticle to the cosmic density today is (Dodelson Eq. 2.80)

$$\Omega_\nu = \frac{m_\nu}{94h^2\mathrm{eV}} \quad .$$

(b) Verify that $\eta_b \equiv n_b/n_\gamma$ is given by (Dodelson Eq. 3.11)

$$\eta_b = 5.5 \times 10^{-10} \left(\frac{\Omega_b h^2}{0.020} \right) \quad .$$

(c) Verify the time-temperature relation (Dodelson Eq. 3.30)

$$t = 132\,\mathrm{sec}\,(0.1\mathrm{MeV}/T)^2 \quad .$$

3. A recent cosmological speculation is that the universe may contain a quantum field called "quintessence" which has an equation of state parameter $w_Q = p_Q/\rho_Q$ with energy density ρ_Q positive (of course) but pressure p_Q negative. Suppose that the universe contains nothing but pressureless matter, i.e. with $w_m = 0$, and quintessence, with $w_Q = -3/4$. The current density parameter of matter is $\Omega_m \approx 0.3$ and that of quintessence is $\Omega_Q = 1 - \Omega_m$. At what scale factor a_{mQ} will the energy density of quintessence and matter be equal? Solve the Friedmann equation to find $a(t)$ for this universe. What is $a(t)$ in the limit $a \gg a_{mQ}$? What is the current age of the universe, expressed in terms of H_0 and $\Omega_{m,0}$?

4. Suppose that the neutron decay time were $\tau_n = 89$ s instead of $\tau_n = 890$ s, with all other physical parameters unchanged. Estimate Y_p, the primordial mass fraction of nucleons in ^{4}He, assuming that all available neutrons are incorporated into ^{4}He.

5. Suppose that there were no baryon asymmetry so that the number density of baryons exactly equaled that of anti-baryons. Determine the final relic density of (baryons + anti-baryons). At what temperature is this relic density reached?

6. There is a fundamental limitation on the annihilation cross section of a particle χ of mass m: because of unitarity, $< \sigma v >$ must be less than or equal to $\sim m^{-2}$, give or take a factor of order unity. Determine Ω_χ for a particle with $< \sigma v >= m^{-2}$. For what value of m is $\Omega_\chi = 1$? (Assume $x_f = 10$ and $g_*(m) = 100$, the nominal values in Dodelson Eq. 3.60.) Note that if m exceeds this value, $\Omega_\chi > 1$, which is ruled out. This is a strong argument against stable particles (and therefore dark matter candidates) with masses above this critical value. (See Griest and Kamionkowski 1990, Phys. Rev. Lett. 64, 615.)

7. Please write the title and a brief description of the term project that you are considering doing for this course. You are encouraged to consult the list of projects handed out in class (also available under Handouts at physics.ucsc.edu/~joel/224.html), but you are welcome to propose topics not on that list. It would be good to discuss project ideas with Joel Primack (office hours Wednesdays 2-3:30 or by appointment) if you have not already done so.