Physics 224 Spring 2008

Origin and Evolution of the Universe

Cosmic Inflation

Lecture 16 - Monday Mar 10

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Outline

L15 WMAP 5-year Data and Papers Released
Grand Unification of Forces
Phase Transitions in the Early Universe
Topological Defects: Strings, Monopoles
L16 Problems Solved by Cosmic Inflation
Simple Models of Cosmic Inflation
Generic Predictions of Inflation
Details on Some Simple Inflation Models

Note: I edited much of the material in the Topological Defects slides from the website http://www.damtp.cam.ac.uk/user/gr/public/cs_top.html

GUT Monopoles

A simple SO(3) GUT illustrates how nonsingular monopoles arise. The Lagragian is

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}D_\mu\Phi^a D^\mu\Phi^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{8}\lambda(\Phi^a\Phi^a - \sigma^2)^2, \\
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - e\varepsilon_{abc}A^b_\mu A^c_\nu, \\
D_\mu\Phi^a &= \partial_\mu\Phi^a - e\varepsilon_{abc}A^b_\mu\Phi^c.
\end{aligned}$$

The masses of the resulting charged vector and Higgs bosons after spontaneous symmetry breaking are

$$\begin{aligned}
M_V^2 &= e^2\sigma^2, \\
M_S^2 &= \lambda\sigma^2.
\end{aligned}$$

If the Higgs field Φ^a happens to rotate about a sphere in SO(3) space as one moves around a sphere about any particular point in **x**-space, then it must vanish at the particular point. Remarkably, if we identify the massless vector field as the photon, this configuration corresponds to a nonsingular magnetic monopole, as was independently discovered by 'tHooft and Polyakov. The monopole has magnetic charge twice the minimum Dirac value, $g = 2\pi/e = (4\pi/e^2)(e/2) \approx 67.5$ e. The singular magnetic field is cut off at scale σ, and as a result the GUT monopole has mass $M_{\text{monopole}} \approx M_V/\alpha \approx M_{GUT}/\alpha \approx 10^{18}$ GeV, which is about $0.5\text{x}10^{16}$ times the mass of a gold atom!

GUT Monopole Problem

The Kibble mechanism produces ~ one GUT monopole per horizon volume when the GUT phase transition occurs. These GUT monopoles have a number density over entropy

$$n_M/s \sim 10^2 (T_{GUT}/M_{Pl})^3 \sim 10^{-13}$$

(compared to $n_B/s \sim 10^{-9}$ for baryons) Their annihilation is inefficient since they are so massive, and as a result they are about as abundant as gold atoms but 10^{16} times more massive, so they "overclose" the universe. This catastrophe must be avoided! **This was Alan Guth's initial motivation for inventing cosmic inflation.**

Inflation

I will summarize the key ideas of inflation theory, following my lectures at the Jerusalem Winter School, published as the first chapter in Avishai Dekel & Jeremiah Ostriker, eds., *Formation of Structure in the Universe* (Cambridge University Press, 1999), and Dierck-Ekkehard Liebscher, *Cosmology* (Springer, 2005) (available electronically through the UCSC library).

Motivations for Inflation

	PROBLEM SOLVED
Horizon	Homogeneity, Isotropy, Uniform T
Flatness/Age	Expansion and gravity balance
“Dragons”	Monopoles, doman walls,…banished
Structure	Small fluctuations to evolve into galaxies, clusters, voids

Cosmological constant $\Lambda > 0 \Rightarrow$ space repels space, so the more space the more repulsion, $\Rightarrow$ de Sitter exponential expansion $a \propto e^{\sqrt{\Lambda}t}$.

Inflation is exponentially accelerating expansion caused by effective cosmological constant (“false vacuum” energy) associated with hypothetical scalar field (“inflaton”).

	FORCES OF NATURE	**Spin**
Known	Gravity	2
	Strong, weak, and electromagnetic	1
Goal of LHC	Mass (Higgs Boson)	0
Early universe	Inflation (Inflaton)	0

Inflation lasting only $\sim 10^{-32}$s suffices to solve all the problems listed above. Universe must then convert to ordinary expansion through conversion of false to true vacuum (“re-”heating).

Horizons

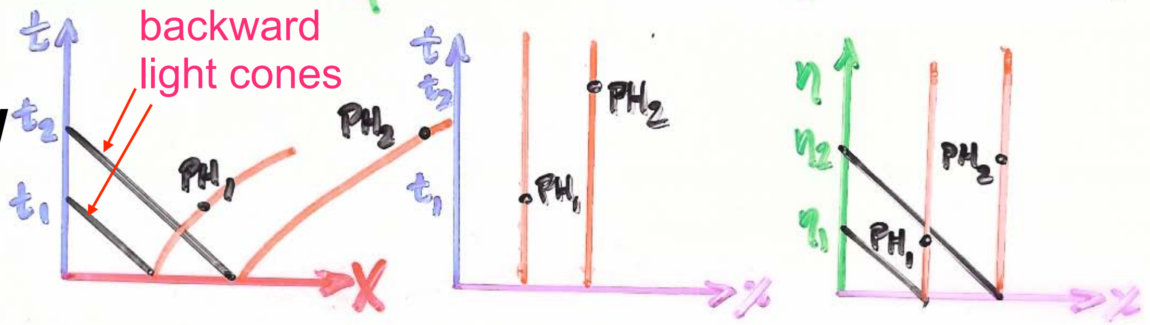
$$ds^2 = dt^2 - dX^2 = dt^2 - R^2 dx^2 = R^2(d\eta^2 - dx^2)$$

conformal time $d\eta = dt/R$ comoving coord. $dx = dX/R$

PARTICLE HORIZON

Spherical surface that at time t separates ***worldlines*** into observed vs. unobserved

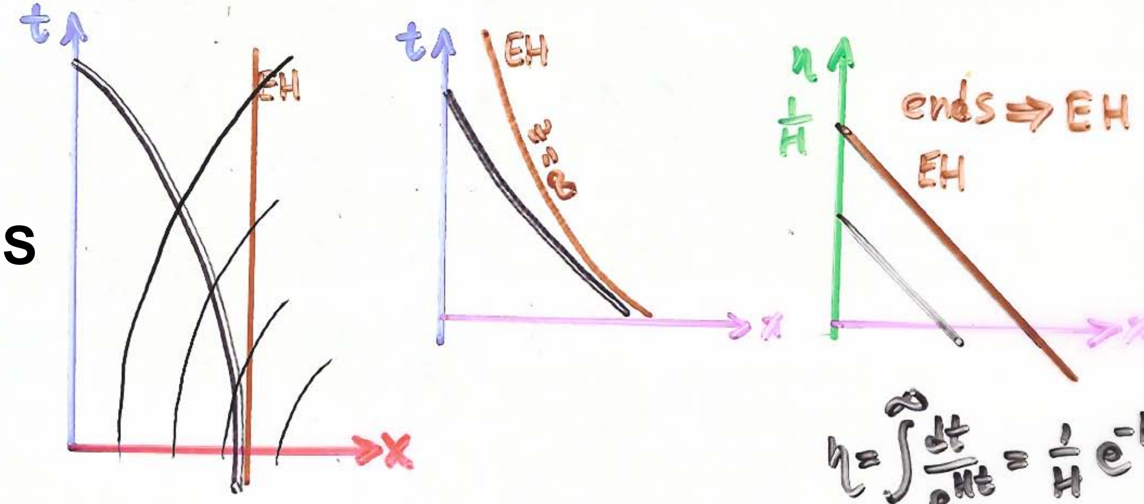
FRW


backward light cones

EVENT HORIZON

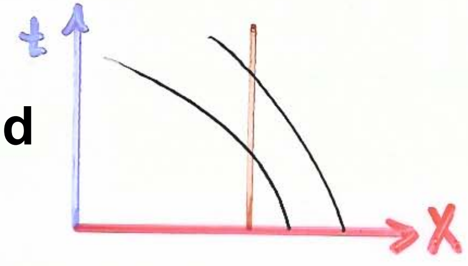
Backward lightcone that separates ***events*** that will someday be observed from those never observed

deS



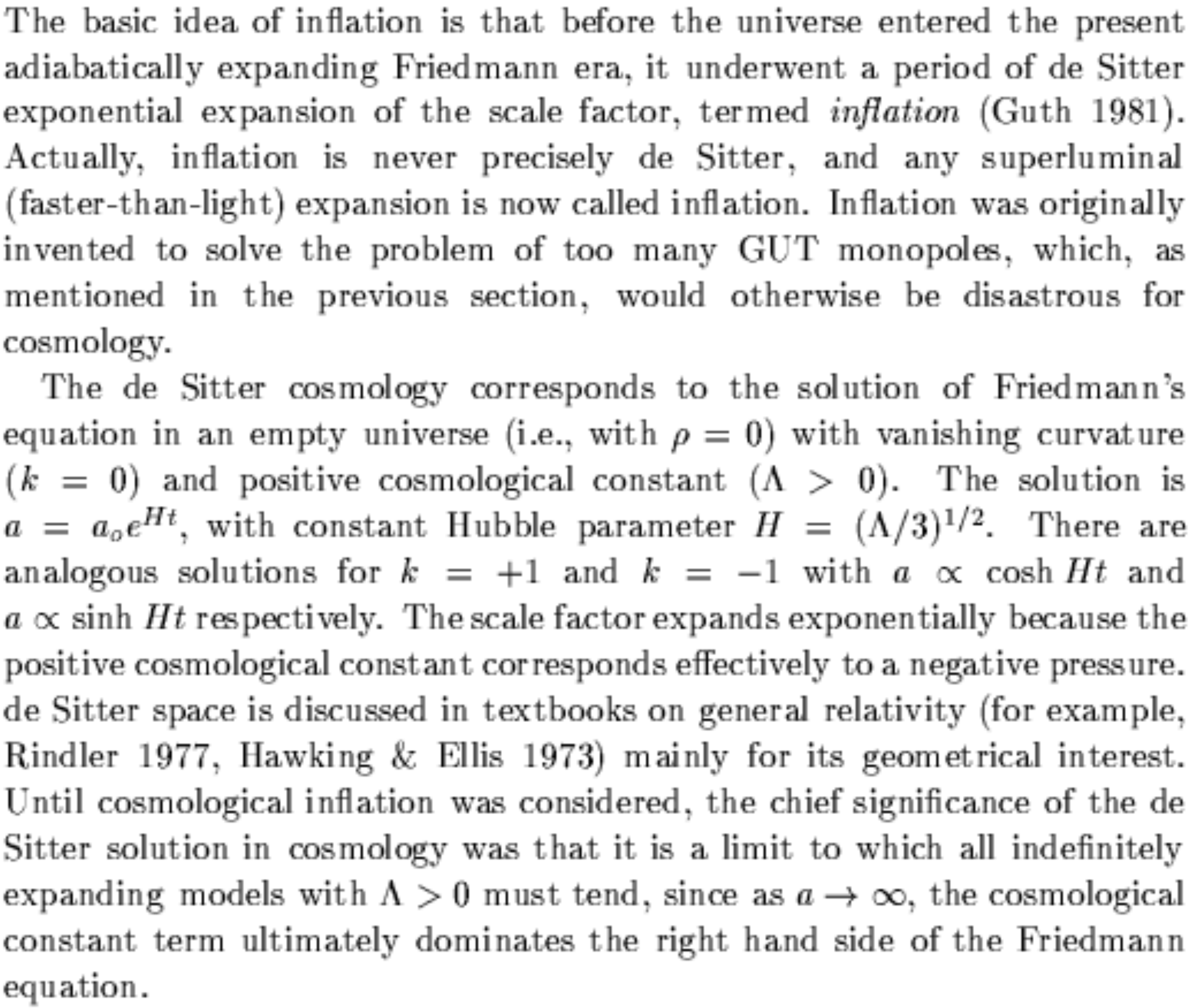
$$\eta = \int_{t_0}^{\infty} \frac{dt}{e^{Ht}} = \frac{1}{H} e^{-Ht_0} \leq \frac{1}{H}$$

Schwarzschild



See Harrison, *Cosmology*
Rindler, *Relativity*

Inflation Basics



The basic idea of inflation is that before the universe entered the present adiabatically expanding Friedmann era, it underwent a period of de Sitter exponential expansion of the scale factor, termed *inflation* (Guth 1981). Actually, inflation is never precisely de Sitter, and any superluminal (faster-than-light) expansion is now called inflation. Inflation was originally invented to solve the problem of too many GUT monopoles, which, as mentioned in the previous section, would otherwise be disastrous for cosmology.

The de Sitter cosmology corresponds to the solution of Friedmann's equation in an empty universe (i.e., with $\rho = 0$) with vanishing curvature ($k = 0$) and positive cosmological constant ($\Lambda > 0$). The solution is $a = a_o e^{Ht}$, with constant Hubble parameter $H = (\Lambda/3)^{1/2}$. There are analogous solutions for $k = +1$ and $k = -1$ with $a \propto \cosh Ht$ and $a \propto \sinh Ht$ respectively. The scale factor expands exponentially because the positive cosmological constant corresponds effectively to a negative pressure. de Sitter space is discussed in textbooks on general relativity (for example, Rindler 1977, Hawking & Ellis 1973) mainly for its geometrical interest. Until cosmological inflation was considered, the chief significance of the de Sitter solution in cosmology was that it is a limit to which all indefinitely expanding models with $\Lambda > 0$ must tend, since as $a \to \infty$, the cosmological constant term ultimately dominates the right hand side of the Friedmann equation.

As Guth (1981) emphasized, the de Sitter solution might also have been important in the very early universe because the vacuum energy that plays such an important role in spontaneously broken gauge theories also acts as an effective cosmological constant. A period of de Sitter inflation preceding ordinary radiation-dominated Friedmann expansion could explain several features of the observed universe that otherwise appear to require very special initial conditions: the horizon, flatness/age, monopole, and structure formation problems. (See Table 1.6.)

Let us illustrate how inflation can help with the horizon problem. At recombination ($p^+ + e^- \rightarrow H$), which occurs at $a/a_o \approx 10^{-3}$, the mass encompassed by the horizon was $M_H \approx 10^{18} M_\odot$, compared to $M_{H,o} \approx 10^{22} M_\odot$ today. Equivalently, the angular size today of the causally connected regions at recombination is only $\Delta\theta \sim 3°$. Yet the fluctuation in temperature of the cosmic background radiation from different regions is very small: $\Delta T/T \sim 10^{-5}$. How could regions far out of causal contact have come to temperatures that are so precisely equal? This is the "horizon problem". With inflation, it is no problem because the entire observable universe initially lay inside a single causally connected region that subsequently inflated to a gigantic scale. Similarly, inflation exponentially dilutes any preceeding density of monopoles or other unwanted relics (a modern version of the "dragons" that decorated the unexplored borders of old maps).

In the first inflationary models, the dynamics of the very early universe was typically controlled by the self-energy of the Higgs field associated with the breaking of a Grand Unified Theory (GUT) into the standard 3-2-1 model: GUT$\rightarrow SU(3)_{color} \otimes [SU(2) \otimes U(1)]_{electroweak}$. This occurs when the cosmological temperature drops to the unification scale $T_{GUT} \sim 10^{14}$ GeV at about 10^{-35} s after the Big Bang. Guth (1981) initially considered a scheme in which inflation occurs while the universe is trapped in an unstable state (with the GUT unbroken) on the wrong side of a maximum in the Higgs potential. This turns out not to work: the transition from a de Sitter to a Friedmann universe never finishes (Guth & Weinberg 1981). The solution in the "new inflation" scheme (Linde 1982; Albrecht and Steinhardt 1982) is for inflation to occur *after* barrier penetration (if any). It is necessary that the potential of the scalar field controlling inflation ("*inflaton*") be nearly flat (i.e., decrease very slowly with increasing inflaton field) for the inflationary period to last long enough. This nearly flat part of the potential must then be followed by a very steep minimum, in order that the energy contained in the Higgs potential be rapidly shared with the other degrees of freedom ("reheating"). A more general approach, "chaotic" inflation, has been worked out by Linde (1983, 1990) and others; this works for a wide range of inflationary potentials, including simple power laws such as $\lambda\phi^4$. However, for the amplitude of the fluctuations to be small enough for consistency with observations, it is necessary that the inflaton self-coupling be very small, for example $\lambda \sim 10^{-14}$ for the ϕ^4 model. This requirement prevents a Higgs field from being the inflaton, since Higgs fields by definition have gauge couplings to the gauge field (which are expected to be of order unity), and these would generate self-couplings of similar magnitude even if none were present.

It turns out to be necessary to inflate by a factor $\gtrsim e^{66}$ in order to solve the flatness problem, i.e., that $\Omega_0 \sim 1$. (With $H^{-1} \sim 10^{-34}$ s during the de Sitter phase, this implies that the inflationary period needs to last for only a relatively small time $\tau \gtrsim 10^{-32}$ s.) The "flatness problem" is essentially the question why the universe did not become curvature dominated long ago. Neglecting the cosmological constant on the assumption that it is unimportant after the inflationary epoch, the Friedmann equation can be written

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\pi^2}{30}g(T)T^4 - \frac{kT^2}{(aT)^2}$$

where the first term on the right hand side is the contribution of the energy density in relativistic particles and $g(T)$ is the effective number of degrees of freedom. The second term on the right hand side is the curvature term. Since $aT \approx$ constant for adiabatic expansion, it is clear that as the temperature T drops, the curvature term becomes increasingly important. The quantity $K \equiv k/(aT)^2$ is a dimensionless measure of the curvature. Today, $|K| = |\Omega - 1|\, H_o^2/T_o^2 \leq 2 \times 10^{-58}$. Unless the curvature exactly vanishes, the most "natural" value for K is perhaps $K \sim 1$. Since inflation increases a by a tremendous factor $e^{H\tau}$ at essentially constant T (after reheating), it increases aT by the same tremendous factor and thereby decreases the curvature by that factor squared. Setting $e^{-2H\tau} \lesssim 2{\times}10^{-58}$ gives the needed amount of inflation: $H\tau \gtrsim 66$. This much inflation turns out to be enough to take care of the other cosmological problems mentioned above as well.

Inflationary Fluctuations

Thus far, it has been sketched how inflation stretches, flattens, and smooths out the universe, thus greatly increasing the domain of initial conditions that could correspond to the universe that we observe today. But inflation also can explain the origin of the fluctuations necessary in the gravitional instability picture of galaxy and cluster formation. Recall that the very existence of these fluctuations is a problem in the standard Big Bang picture, since these fluctuations are much larger than the horizon at early times. How could they have arisen?

The answer in the inflationary universe scenario is that they arise from quantum fluctuations in the inflaton field ϕ whose vacuum energy drives inflation. The scalar fluctuations $\delta\phi$ during the de Sitter phase are of the order of the Hawking temperature $H/2\pi$. Because of these fluctuations, there is a time spread $\Delta t \approx \delta\phi/\dot{\phi}$ during which different regions of the same size complete the transition to the Friedmann phase. The result is that the density fluctuations when a region of a particular size re-enters the horizon are equal to (Guth & Pi 1982; see Linde 1990 for alternative approaches) $\delta_H \equiv (\delta\rho/\rho)_H \sim \Delta t/t_H = H\Delta t$. The time spread Δt can be estimated from the equation of motion of ϕ (the free Klein-Gordon equation in an expanding universe): $\ddot{\phi} + 3H\dot{\phi} = -(\partial V/\partial\phi)$. Neglecting the $\ddot{\phi}$ term, since the scalar potential V must be very flat in order for enough inflation to occur (this is called the "slow roll" approximation), $\dot{\phi} \approx -V'/(3H)$, so $\delta_H \sim H^3/V' \sim V^{3/2}/V'$. Unless there is a special feature in the potential $V(\phi)$ as ϕ rolls through the scales of importance in cosmology (producing such "designer inflation" features generally requires fine tuning — see e.g. Hodges et al. 1990), V and V' will hardly vary there and hence δ_H will be essentially constant. These are fluctuations of all the contents of the universe, so they are adiabatic fluctuations.

Thus *inflationary models typically predict a nearly constant curvature spectrum* $\delta_H =$ constant *of adiabatic fluctuations*. Some time ago Harrison (1970), Zel'dovich (1972), and others had emphasized that this is the only scale-invariant (i.e., power-law) fluctuation spectrum that avoids trouble at both large and small scales. If $\delta_H \propto M_H^{-\alpha}$, where M_H is the mass inside the horizon, then if $-\alpha$ is too large the universe will be less homogeneous on large than small scales, contrary to observation; and if α is too large, fluctuations on sufficiently small scales will enter the horizon with $\delta_H \gg 1$ and collapse to black holes (see e.g. Carr, Gilbert, & Lidsey 1995, Bullock & Primack 1996); thus $\alpha \approx 0$. The $\alpha = 0$ case has come to be known as the Zel'dovich spectrum.

Inflation predicts more: it allows the calculation of the value of the constant δ_H in terms of the properties of the scalar potential $V(\phi)$. Indeed, this proved to be embarrassing, at least initially, since the Coleman-Weinberg potential, the first potential studied in the context of the new inflation scenario, results in $\delta_H \sim 10^2$ (Guth & Pi 1982) some six orders of magnitude too large. But this does not seem to be an insurmountable difficulty; as was mentioned above, chaotic inflation works, with a sufficiently small self-coupling. Thus inflation at present appears to be a plausible solution to the problem of providing reasonable cosmological initial conditions (although it sheds no light at all on the fundamental question why the cosmological constant is so small now). Many variations of the basic idea of inflation have been worked out

Many Inflation Models

HOW INFLATION BEGINS

Old Inflation T_{initial} high, $\phi_{\text{in}} \approx 0$ is false vacuum until phase transition
Ends by bubble creation; Reheat by bubble collisions

New Inflation Slow roll down $V(\phi)$, no phase transition

Chaotic Inflation Similar to New Inflation, but ϕ_{in} essentially arbitrary:
any region with $\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\partial_i\phi)^2 \lesssim V(\phi)$ inflates

Extended Inflation Like Old Inflation, but slower (e.g., power $a \propto t^p$),
so phase transition can finish

POTENTIAL $V(\phi)$ DURING INFLATION

Chaotic typically $V(\phi) = \Lambda\phi^n$, can also use $V = V_0 e^{\alpha\phi}$, etc.

$$\Rightarrow a \propto t^p, \; p = 16\pi/\alpha^2 \gg 1$$

HOW INFLATION ENDS

First-order phase transition — e.g., Old or Extended inflation
Faster rolling → oscillation — e.g., Chaotic $V(\phi)^2\Lambda\phi^n$
"Waterfall" — rapid roll of σ triggered by slow roll of ϕ

(RE)HEATING

Decay of inflatons
"Preheating" by parametric resonance, then decay

BEFORE INFLATION?

Eternal Inflation? Can be caused by

- Quantum $\delta\phi \sim H/2\pi >$ rolling $\Delta\phi = \phi\Delta t = \phi H^{-1} \approx V'/V$
- Monopoles or other topological defects

Inflaton Theory in More Detail

Action of gravity + scalar inflaton field:

$$S = -\frac{c^3}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-\det\, g_{mn}}\; R$$
$$+\int \mathrm{d}^4x\,\sqrt{-\det\, g_{mn}}\;\hbar\;\left(\frac{1}{2}\frac{\partial\phi}{\partial x^i}\frac{\partial\phi}{\partial x^k}g^{ik} - V[\phi]\right)$$

The simplest V is just quadratic $V[\phi] = \frac{1}{2}\frac{m^2c^2}{\hbar^2}\phi^2$

which just gives the inflaton field a mass m. The model of symmetry breakdown requires a more complicated potential V [φ]. It must contain degenerate minima that allow ground states with φ = 0. In such a ground state, the mass is defined for small perturbations by

$$m^2 = \frac{\hbar^2}{c^2}\frac{\mathrm{d}^2V}{\mathrm{d}\phi^2}\;.$$

The energy–momentum tensor is given by

$$T_{ik} = \hbar c \left(\frac{\partial \phi}{\partial x^i} \frac{\partial \phi}{\partial x^k} - g_{ik} \left(\frac{1}{2} g^{lm} \frac{\partial \phi}{\partial x^l} \frac{\partial \phi}{\partial x^m} - V[\phi] \right) \right)$$

which implies that the energy density and pressure are given by

$$\varepsilon = \hbar c \left(V + \frac{1}{2c^2} \dot{\phi}^2 + \frac{1}{2} \frac{1}{a^2[t]} (\nabla \phi)^2 \right)$$

and

$$p = \hbar c \left(-V + \frac{1}{2c^2} \dot{\phi}^2 - \frac{1}{6} \frac{1}{a^2[t]} (\nabla \phi)^2 \right) .$$

Thus a scalar field with a nearly constant potential V corresponds to

$$\varrho c^2 = \varepsilon = -p \ (= \hbar c V[\phi]).$$

Since w = p/ε = -1, this is effectively a cosmological constant. More generally, a scalar field that is not at the minimum of its potential generates generates “*dark energy*”.

The field equation for the inflaton in expanding space is

$$\frac{\partial^2 \phi}{c^2 \partial t^2} - \frac{1}{a^2}\nabla^2\phi + \frac{3\dot{a}\dot{\phi}}{c^2 a} + \frac{\mathrm{d}V}{\mathrm{d}\phi} \quad (+3H\Gamma\dot{\phi}^2) = 0 \, .$$

This becomes the following equation if the spatial variations of ϕ (and the last term) can be neglected

$$\ddot{\phi} + 3H[t]\dot{\phi} = -c^2 \frac{\mathrm{d}V[\phi]}{\mathrm{d}\phi} \, .$$

This equation must be solved along with the Einstein equations:

$$H^2 = \frac{8\pi G}{3}\frac{\hbar}{c}\left(V + \frac{1}{2c^2}\dot{\phi}^2\right) \quad \text{and } \dot{H} = -4\pi\frac{\hbar G}{c^3}\dot{\phi}^2$$

With a suitably chosen potential V, the inflaton will quickly reach its ground state and inflation will end. The term in parenthesis allows the inflaton to decay into other fields at the end of inflation, thus *reheating* the universe.

The last equation leads to

$$H' = \frac{\mathrm{d}H[\phi]}{\mathrm{d}\phi} = -4\pi\frac{\hbar G}{c^3}\dot{\phi}$$

which allows us to write the Friedmann equation as

$$\left(\frac{\mathrm{d}H}{\mathrm{d}\phi}\right)^2 = 12\pi\frac{\hbar G}{c^3}H^2 - 32\pi^2\frac{\hbar^2 G^2}{c^4}V[\phi]\ .$$

When the inflaton is rolling slowly, the evolution of the inflaton is governed by

$$\dot{\phi} = -\frac{c^2}{3H}\frac{\mathrm{d}V}{\mathrm{d}\phi}\ , \quad H^2 = \frac{8\pi\hbar G}{3c}V\ .$$

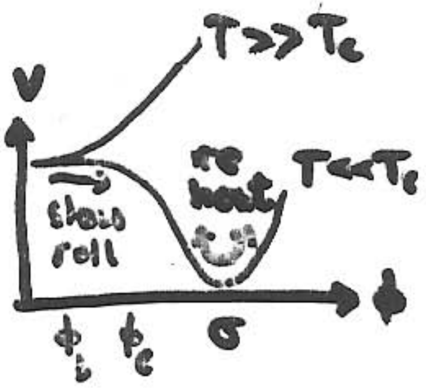
Then the number N of e-folds of the scale factor a is given by

$$N = \ln\frac{a}{a_1} = \int_{t_1}^{t} H\,\mathrm{d}t = \int_{\phi_1}^{\phi}\mathrm{d}\phi\frac{H}{\dot{\phi}} = 4\pi\frac{\hbar G}{c^3}\int_{\phi}^{\phi_1}\mathrm{d}\phi\frac{H}{H'} \approx 8\pi\frac{\hbar G}{c^3}\int_{\phi}^{\phi_1}\mathrm{d}\phi\frac{V}{V'}\ .$$

Inflationary Models in More Detail

PROTOTYPE MODEL FOR NEW INFLATION

$$V = \lambda(\phi^2 - \sigma^2)^2$$



Assume that in some region R_S, $\phi \approx 0$. Transition to $\phi \to \sigma$ is governed by KG eqn $\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} - \frac{1}{R^2}\cancel{\nabla^2\phi} = -V'$

neglect

where $H^2 = \frac{8\pi}{3m_{Pl}^2}\rho$, $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_R$, $p = \frac{1}{2}\dot{\phi}^2 - V(\phi) + \frac{1}{3}\rho_R$

If $V(\phi)$ is large and flat enough, $V(\phi)$ will be $\gg \dot{\phi}^2$ and ρ_R, $\ddot{\phi}$ will be $\ll 3H\dot{\phi}$, and

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2 \approx \frac{8\pi V}{3m_{Pl}^2} \Rightarrow R = R_0 e^N, \quad N = \int H dt$$

For example, take $\sigma = 10^{14}$ GeV, $\lambda = 1$, $R_i = H^{-1} = \frac{M_{pl}}{\sigma^2} = \frac{10^{19}\,\text{GeV}}{(10^{14}\,\text{GeV})^2}$

$= R_i = 10^{-9}\,\text{GeV}^{-1} = 10^{-23}$ cm, $\Rightarrow S_i = (R_i T_i)^3 = \left(\frac{m_{pl}}{\sigma^2}\sigma\right)^3 = 10^{15}$

Then $S_F = (R_f T_{RH})^3 = (e^N R_i T_{RH})^3$

If $N = 100$, $e^{300} = 10^{130}$, & this gives $S_F = 10^{145}$

Requiring $e^{3N}\frac{m_{pl}}{\sigma} = S_F \geq 10^{88} \Rightarrow 3N \geq \underbrace{\ln 10^{88}}_{202.6} + \ln \sigma/m_{pl}$

$\Rightarrow N \geq 67.5 - 9.2 + \ln \sigma/10^{15}\,\text{GeV} = 58 + \ln \sigma/10^{15}\,\text{GeV}$

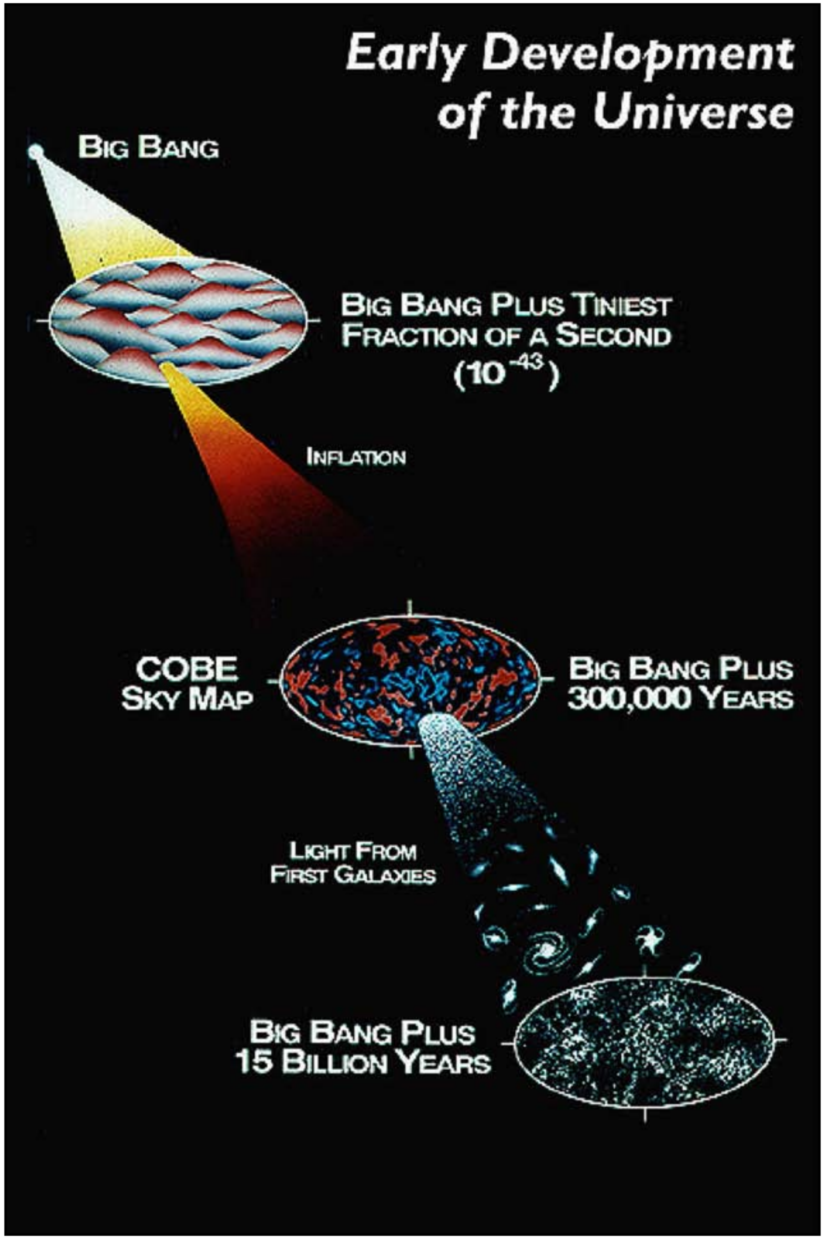
Solves: Horizon

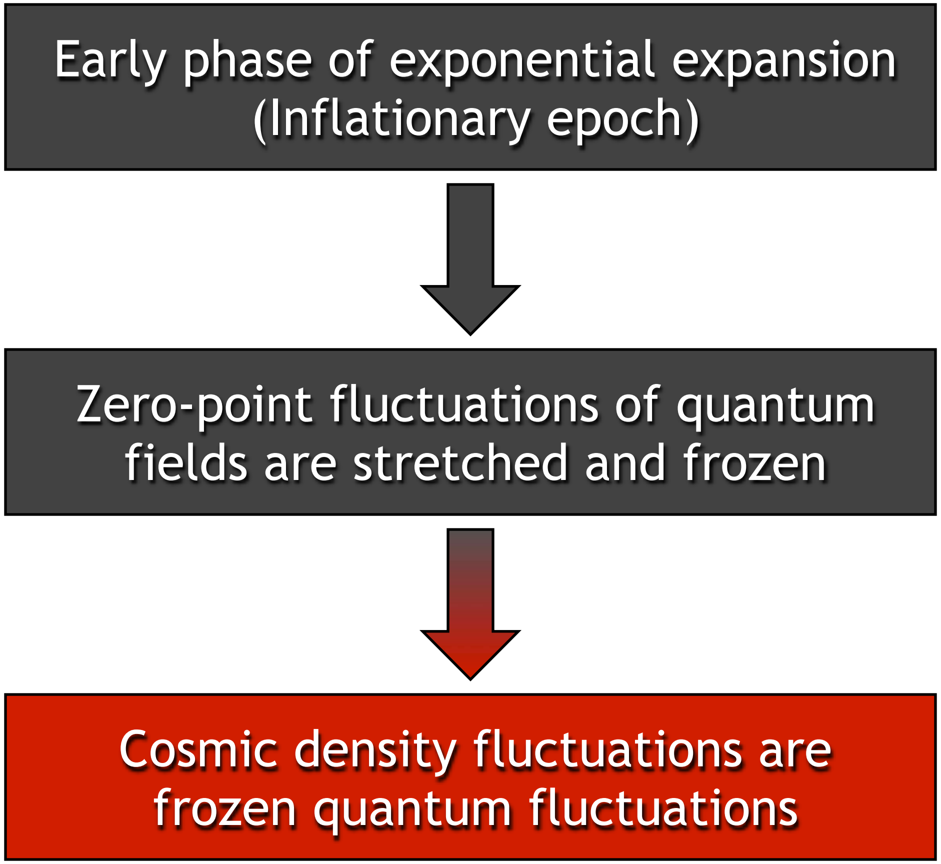
Flatness / Age: k/R^2 decreases by e^{-2N}

Relics: density $\sim R^{-3}$ " " e^{-3N}

Fluctuations?

Generating the Primordial Density Fluctuations



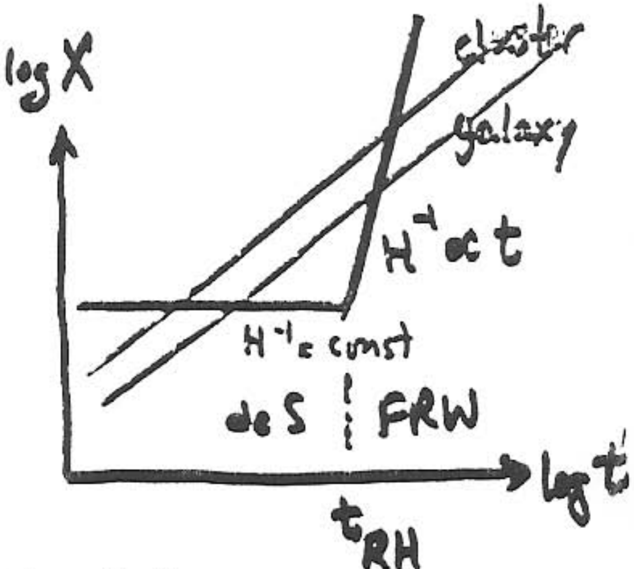


Early phase of exponential expansion
(Inflationary epoch)

Zero-point fluctuations of quantum fields are stretched and frozen

Cosmic density fluctuations are frozen quantum fluctuations

FLUCTUATIONS IN INFLATION



LOFI last scales to cross outside horizon
in de S
are first to cross inside in FRW

If present horizon crossed outside 60 e-folds before end of inflation, galaxies crossed 52 efolds before end, and any mass M_H crossed at $60 + \frac{1}{3} \ln M_H / 10^{22} M_\odot = N_\ell$.

There are quantum fluctuations in a deSitter universe corresponding to the Hawking radiation temperature $T_H = \frac{H}{2\pi}$: $\langle (\Delta\phi)^2 \rangle = \left(\frac{H}{2\pi}\right)^2$

[Guth + Pi: PRL $\underline{49}$, 1110 (1982) got the same answer using de S Green's fcns.]

These lead to density fluctuations $\delta\rho = \frac{\partial V}{\partial \phi} \Delta\phi = -3H\dot\phi\Delta\phi$ slow roll!
$V' = -3H\dot\phi$

In $\underline{\text{de S}}$ phase, $\rho = \frac{1}{2}\dot\phi^2 + V(\phi) + \underset{\text{negligible}}{\rho_R}$, $p = \frac{1}{2}\dot\phi^2 - V(\phi) + \underset{\text{negligible}}{\frac{1}{3}\rho_R}$.

Bardeen's gauge invariant parameter $\zeta = \frac{\delta\rho}{\rho + p}$ is constant outside horizon.

deS: $\zeta = \frac{\delta\rho}{\dot\phi^2} = \frac{V'\Delta\phi}{\dot\phi^2} = \frac{-3H\dot\phi\Delta\phi}{\dot\phi^2} = -\frac{3H^2}{2\pi\dot\phi}$

$\quad\;\; \|$

FRW: $\zeta = \frac{\delta\rho}{\frac{4}{3}\rho} \Rightarrow \left(\frac{\delta\rho}{\rho}\right)_H = \frac{-2H^2}{\pi\dot\phi} = \frac{6H^3}{\pi V'(\phi)}$

when given scale crosses inside H in FRW era

when scale crossed outside H in dS era

CHAOTIC INFLATION $\quad V(\phi) = \lambda\phi^4 \quad \phi$ = inflaton $\quad$ Linde 1983

initial condition: ϕ large and (in inflating patch) homogeneous $\Rightarrow$

$$N_\lambda(\phi\to 0) = \int H\,dt = \int_\phi^0 H\dot\phi^{-1}dt = \int_\phi^0 \frac{-3\overset{\frac{8}{3}\pi G\lambda\phi^4}{H^2}}{4\lambda\phi^3}\,d\phi = \int_0^\phi \frac{2\pi}{m_{Pl}^2}\phi\,d\phi = \pi\left(\frac{\phi}{m_{Pl}}\right)^2$$

$$\Rightarrow \left(\frac{\delta\rho}{\rho}\right)_H = \frac{6H^3}{\pi V'(\phi)} = \frac{6}{\pi}\left(\frac{8\pi}{3m_{Pl}^2}\lambda\phi^4\right)^{3/2}\frac{1}{4\lambda\phi^3} = 11.6\,\lambda^{1/2}\left(\frac{\phi}{m_{Pl}}\right)^3 = 2.1\,\lambda^{1/2}N_\lambda^{3/2}$$

$\therefore \left(\frac{\delta\rho}{\rho}\right)_H \approx 10^{-4}$ and $N_\lambda \approx 50$ for galaxies $\Rightarrow \underline{\lambda \simeq 10^{-12}}$

$N_{tot} = N_\lambda(\phi_i \to 0) > 60 \Rightarrow \frac{\phi_i}{m_{Pl}} > \left(\frac{60}{\pi}\right)^{1/2} = 4.3$ (Is classical gravity ok?)

Again, T_{RH} depends on ϕ coupling to other fields, & is low but b. asym. could be generated by decay.

Eternal Inflation

Vilenkin (1983) and Linde (1986, 1990) pointed out that if one extrapolates inflation backward to try to imagine what might have preceeded it, in many versions of inflation the answer is "eternal inflation": in most of the volume of the universe inflation is still happening, and our part of the expanding universe (a region encompassing far more than our entire cosmic horizon) arose from a tiny part of such a region. To see how eternal inflation works, consider the simple chaotic model with $V(\phi) = (m^2/2)\phi^2$. During the de Sitter Hubble time H^{-1}, where as usual $H^2 = (8\pi G/3)V$, the slow rolling of ϕ down the potential will reduce it by

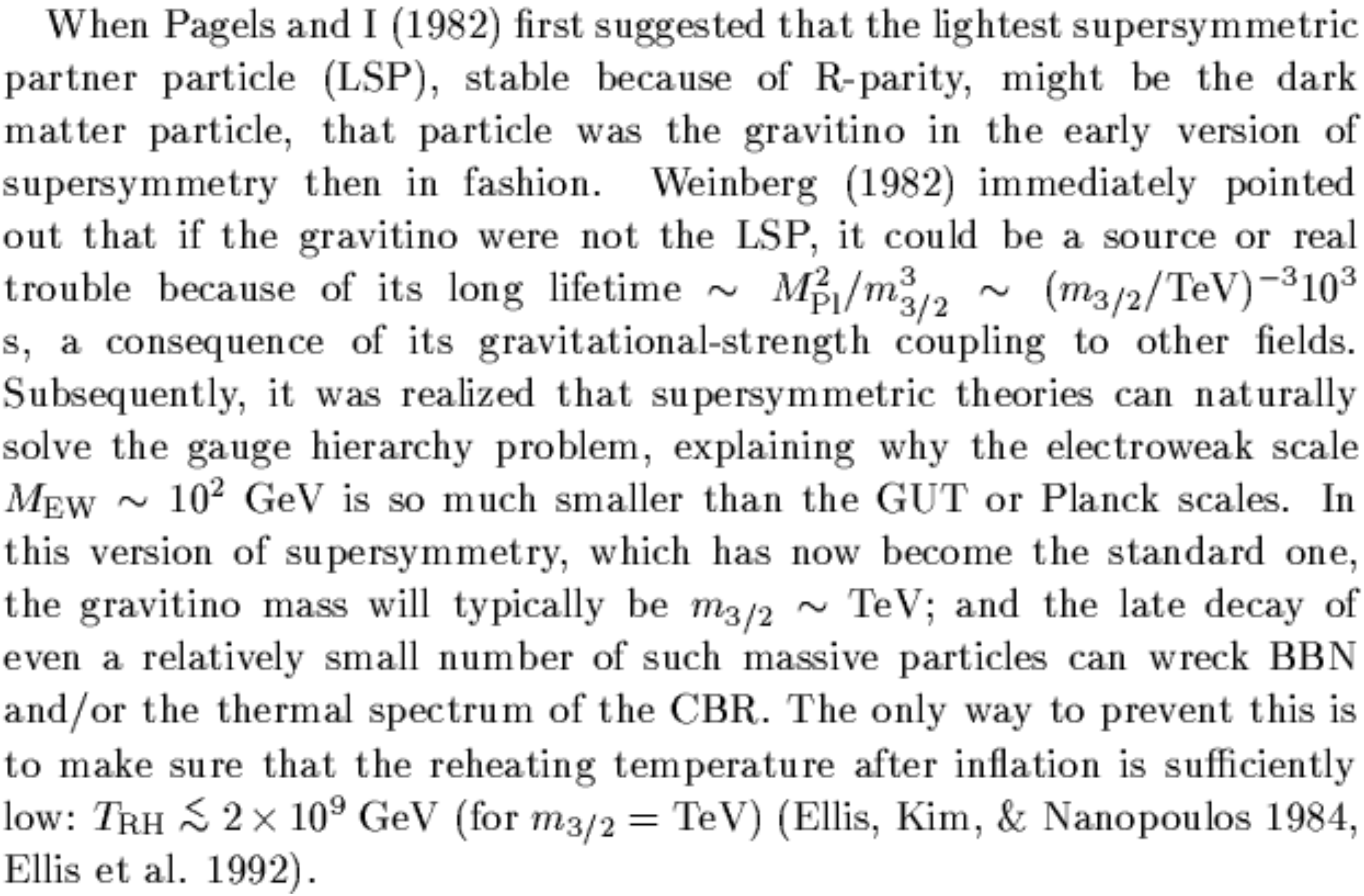
$$\Delta\phi = \dot{\phi}\Delta t = -\frac{V'}{3H}\Delta t = \frac{m_{pl}^2}{4\pi\phi}. \qquad (1.7)$$

Here m_{Pl} is the Planck mass ($m_{Planck} = 1/G^{1/2}$). But there will also be quantum fluctuations that will change ϕ up or down by

$$\delta\phi = \frac{H}{2\pi} = \frac{m\phi}{\sqrt{3\pi}m_{Pl}} \qquad (1.8)$$

These will be equal for $\phi_* = m_{pl}^{3/2}/2m^{1/2}$, $V(\phi_*) = (m/8m_{Pl})m_{Pl}^4$. If $\phi \gtrsim \phi_*$, *positive quantum fluctuations dominate* the evolution: after $\Delta t \sim H^{-1}$, an initial region becomes $\sim e^3$ regions of size $\sim H^{-1}$, in half of which ϕ increases to $\phi + \delta\phi$. Since $H \propto \phi$, this drives inflation faster in these regions.

Supersymmetric Inflation



When Pagels and I (1982) first suggested that the lightest supersymmetric partner particle (LSP), stable because of R-parity, might be the dark matter particle, that particle was the gravitino in the early version of supersymmetry then in fashion. Weinberg (1982) immediately pointed out that if the gravitino were not the LSP, it could be a source or real trouble because of its long lifetime $\sim M_{\rm Pl}^2/m_{3/2}^3 \sim (m_{3/2}/{\rm TeV})^{-3}10^3$ s, a consequence of its gravitational-strength coupling to other fields. Subsequently, it was realized that supersymmetric theories can naturally solve the gauge hierarchy problem, explaining why the electroweak scale $M_{\rm EW} \sim 10^2$ GeV is so much smaller than the GUT or Planck scales. In this version of supersymmetry, which has now become the standard one, the gravitino mass will typically be $m_{3/2} \sim$ TeV; and the late decay of even a relatively small number of such massive particles can wreck BBN and/or the thermal spectrum of the CBR. The only way to prevent this is to make sure that the reheating temperature after inflation is sufficiently low: $T_{\rm RH} \lesssim 2 \times 10^9$ GeV (for $m_{3/2} =$ TeV) (Ellis, Kim, & Nanopoulos 1984, Ellis et al. 1992).

Basic Predictions of Inflation

1. **Flat universe.** This is perhaps the most fundamental prediction of inflation. Through the Friedmann equation it implies that the total energy density is always equal to the critical energy density; it does not however predict the form (or forms) that the critical density takes on today or at any earlier or later epoch.

2. **Nearly scale-invariant spectrum of Gaussian density perturbations.** These density perturbations (scalar metric perturbations) arise from quantum-mechanical fuctuations in the field that drives inflation; they begin on very tiny scales (of the order of 10^{-23} cm, and are stretched to astrophysical size by the tremendous growth of the scale factor during inflation (factor of e^{60} or greater). Scale invariant refers to the fact that the fuctuations in the gravitational potential are independent of length scale; or equivalently that the horizon-crossing amplitudes of the density perturbations are independent of length scale. While the shape of the spectrum of density perturbations is common to all models, the overall amplitude is model dependent. Achieving density perturbations that are consistent with the observed anisotropy of the CBR and large enough to produce the structure seen in the Universe today requires a horizon crossing amplitude of around 2×10^{-5}.

3. **Nearly scale-invariant spectrum of gravitational waves**, from quantum-mechanical fluctuations in the metric itself . These can be detected as CMB “B-mode” polarization, or using special gravity wave detectors such as LIGO and LISA.

Density Fluctuations from Inflation

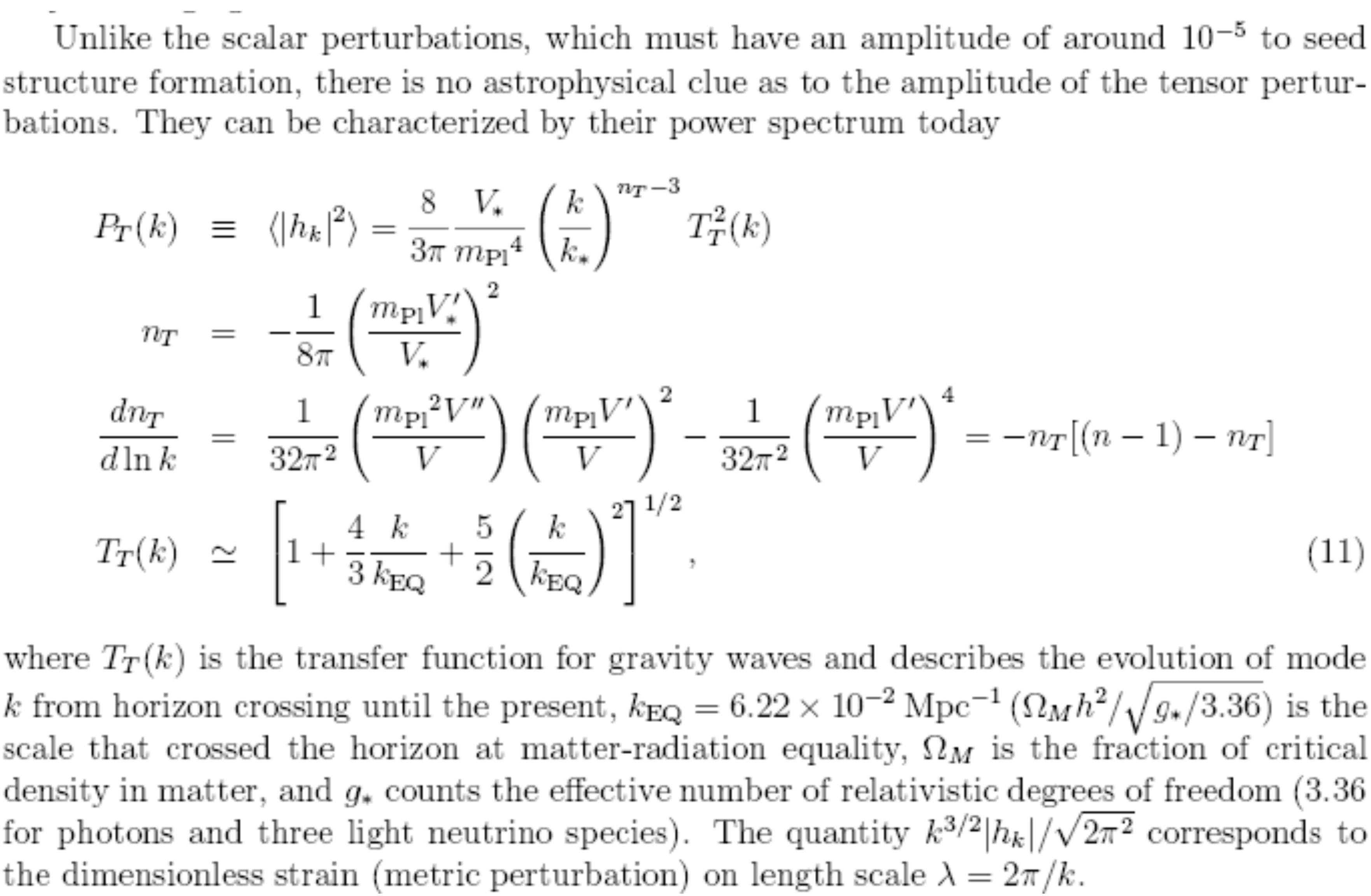
The relationship between the inflationary potential and the power spectrum of density perturbations today ($P(k) \equiv \langle |\delta_k|^2 \rangle$) is given by

$$\begin{aligned}
P(k) &= \frac{1024\pi^3}{75}\,\frac{k}{H_0^4}\,\frac{V_*^3}{{m_{\rm Pl}}^6 V_*'^2}\left(\frac{k}{k_*}\right)^{n-1} T^2(k) \\
n-1 &= -\frac{1}{8\pi}\left(\frac{m_{\rm Pl}V_*'}{V_*}\right)^2 + \frac{m_{\rm Pl}}{4\pi}\left(\frac{m_{\rm Pl}V_*'}{V_*}\right)' \\
\frac{dn}{d\ln k} &= -\frac{1}{32\pi^2}\left(\frac{{m_{\rm Pl}}^3 V_*'''}{V_*}\right)\left(\frac{m_{\rm Pl}V_*'}{V_*}\right) \\
&\quad + \frac{1}{8\pi^2}\left(\frac{{m_{\rm Pl}}^2 V_*''}{V_*}\right)\left(\frac{m_{\rm Pl}V_*'}{V_*}\right)^2 - \frac{3}{32\pi^2}\left(m_{\rm Pl}\frac{V_*'}{V_*}\right)^4 \\
T(q) &= \frac{\ln\left(1+2.34q\right)/2.34q}{\left[1+3.89q+(16.1q)^2+(5.46q)^3+(6.71q)^4\right]^{1/4}},
\end{aligned}$$

generally nonzero

where $V(\phi)$ is the inflationary potential, prime denotes $d/d\phi$, V_* is the value of the scalar potential when the scale k_* crossed outside the horizon during inflation, $T(k)$ is the transfer function which accounts for the evolution of the mode k from horizon crossing until the present, $q = k/h\Gamma$, and $\Gamma \simeq \Omega_M h$ is the "shape" parameter

Gravity Waves from Inflation



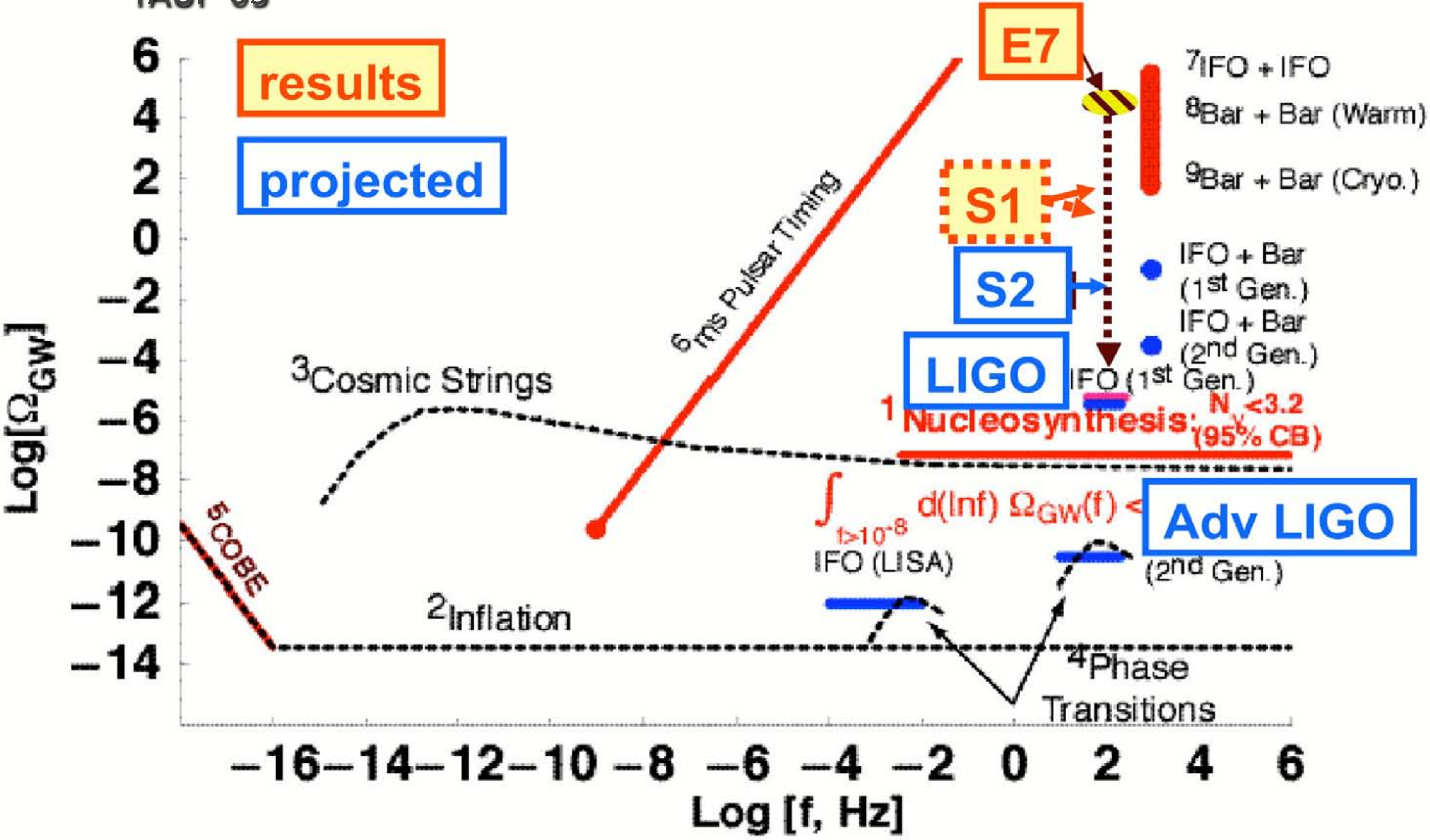
Unlike the scalar perturbations, which must have an amplitude of around 10^{-5} to seed structure formation, there is no astrophysical clue as to the amplitude of the tensor perturbations. They can be characterized by their power spectrum today

$$
\begin{aligned}
P_T(k) &\equiv \langle |h_k|^2 \rangle = \frac{8}{3\pi} \frac{V_*}{{m_{\rm Pl}}^4} \left(\frac{k}{k_*} \right)^{n_T - 3} T_T^2(k) \\
n_T &= -\frac{1}{8\pi} \left(\frac{m_{\rm Pl} V_*'}{V_*} \right)^2 \\
\frac{dn_T}{d\ln k} &= \frac{1}{32\pi^2} \left(\frac{{m_{\rm Pl}}^2 V''}{V} \right) \left(\frac{m_{\rm Pl} V'}{V} \right)^2 - \frac{1}{32\pi^2} \left(\frac{m_{\rm Pl} V'}{V} \right)^4 = -n_T[(n-1) - n_T] \\
T_T(k) &\simeq \left[1 + \frac{4}{3} \frac{k}{k_{\rm EQ}} + \frac{5}{2} \left(\frac{k}{k_{\rm EQ}} \right)^2 \right]^{1/2} , \qquad (11)
\end{aligned}
$$

where $T_T(k)$ is the transfer function for gravity waves and describes the evolution of mode k from horizon crossing until the present, $k_{\rm EQ} = 6.22 \times 10^{-2}\,{\rm Mpc}^{-1}\,(\Omega_M h^2/\sqrt{g_*/3.36})$ is the scale that crossed the horizon at matter-radiation equality, Ω_M is the fraction of critical density in matter, and g_* counts the effective number of relativistic degrees of freedom (3.36 for photons and three light neutrino species). The quantity $k^{3/2}|h_k|/\sqrt{2\pi^2}$ corresponds to the dimensionless strain (metric perturbation) on length scale $\lambda = 2\pi/k$.

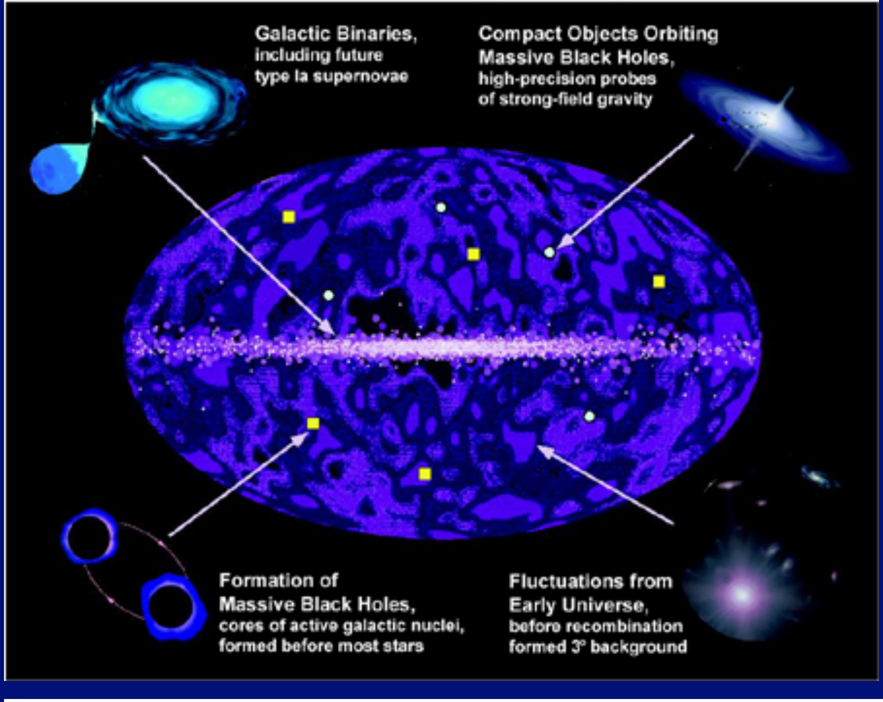
Stochastic Background

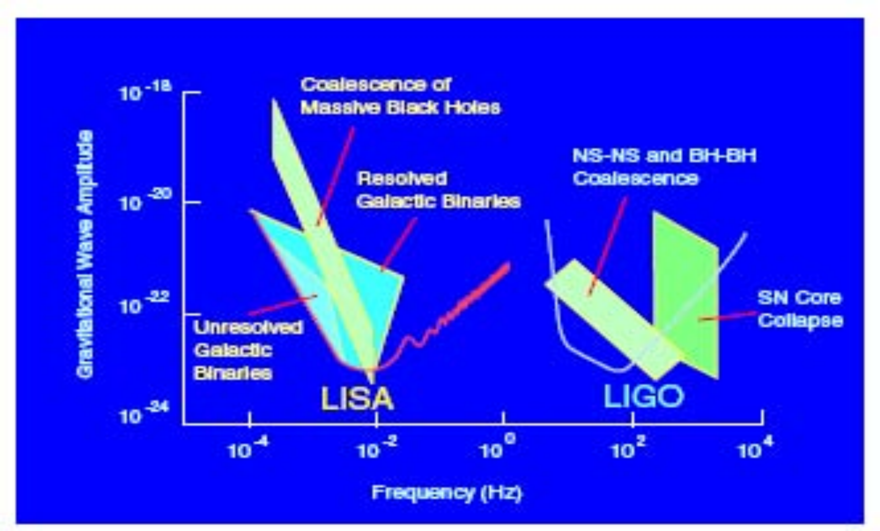
B. Barish
TAUP 03



LISA: Science Goals

- Beyond Einstein science
  - determine how and when massive black holes form
  - investigate whether general relativity correctly describes gravity under extreme conditions
  - determine how black hole growth is related to galaxy evolution
  - determine if black holes are correctly described by general relativity
  - investigate whether there are gravitational waves from the early universe
  - determine the distance scale of the universe
- Broader science
  - determine the distribution of binary systems of white dwarfs and neutron stars in our Galaxy





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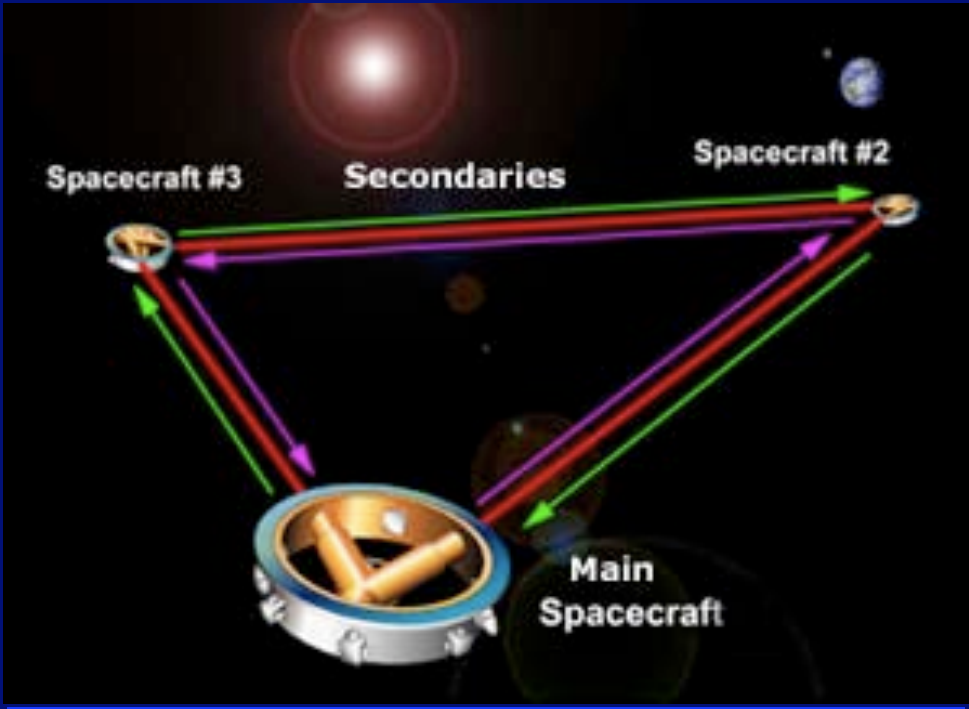


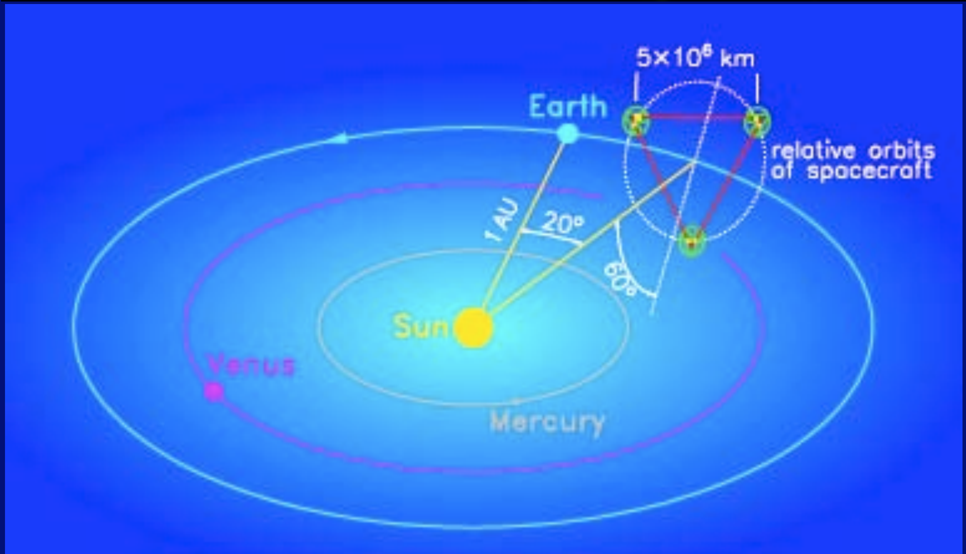
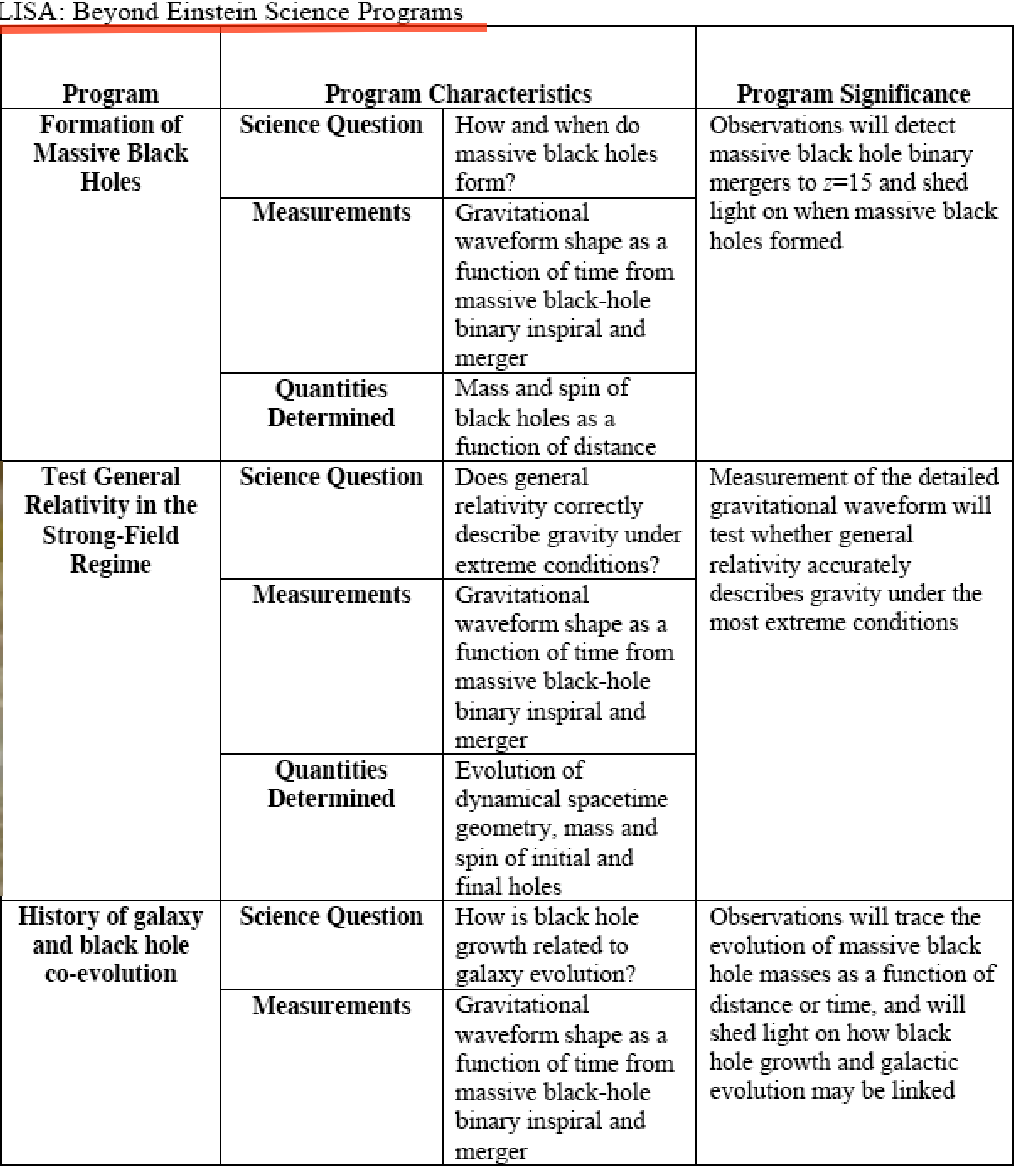
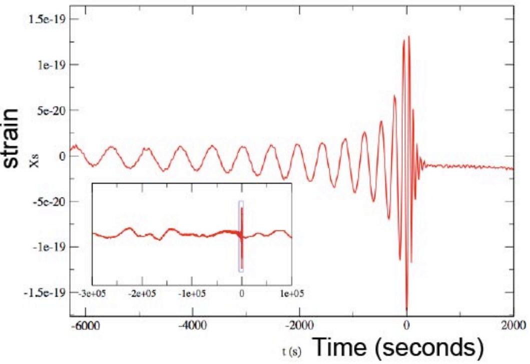
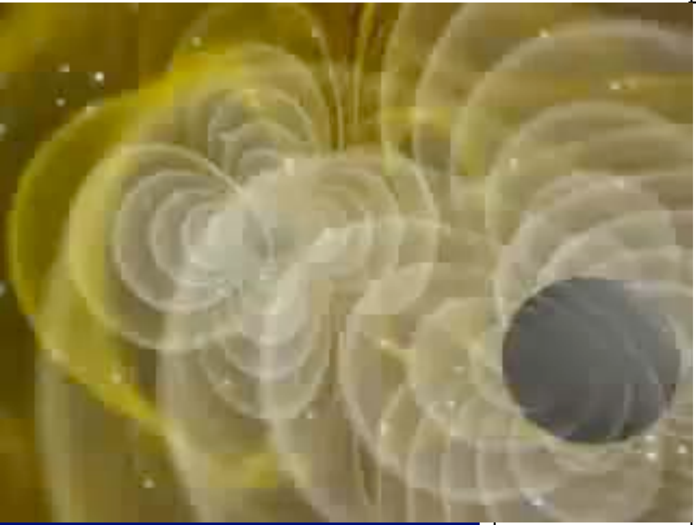


TABLE 2.F.3 LISA: Beyond Einstein Science Programs

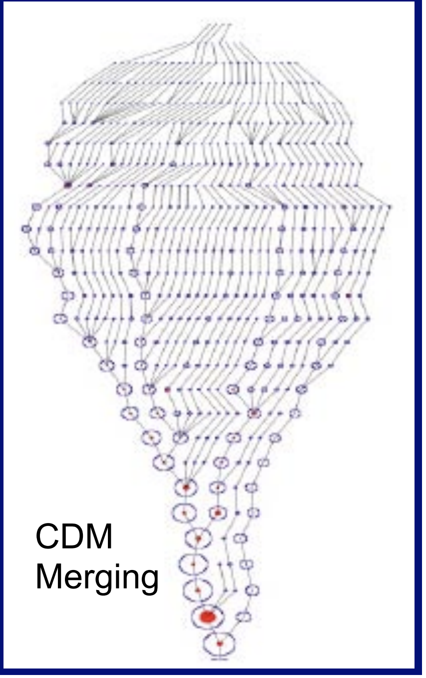


Science	Program	Program Characteristics		Program Significance
Science Definition Programs	Formation of Massive Black Holes	Science Question	How and when do massive black holes form?	Observations will detect massive black hole binary mergers to z=15 and shed light on when massive black holes formed
		Measurements	Gravitational waveform shape as a function of time from massive black-hole binary inspiral and merger	
		Quantities Determined	Mass and spin of black holes as a function of distance	
	Test General Relativity in the Strong-Field Regime	Science Question	Does general relativity correctly describe gravity under extreme conditions?	Measurement of the detailed gravitational waveform will test whether general relativity accurately describes gravity under the most extreme conditions
		Measurements	Gravitational waveform shape as a function of time from massive black-hole binary inspiral and merger	
		Quantities Determined	Evolution of dynamical spacetime geometry, mass and spin of initial and final holes	
	History of galaxy and black hole co-evolution	Science Question	How is black hole growth related to galaxy evolution?	Observations will trace the evolution of massive black hole masses as a function of distance or time, and will shed light on how black hole growth and galactic evolution may be linked
		Measurements	Gravitational waveform shape as a function of time from massive black-hole binary inspiral and merger	

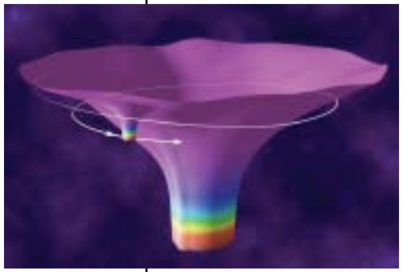




Joan Centrella et al. NASA/GSFC
Visualization: Chris Henze, NASA/Ames

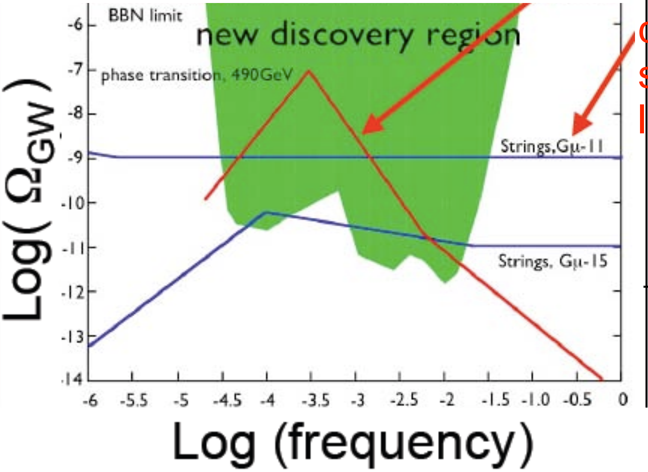


Science	Program	Program Characteristics		Program Significance
		Quantities Determined	Mass as a function of distance	
Additional Beyond Einstein Science	Map black-hole spacetimes	Science Question	Are black holes correctly described by general relativity?	Observations will yield maps of the spacetime geometry surrounding massive black holes, and will test whether they are described by the Kerr geometry predicted by general relativity. They will also measure the parameters (mass, spin, shape) of the holes, and test whether they obey the no-hair theorems of GR
		Measurements	Gravitational waveform shape from small bodies spiraling into massive black holes (EMRI)	
		Quantities Determined	Mass, spin, multipole moments, spacetime geometry close to hole	
	Cosmological backgrounds	Science Question	Are there gravitational waves from the early universe?	First-order phase transitions or cosmic strings in the early universe could leave a background of detectable waves
		Measurements	Stochastic background of gravitational waves	
		Quantities Determined	Effective energy density of waves vs. frequency	
	Cosmography, Dark energy	Science Question	What is the distance scale of the universe?	If redshift of source or host galaxy can be determined, then precise, calibration-free measurements of the Hubble parameter and other cosmological parameters could be done, significantly constraining dark energy
		Measurements	Gravitational waveform shape and amplitude measurements yield luminosity distance of sources directly	
		Quantities Determined	Luminosity distance	



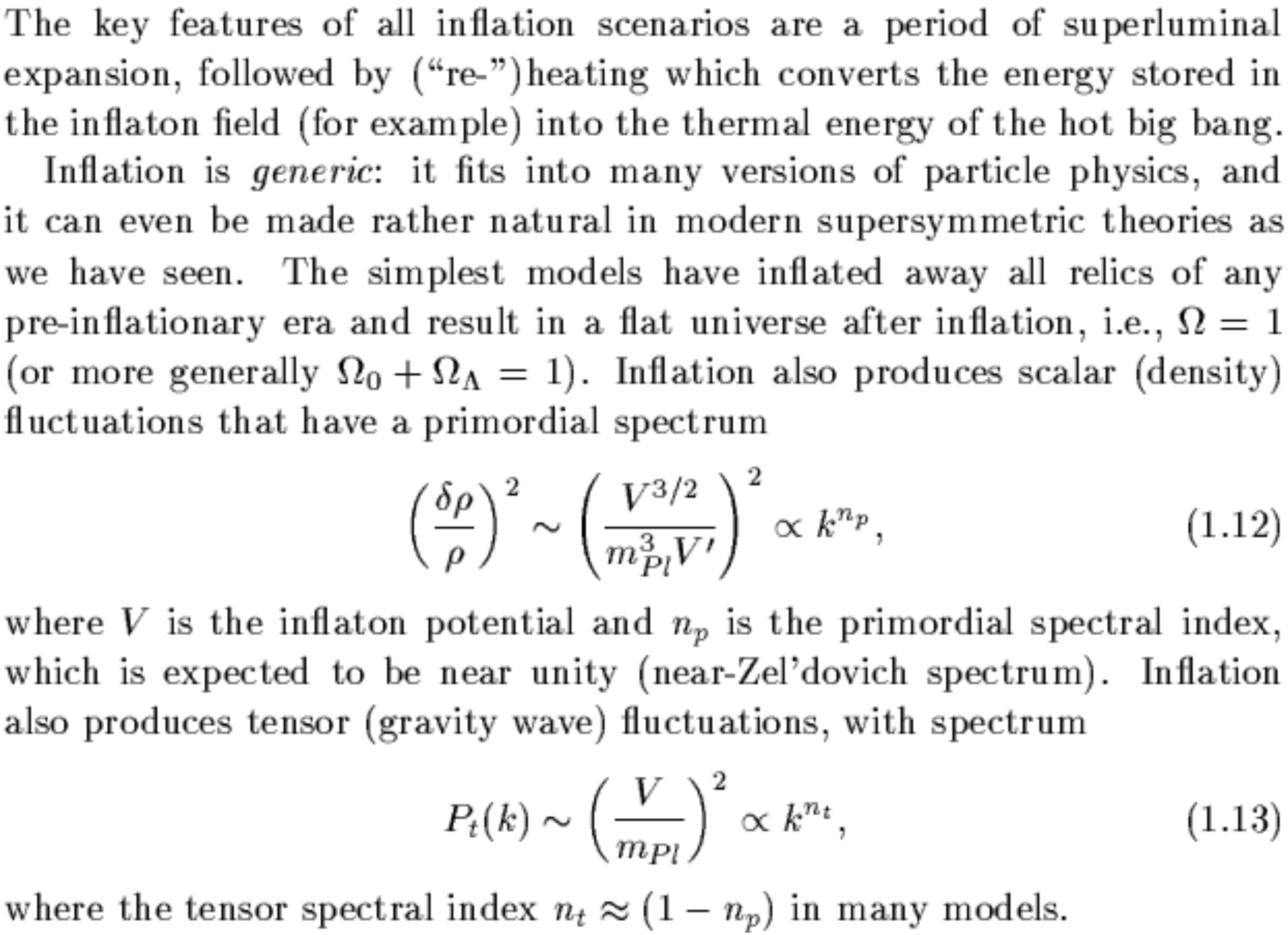
1st order PT

cosmic superstring loop bursts



$$\text{Distance} \cong c\frac{1}{\text{frequency}^2 \times t_{\text{chirp}} \times \text{amplitude}}$$

Inflation Summary



The key features of all inflation scenarios are a period of superluminal expansion, followed by ("re-")heating which converts the energy stored in the inflaton field (for example) into the thermal energy of the hot big bang.

Inflation is *generic*: it fits into many versions of particle physics, and it can even be made rather natural in modern supersymmetric theories as we have seen. The simplest models have inflated away all relics of any pre-inflationary era and result in a flat universe after inflation, i.e., $\Omega = 1$ (or more generally $\Omega_0 + \Omega_\Lambda = 1$). Inflation also produces scalar (density) fluctuations that have a primordial spectrum

$$\left(\frac{\delta\rho}{\rho}\right)^2 \sim \left(\frac{V^{3/2}}{m_{Pl}^3 V'}\right)^2 \propto k^{n_p}, \tag{1.12}$$

where V is the inflaton potential and n_p is the primordial spectral index, which is expected to be near unity (near-Zel'dovich spectrum). Inflation also produces tensor (gravity wave) fluctuations, with spectrum

$$P_t(k) \sim \left(\frac{V}{m_{Pl}}\right)^2 \propto k^{n_t}, \tag{1.13}$$

where the tensor spectral index $n_t \approx (1 - n_p)$ in many models.

The quantity $(1-n_p)$ is often called the "tilt" of the spectrum; the larger the tilt, the more fluctuations on small spatial scales (corresponding to large k) are suppressed compared to those on larger scales. The scalar and tensor waves are generated by independent quantum fluctuations during inflation, and so their contributions to the CMB temperature fluctuations add in quadrature. The ratio of these contributions to the quadrupole anisotropy amplitude Q is often called $T/S \equiv Q_t^2/Q_s^2$; thus the primordial scalar fluctuation power is decreased by the ratio $1/(1+T/S)$ for the same COBE normalization, compared to the situation with no gravity waves $(T=0)$. In power-law inflation, $T/S = 7(1-n_p)$. This is an approximate equality in other popular inflation models such as chaotic inflation with $V(\phi) = m^2\phi^2$ or $\lambda\phi^4$. But note that the tensor wave amplitude is just the inflaton potential during inflation divided by the Planck mass, so the gravity wave contribution is negligible in theories like the supersymmetric model discussed above in which inflation occurs at an energy scale far below m_{Pl}. Because gravity waves just redshift after they come inside the horizon, the tensor contributions to CMB anisotropies corresponding to angular wavenumbers $\ell \gg 20$, which came inside the horizon long ago, are strongly suppressed compared to those of scalar fluctuations.