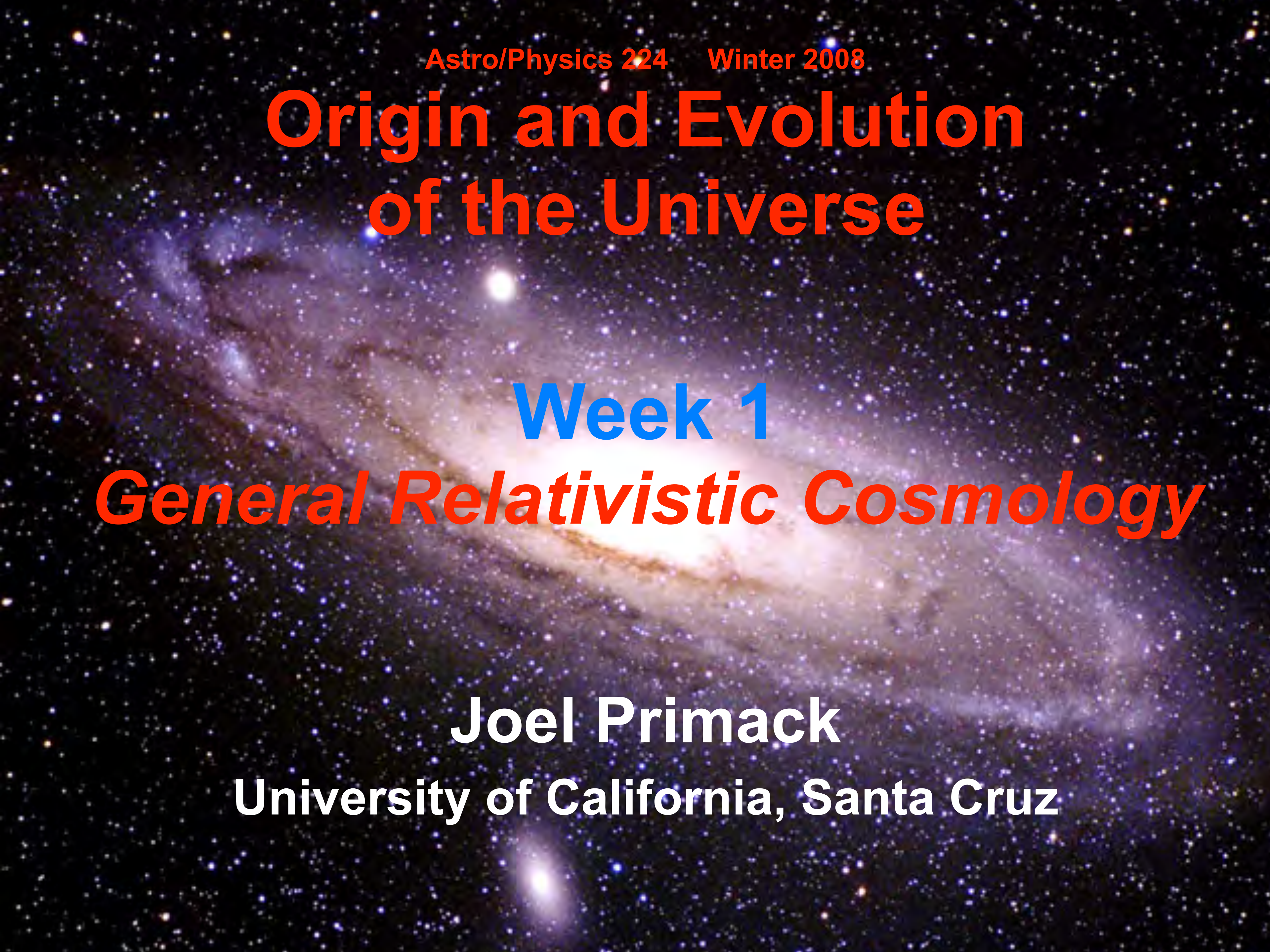


Astro/Physics 224 Winter 2008

Origin and Evolution of the Universe

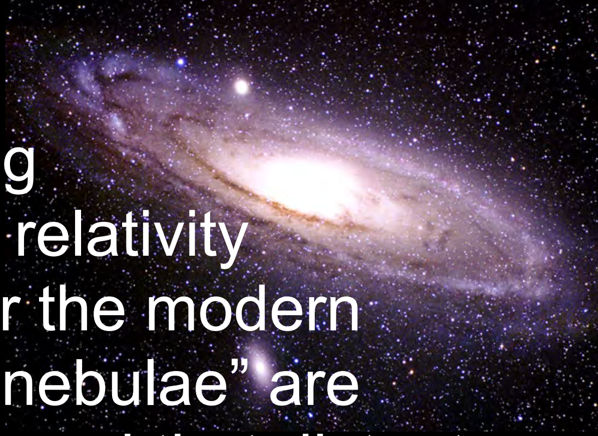
Week 1

General Relativistic Cosmology

Joel Primack

University of California, Santa Cruz

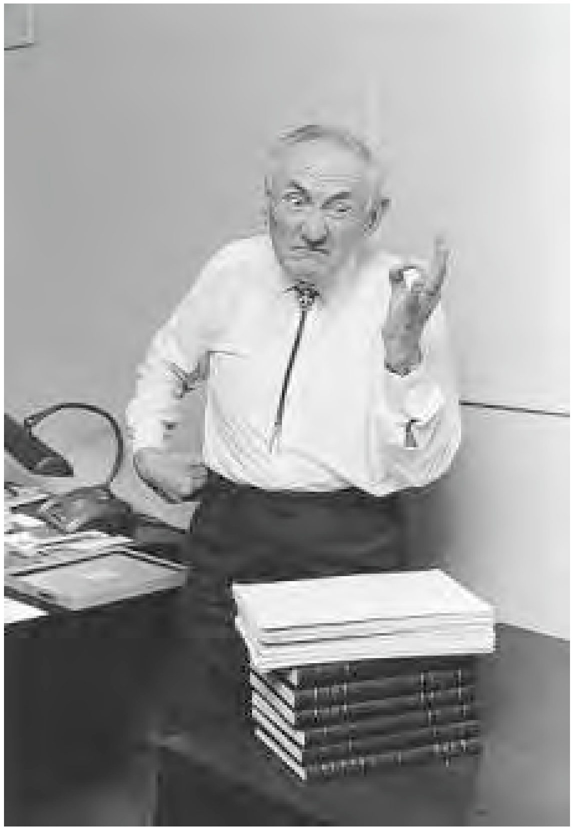
Modern Cosmology



A series of major discoveries has laid a lasting foundation for cosmology. Einstein's general relativity (1916) provided the conceptual foundation for the modern picture. Then Hubble discovered that "spiral nebulae" are large galaxies like our own Milky Way (1922), and that distant galaxies are receding from the Milky Way with a speed proportional to their distance (1929), which means that we live in an expanding universe. The discovery of the cosmic background radiation (1965) showed that the universe began in a very dense, hot, and homogeneous state: the Big Bang. This was confirmed by the discovery that the cosmic background radiation has exactly the same spectrum as heat radiation (1989), and the measured abundances of the light elements agree with the predictions of Big Bang theory if the abundance of ordinary matter is about 4% of critical density. Most of the matter in the universe is invisible particles which move very sluggishly in the early universe ("Cold Dark Matter").

The Dark Matter Rap: Cosmological History for the MTV Generation by David Weinberg*

My name is Fritz Zwicky,
I can be kind of prickly,
This song had better start
by giving me priority.
Whatever anybody says,
I said in 1933.
Observe the Coma cluster,
the redshifts of the galaxies
imply some big velocities.
They're moving so fast,
there must be missing mass!
Dark matter.



Dark matter: Do we need it? What is it? Where is it? How much?
Do we need it? Do we need it? Do we need it? Do we need it?

* Written in 1992. http://www.astronomy.ohio-state.edu/~dhw/Silliness/silliness.html

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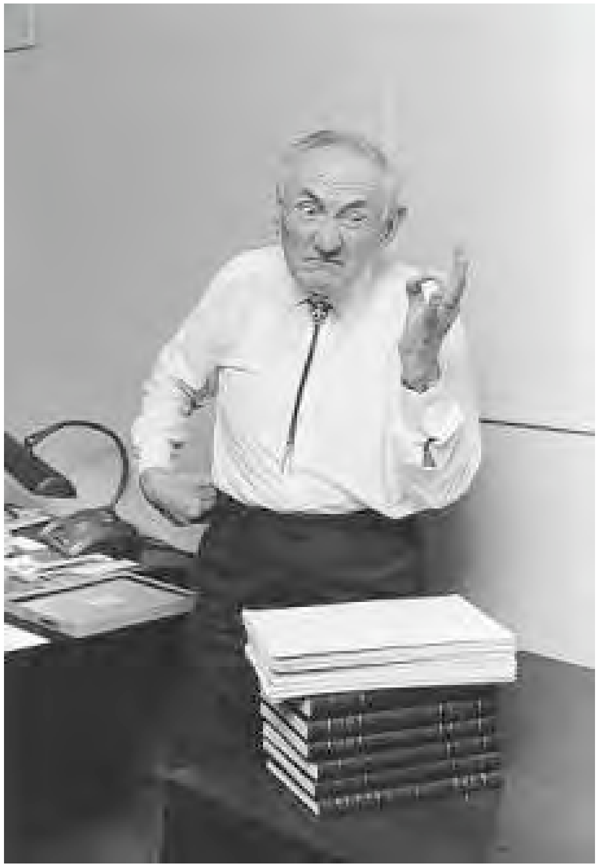
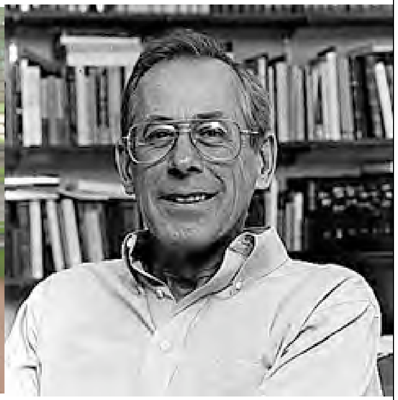


Dark matter: Do we need it? What is it? Where is it? How much?
Do we need it? Do we need it? Do we need it? Do we need it?

For nearly forty years, the dark matter problem sits.
Nobody gets worried 'cause, "It's only crazy Fritz."
The next step's not 'til the early 1970s,
Ostriker and Peebles, dynamics of the galaxies,
cold disk instabilities.
They say: "If the mass, were sitting in the stars,
all those pretty spirals, ought to be bars!
Self-gravitating disks? Uh-uh, oh no.
What those spirals need is a massive halo.



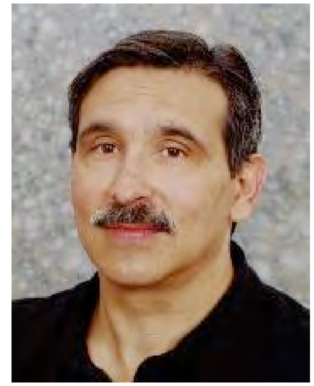
And hey, look over here, check out these observations,
Vera Rubin's optical curves of rotation,
they can provide our needed confirmation:
Those curves aren't falling, they're FLAT!
Dark matter's where it's AT!

Dark matter: Do we need it? What is it? Where is it? How much?
What is it? What is it? What is it? What is it?

And so the call goes out for the dark matter candidates:
black holes, snowballs, gas clouds, low mass stars, or planets.
But we quickly hit a snag because galaxy formation
requires too much structure in the background radiation
if there's only baryons and adiabatic fluctuations.

The Russians have an answer: "We can solve the impasse.
Lyubimov has shown that the neutrino has mass."
Zel'dovich cries, "Pancakes! The dark matter's HOT."
Carlos Frenk, Simon White, Marc Davis say, "NOT!
Quasars are old, and the pancakes must be young.
Forming from the top down it can't be done."
So neutrinos hit the skids, and the picture's looking black.
But California laid-back, Blumenthal & Primack
say, "Don't have a heart attack.
There's lots of other particles. Just read the physics articles.
Take this pretty theory that's called supersymmetry.
What better for dark matter than the L-S-P?
The mass comes in at a ~ keV, and that's not hot, that's warm."
Jim Peebles says, "Warm? Don't be half-hearted.
Let's continue the trend that we have started.
I'll stake out a position that's bold:
dark matter's not hot, not warm, but COLD."
Well cold dark matter causes overnight sensations:
hand-waving calculations,
computer simulations,
detailed computations of the background fluctuations.
Results are good, and the prospects look bright.
Here's a theory that works! Well, maybe not quite.






Dark matter: Do we need it? What is it? Where is it? How much?
Where is it? How much? Where is it? How much?

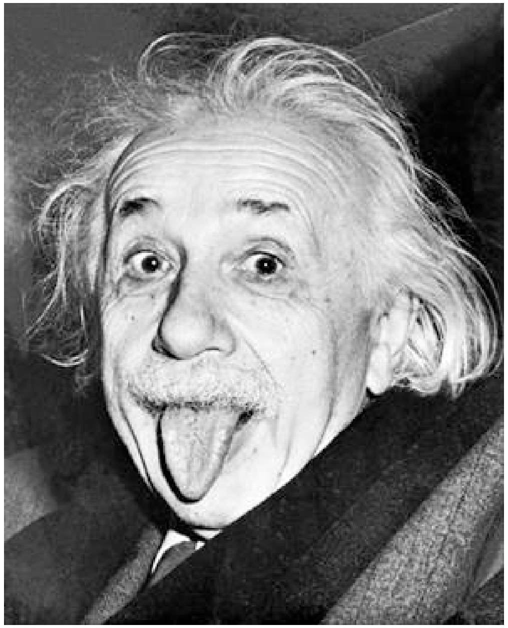
We have another puzzle that goes back to Robert Dicke.
Finding a solution has proven kind of tricky.
The CMB's so smooth, it's as if there'd been a compact
between parts of the universe that aren't in causal contact.
Alan Guth says, "Inflation,
will be our salvation,
give smoothness of the universe a causal explanation,
and even make the galaxies from quantum fluctuations!
There is one prediction, from which it's hard to run.
If inflation is correct, then Omega should be one."
Observers say, "Stop, no, sorry, won't do.
Look at these clusters, Omega's point 2."
The theorists respond, "We have an explanation.
The secret lies in biased galaxy formation.
We're not short of critical mass density.
Just some regions, are missing luminosity."
Observers roll their eyes, and they start to get annoyed,
But the theorists reply, "There's dark matter in the voids."

Dark matter: Do we need it? What is it? Where is it? How much?
Do we need it? Do we need it? Do we need it? Do we need it?

Along comes Moti Milgrom,
who's here to tell us all:
"This dark matter claptrap
has got you on the wrong track.
You're all too mired in conventionality,
wedded to your standard theory of gravity,
seduced by the elegance of General Relativity.
Just change your force law, that's the key.
Give me one free parameter, and I'll explain it all."
"Not so," claim Lake, and Spergel, et al.,
"On dwarf galaxies, your theory does fall."
The argument degenerates; it's soon a barroom brawl.

Dark matter: Do we need it? What is it? Where is it? How much?
What is it? What is it? What is it? What is it?

New observations hit the theory like an ice cold shower.
They show that cold dark matter has too little large scale power.
Says Peebles: "Cold dark matter? My feeblest innovation.
An overly aesthetic, theoretical abberation.
Our theories must have firmer empirical foundation.
Shed all this extra baggage, including the carry-ons.
Use particles we know, i.e., the baryons.
Others aren't convinced, and a few propose a mixture
of matter hot and cold, perhaps with strings or texture.
And nowadays some physicists are beginning to wonder
if it's time to resurrect Einstein's "greatest blunder."
Why seek exotic particles instead of just assume
that the dark matter's all around us -- it's what we call the vacuum?



Who's right? It's hard to know, 'til observation or experiment
gives overwhelming evidence that relieves our predicament.
The search is getting popular as many realize
that the detector of dark matter may well win the Nobel Prize.

So now you've heard my lecture, and it's time to end the session
with the standard closing line: Thank you, any questions?

General Relativity and Cosmology

GR: MATTER TELLS SPACE HOW TO CURVE

$$R^{\mu\nu} - \tfrac{1}{2} R g^{\mu\nu} = 8\pi G T^{\mu\nu} + \Lambda g^{\mu\nu}$$

CURVED SPACE TELLS MATTER HOW TO MOVE

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = 0$$

Cosmological Principle: on large scales, space is uniform and isotropic. COBE-Copernicus Theorem: If all observers observe a nearly-isotropic Cosmic Background Radiation (CBR), then the universe is locally nearly homogeneous and isotropic – i.e., is approximately described by the Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)\,[dr^2 (1 - kr^2)^{-1} + r^2\, d\Omega^2]$$

with curvature constant $k = -1$, 0, or +1. Substituting this metric into the Einstein equation at left above, we get the Friedmann eq.

Friedmann-Robertson-Walker Framework (homogeneous, isotropic universe)

FRW E(00) $\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$ ← Friedmann equation

FRW E(ii) $\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p - \frac{k}{a^2} + \Lambda$

$$H_0 \equiv 100h \,\mathrm{km\,s^{-1}Mpc^{-2}} \equiv 70h_{70}\,\mathrm{km\,s^{-1}Mpc^{-2}}$$

$$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda \text{ with } H \equiv \frac{\dot{a}}{a},\ a_0 \equiv 1,\ \Omega_0 \equiv \frac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2},$$

$$\rho_{c,0} \equiv \frac{3H_0^2}{8\pi G} = 1.36 \times 10^{11} h_{70}^2 M_\odot \mathrm{Mpc}^{-3}$$

$$E(ii) - E(00) \Rightarrow \frac{2\ddot{a}}{a} = -\frac{8\pi}{3} G\rho - 8\pi G p + \frac{2}{3}\Lambda$$

$$\text{Divide by } 2E(00) \Rightarrow \quad q_0 \equiv -\left(\frac{\ddot{a}}{a}\frac{a^2}{\dot{a}^2}\right)_0 = \frac{\Omega_0}{2} - \Omega_\Lambda$$

$$E(00) \Rightarrow t_0 = \int_0^1 \frac{da}{a}\left[\frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}\right]^{-\frac{1}{2}} = H_0^{-1}\int_0^1 \frac{da}{a}\left[\frac{\Omega_0}{a^3} - \frac{k}{H_0^2 a^2} + \Omega_\Lambda\right]^{-\frac{1}{2}}$$

$t_0 = H_0^{-1} f(\Omega_0, \Omega_\Lambda)$ $\qquad H_0^{-1} = 9.78h^{-1}\mathrm{Gyr}$ $= 13.97\ h_{70}^{-1}$ Gyr

$f(1,0) = \frac{2}{3}$
$f(0,0) = 1$
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$f(0.3, 0.7) = 0.964$

$$[E(00)a^3]' \text{ vs. } E(ii) \Rightarrow \quad \frac{\partial}{\partial a}(\rho a^3) = -3pa^2 \ (\text{“continuity”})$$

Given eq. of state $p = p(\rho)$, integrate to determine $\rho(a)$,
integrate $E(00)$ to determine $a(t)$

Matter: $p = 0 \Rightarrow \rho = \rho_0 a^{-3}$ (assumed above in q_0, t_0 eqs.)

Radiation: $p = \frac{\rho}{3}$, $k = 0 \Rightarrow \rho \propto a^{-4}$

Experimental and Historical Sciences

both make predictions about new knowledge, whether from experiments or from the past

Historical Explanation Is Always Inferential

Our age cannot look back to earlier things
Except where reasoning reveals their traces Lucretius

Patterns of Explanation Are the Same in the Historical Sciences as in the Experimental Sciences

Specific conditions + General laws $\Rightarrow$ Particular event

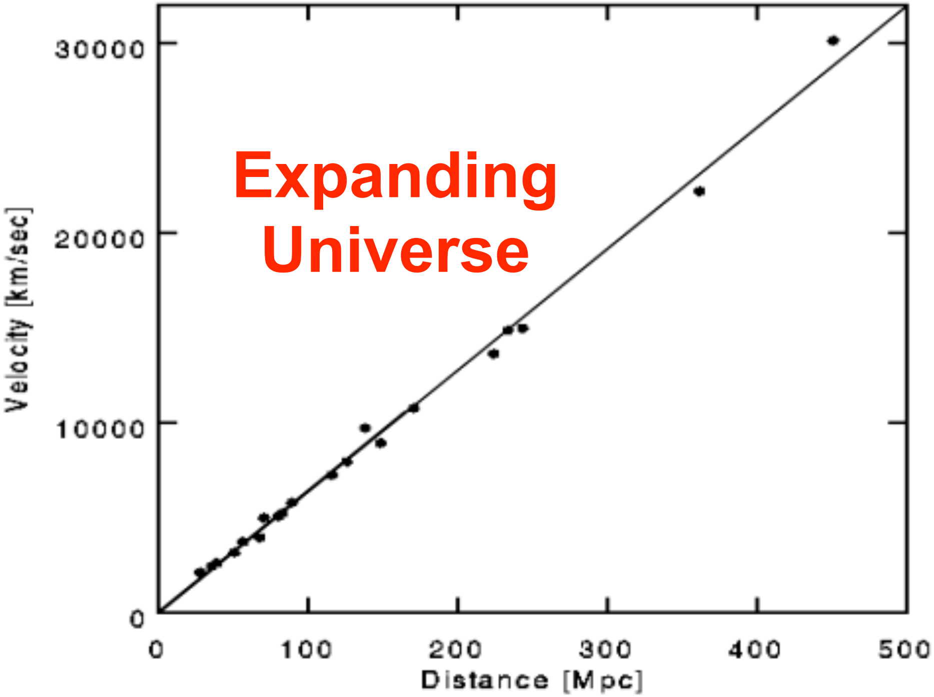
In history as anywhere else in empirical science, the explanation of a phenomenon consists in subsuming it under general empirical laws; and the criterion of its soundness is … exclusively whether it rests on empirically well confirmed assumptions concerning initial conditions and general laws.

C.G. Hempel, *Aspects of Scientific Explanation* (1965), p. 240.

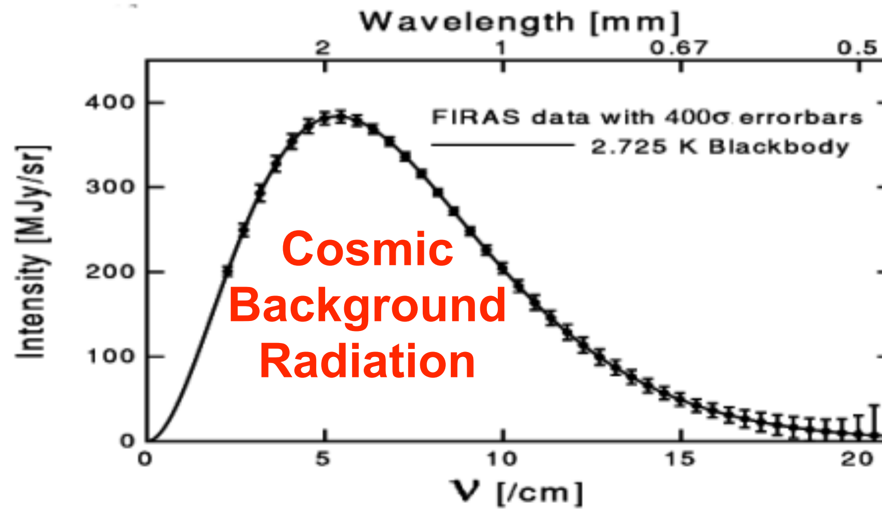
Successful Predictions of the Big Bang

First Prediction	First Confirmation
Expansion of the Universe	
Friedmann 1922, Lemaitre 1927 based on Einstein 1916	Hubble 1929
Cosmic Background Radiation	
Existence of CBR	
Gamow, Alpher, Hermann 1948	Penzias & Wilson 1965
CBR Thermal Spectrum	
Peebles 1966	COBE 1989
CBR Fluctuation Amplitude	
Cold Dark Matter theory 1984	COBE 1992
CBR Acoustic Peak	BOOMERANG 2000 MAXIMA 2000
Light Element Abundances	
Peebles 1966, Wagoner 1967	D/H Tytler et al. 1997

Three Pillars of the Big Bang



A modern illustration of Hubble's Law, displaying the increase of recession speed of galaxies growing in direct proportion to their distance.

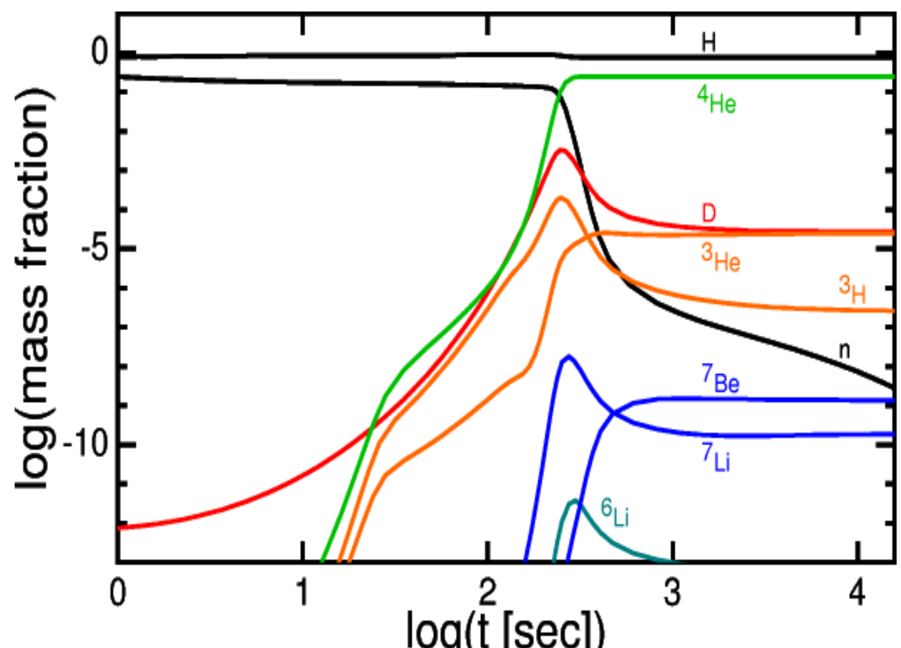


The variation of the intensity of the microwave background radiation with its frequency, as observed by the COBE satellite from above the Earth's atmosphere. The observations (boxes) display a perfect fit with the (solid) curve expected from pure heat radiation with a temperature of 2.73°K.

Big Bang Nucleosynthesis

The detailed production of the lightest elements out of protons and neutrons during the first three minutes of the universe's history. The nuclear reactions occur rapidly when the temperature falls below a billion degrees Kelvin. Subsequently, the reactions are shut down, because of the rapidly falling temperature and density of matter in the expanding universe.

Caution: ^{7}Li is now discordant



Mon. Not. R. astr. Soc. (1991) **251,** 128–136

Dynamical effects of the cosmological constant

Ofer Lahav,[1] Per B. Lilje,[2] Joel R. Primack[3] and Martin J. Rees[1]

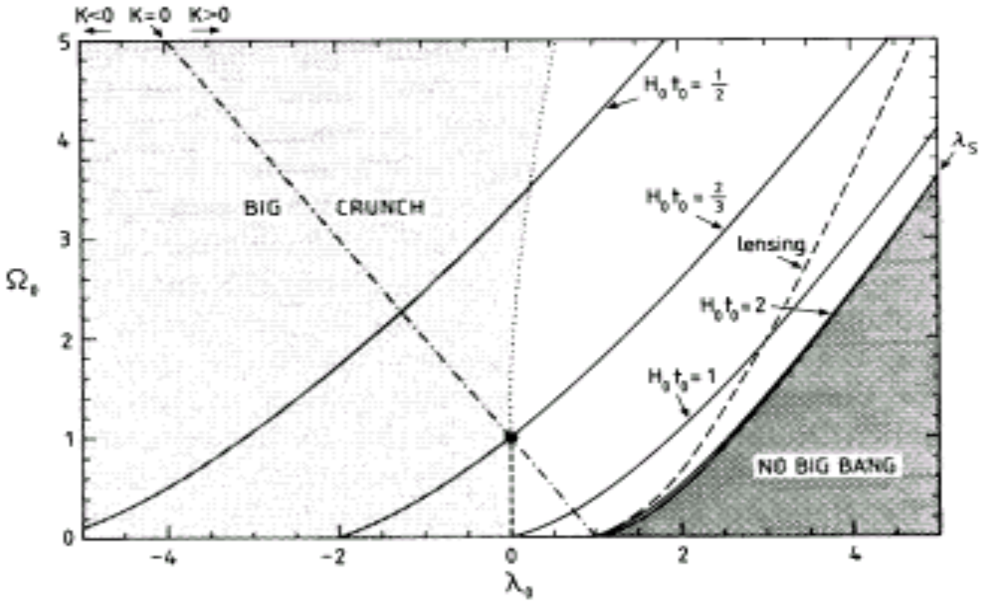
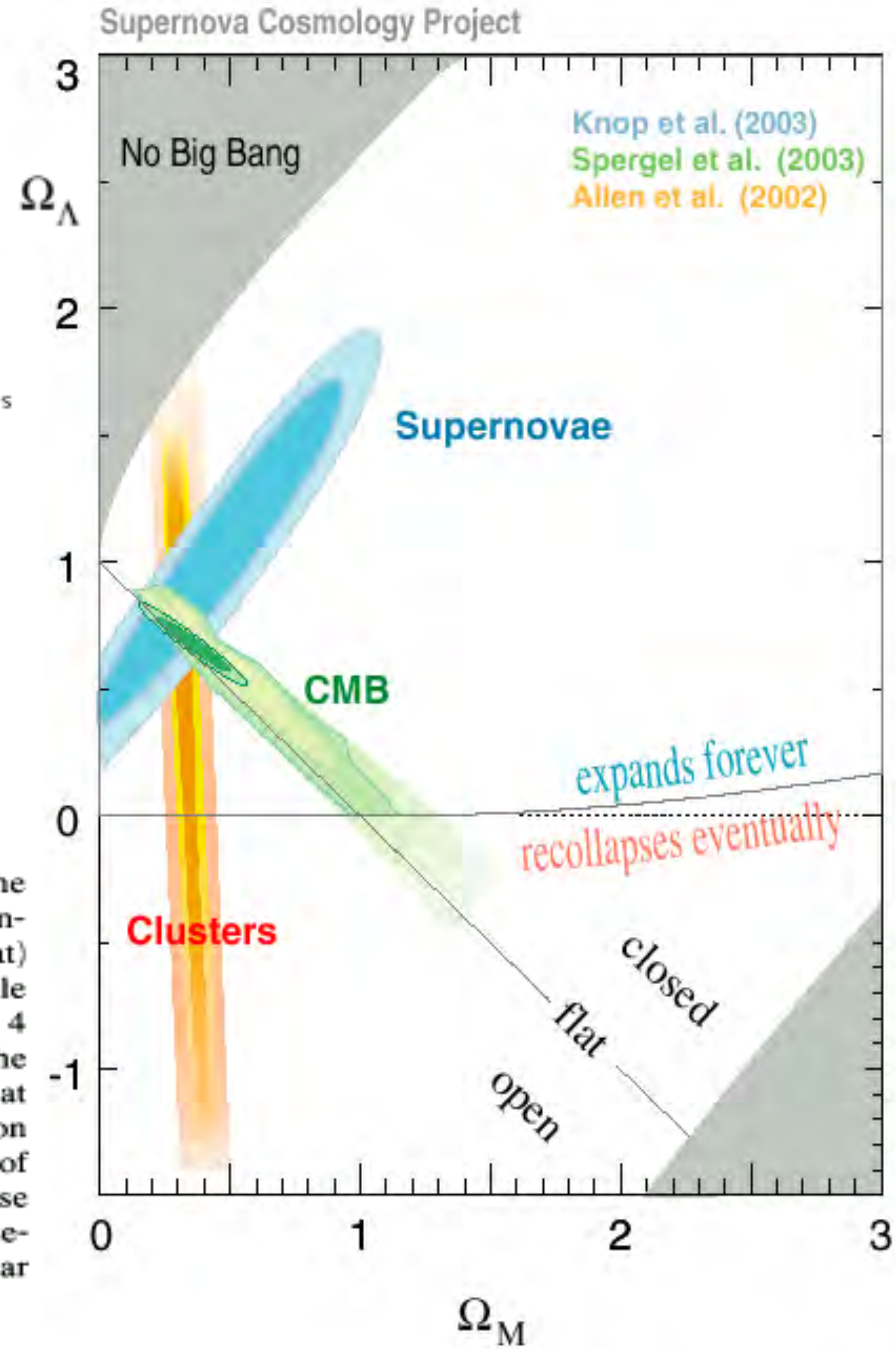


Figure 1. The phase-space of the density parameter Ω_0 and the cosmological constant $\lambda_0 = \Lambda/(3H_0^2)$ with various fundamental constraints. The dashed-dotted line indicates an inflationary (i.e. flat) universe. Note that some open models will have a Big Crunch, while some closed models will expand forever. The solid lines show 4 values for the age of the universe H_0t_0, and the dashed line is the constraint of Gott *et al.* (1989) from a normally lensed quasar at $z = 3.27$. The boundary (λ_s) of the shaded 'No Big Bang' region corresponds to a coasting phase in the past, while the boundary of the 'Big Crunch' (for $\Omega_0 > 1$) region corresponds to a coasting phase in the future. We see that the permitted range in the ($\lambda_0 - \Omega_0$) phase-space is fairly small, but allows values different from the popular point ($\Omega_0 = 1, \lambda_0 = 0$).



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The Age of the Universe

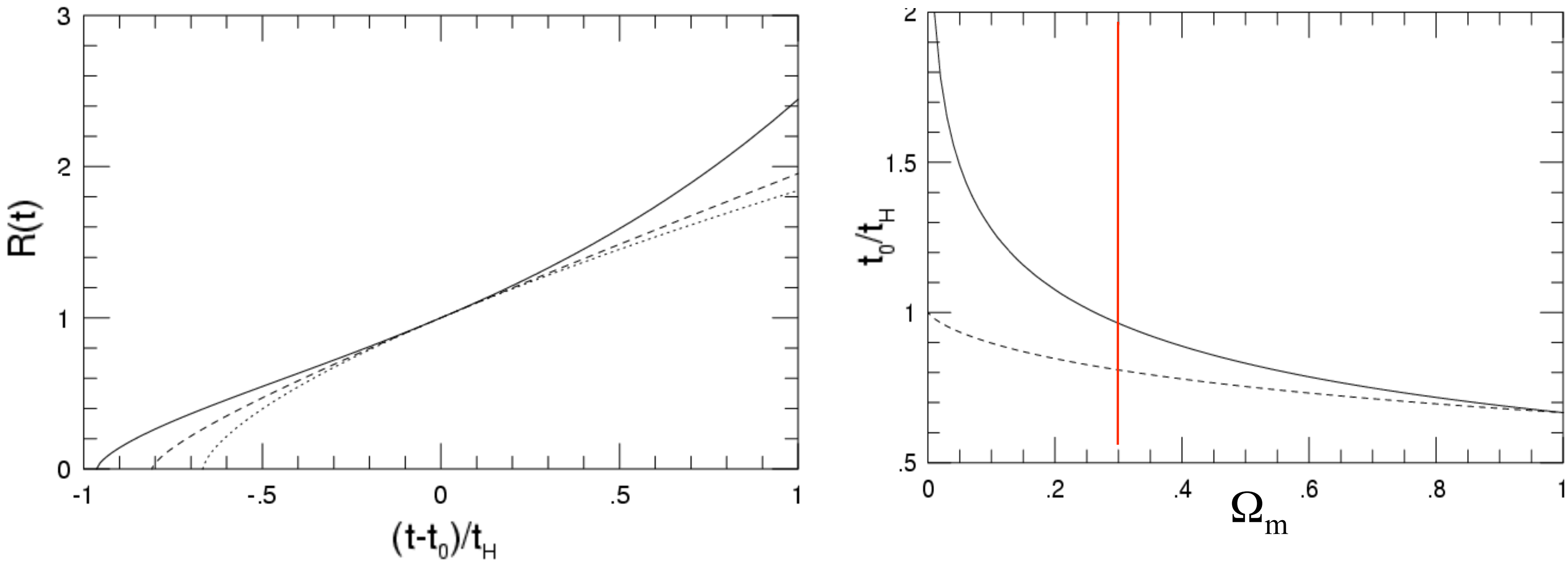
In the mid-1990s there was a crisis in cosmology, because the age of the old Globular Cluster stars in the Milky Way, then estimated to be 16±3 Gyr, was higher than the expansion age of the universe, which for a critical density ($\Omega_m = 1$) universe is 9±2 Gyr (with the Hubble parameter h=0.72±0.07). But when the data from the Hipparcos astrometric satellite became available in 1997, it showed that the distance to the Globular Clusters had been underestimated, which implied that their ages are 12±3 Gyr.

Several lines of evidence now show that the universe does not have $\Omega_m = 1$ but rather $\Omega_{tot} = \Omega_m + \Omega_\Lambda = 1.0$ with $\Omega_m \approx 0.3$, which gives an expansion age of about 14 Gyr.

Moreover, a new type of age measurement based on radioactive decay of Thorium-232 (half-life 14.1 Gyr) measured in a number of stars gives a completely independent age of 14±3 Gyr. A similar measurement, based on the first detection in a star of Uranium-238 (half-life 4.47 Gyr), gives 12.5±3 Gyr.

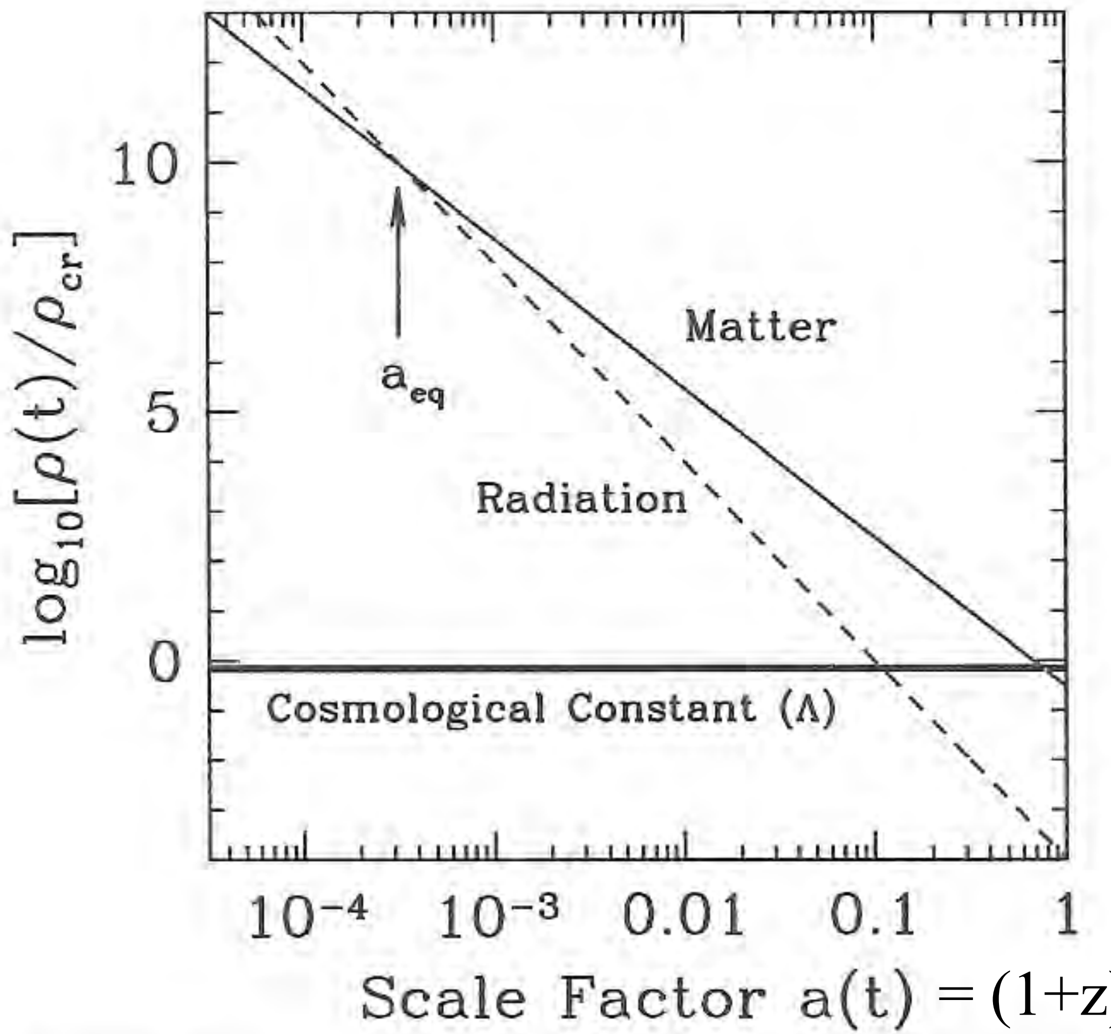
All the recent measurements of the age of the universe are thus in excellent agreement. It is reassuring that three completely different clocks – stellar evolution, expansion of the universe, and radioactive decay – agree so well.

Age of the Universe t_0 in FRW Cosmologies



(a) Evolution of the scale factor $a(t)$ plotted vs. the time after the present $(t - t_0)$ in units of Hubble time $t_H \equiv H_0^{-1} = 9.78h^{-1}$ Gyr for three different cosmologies: Einstein-de Sitter ($\Omega_0 = 1, \Omega_\Lambda = 0$ dotted curve), negative curvature ($\Omega_0 = 0.3, \Omega_\Lambda = 0$: dashed curve), and low-Ω_0 flat ($\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$: solid curve). (b) Age of the universe today t_0 in units of Hubble time t_H as a function of Ω_0 for $\Lambda = 0$ (dashed curve) and flat $\Omega_0 + \Omega_\Lambda = 1$ (solid curve) cosmologies.

Evolution of Densities of Radiation, Matter, & Λ

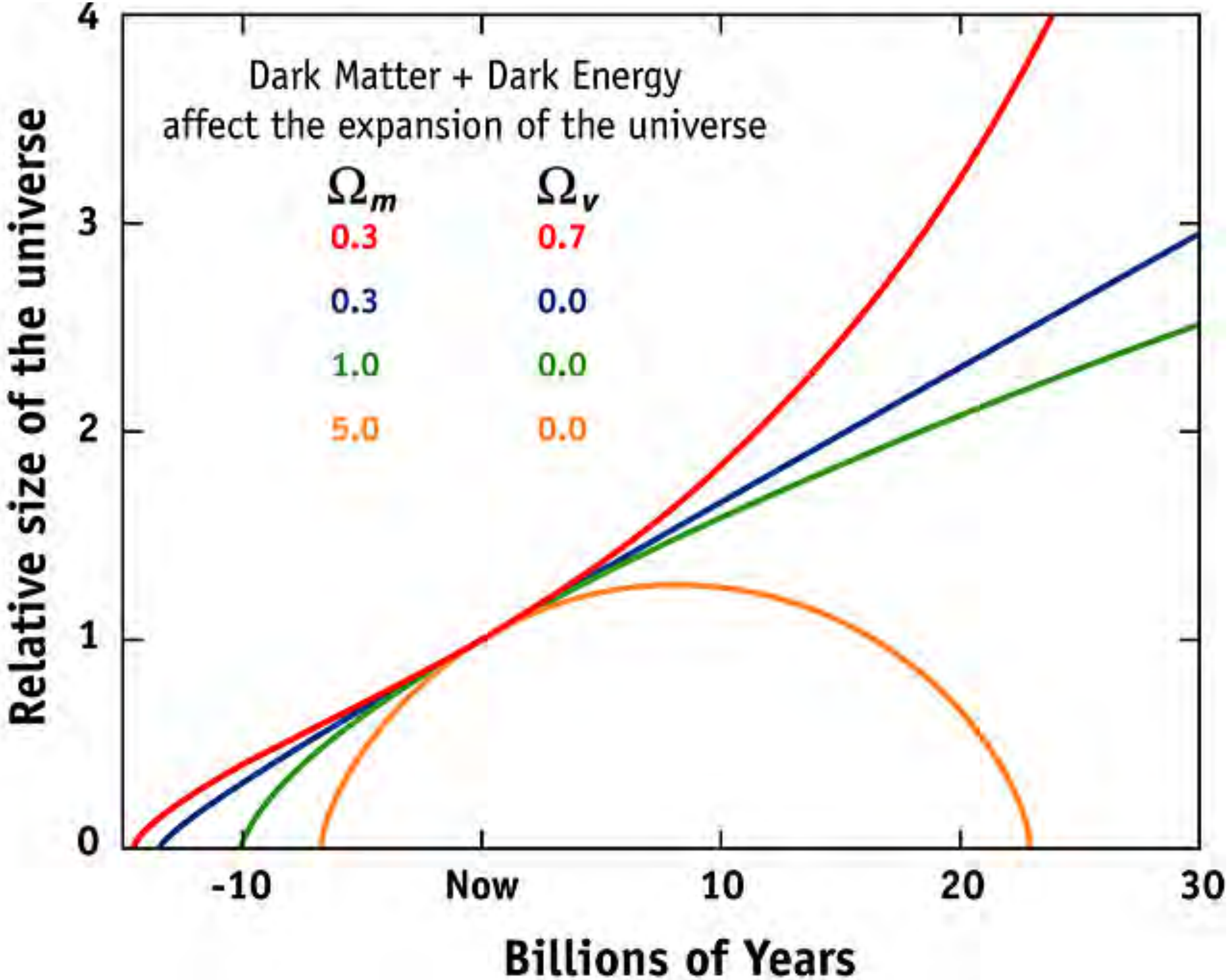


$= (1+z)^{-1}$ z = redshift

Figure 1.3. Energy density vs scale factor for different constituents of a flat universe. Shown are nonrelativistic matter, radiation, and a cosmological constant. All are in units of the critical density today. Even though matter and cosmological constant dominate today, at early times, the radiation density was largest. The epoch at which matter and radiation are equal is a_{eq}.

Dodelson, Chapter 1

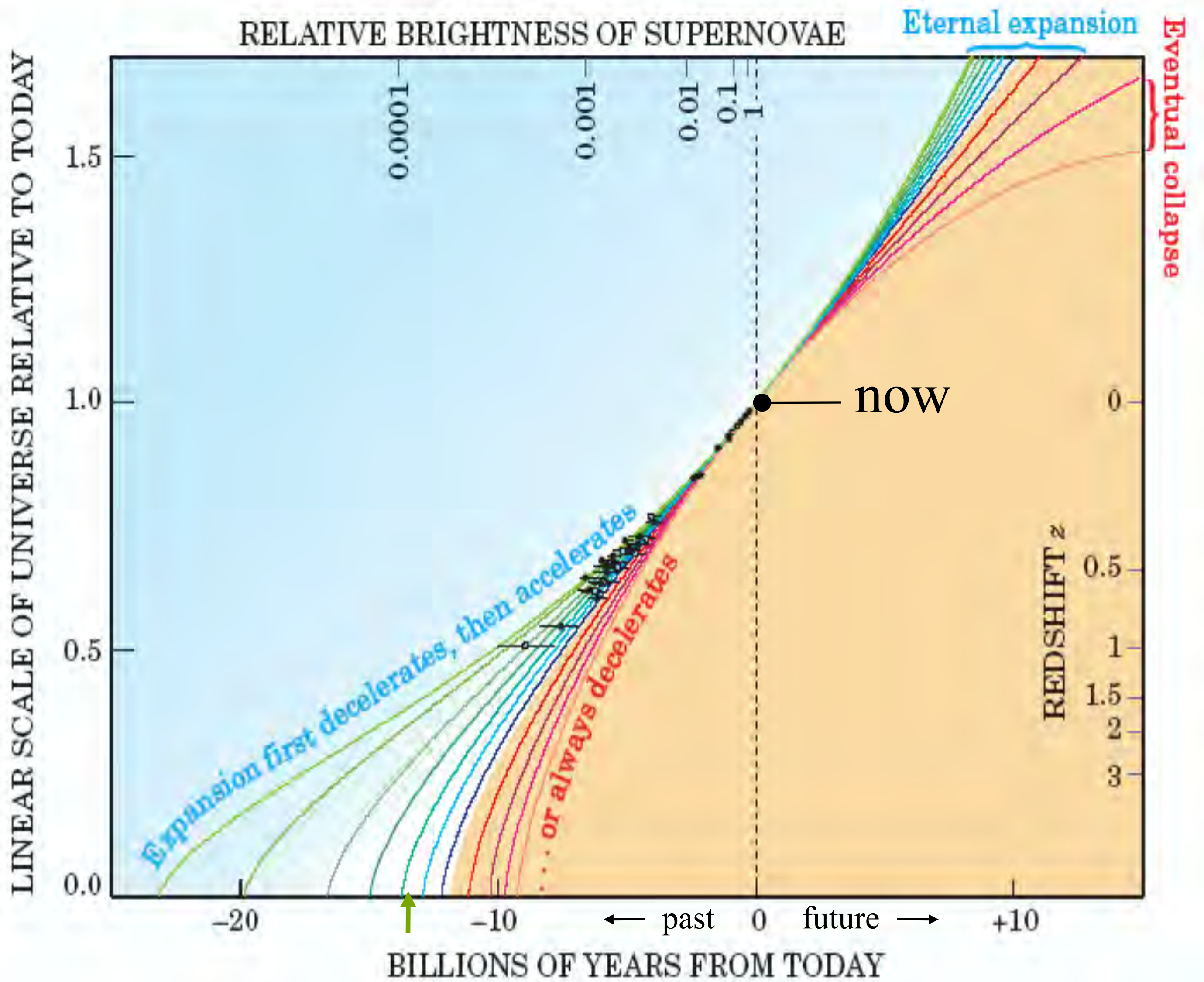
History of Cosmic Expansion for General Ω_M & Ω_Λ



History of Cosmic Expansion for $\Omega_\Lambda = 1 - \Omega_M$

With $\Omega_\Lambda = 0$ the age of the decelerating universe would be only 9 Gyr, but $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$ gives an age of 14Gyr, consistent with stellar and radioactive decay ages

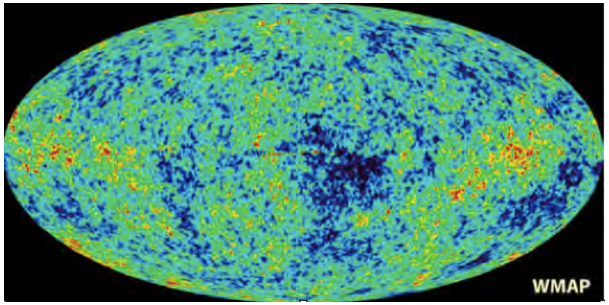
Figure 4. The history of cosmic expansion, as measured by the high-redshift supernovae (the black data points), assuming flat cosmic geometry. The scale factor R of the universe is taken to be 1 at present, so it equals $1/(1 + z)$. The curves in the blue shaded region represent cosmological models in which the accelerating effect of vacuum energy eventually overcomes the decelerating effect of the mass density. These curves assume vacuum energy densities ranging from 0.95 ρ_c (top curve) down to 0.4 ρ_c. In the yellow shaded region, the curves represent models in which the cosmic expansion is always decelerating due to high mass density. They assume mass densities ranging (left to right) from 0.8 ρ_c up to 1.4 ρ_c. In fact, for the last two curves, the expansion eventually halts and reverses into a cosmic collapse.



Saul Perlmutter, ***Physics Today***, Apr 2003

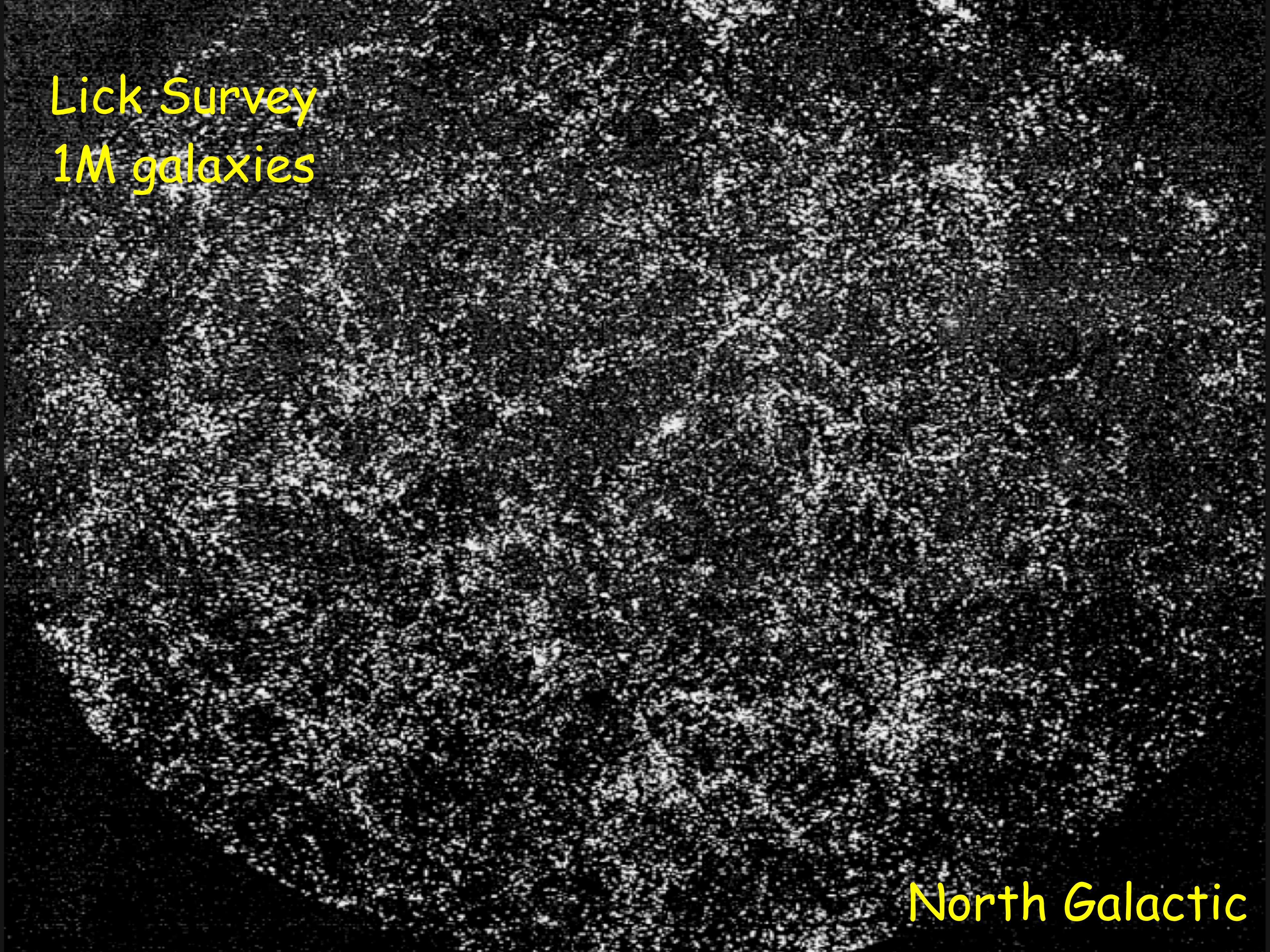
Brief History of the Universe

- Cosmic Inflation generates density fluctuations
- Symmetry breaking: more matter than antimatter
- All antimatter annihilates with almost all the matter (1s)
- Big Bang Nucleosynthesis makes light nuclei (10 min)
- Electrons and light nuclei combine to form atoms, and the cosmic background radiation fills the newly transparent universe (380,000 yr)

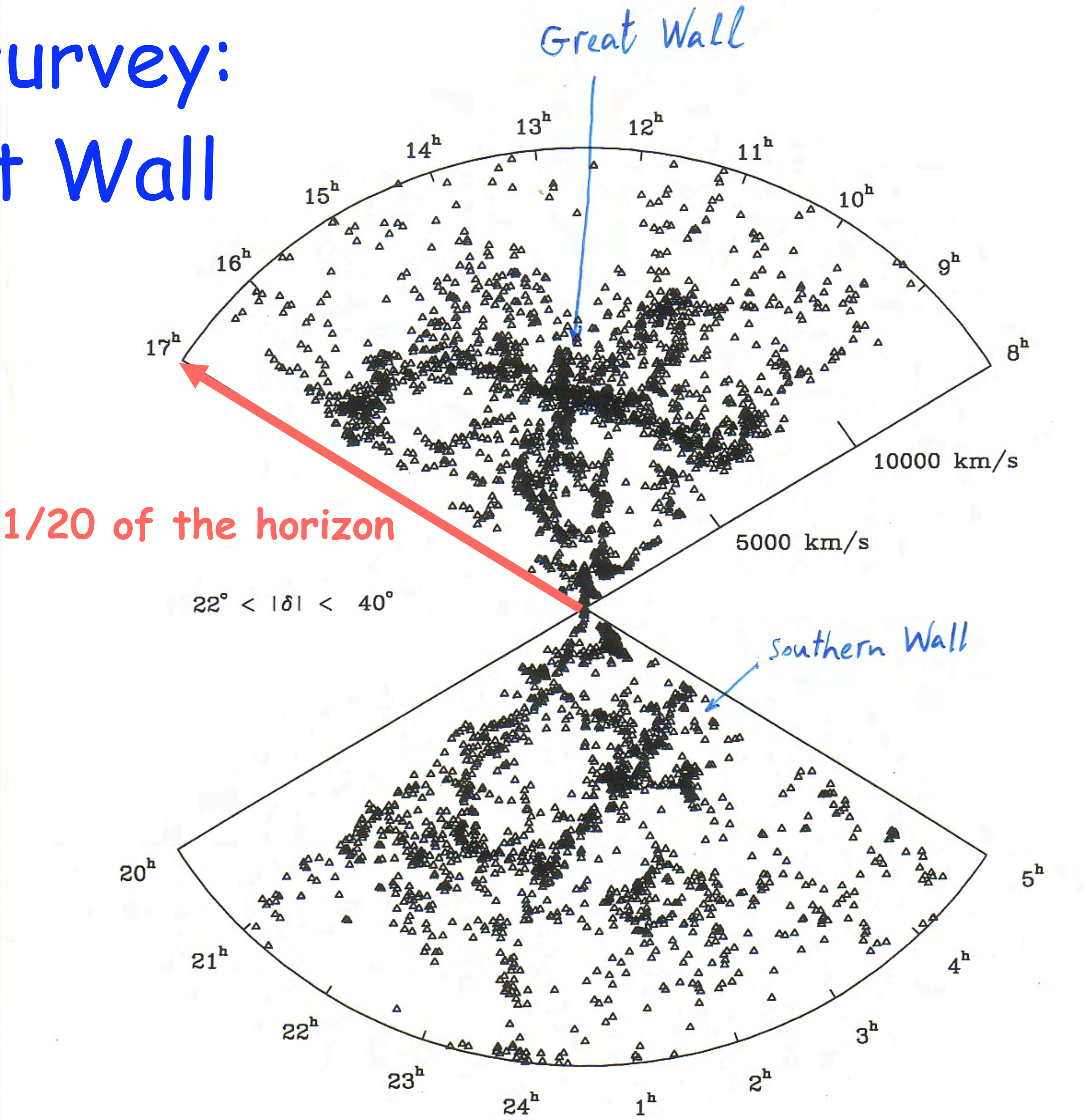


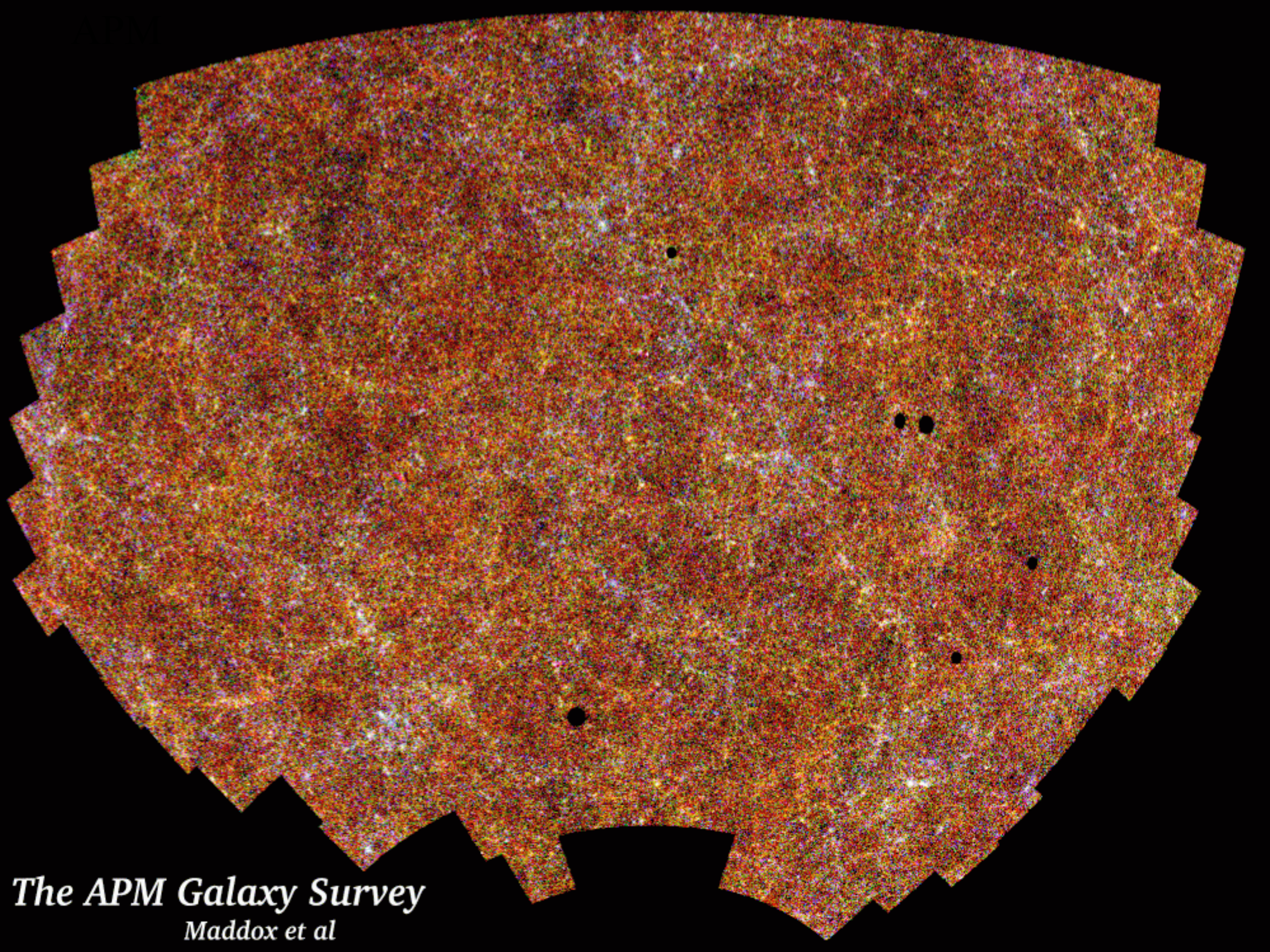
- Galaxies and larger structures form (~1 Gyr)
- Carbon, oxygen, iron, ... are made in stars
- Earth-like planets form around 2^{nd} generation stars
- Life somehow starts (~4 Gyr ago) and evolves on earth

Mapping the large scale structure of the universe ...

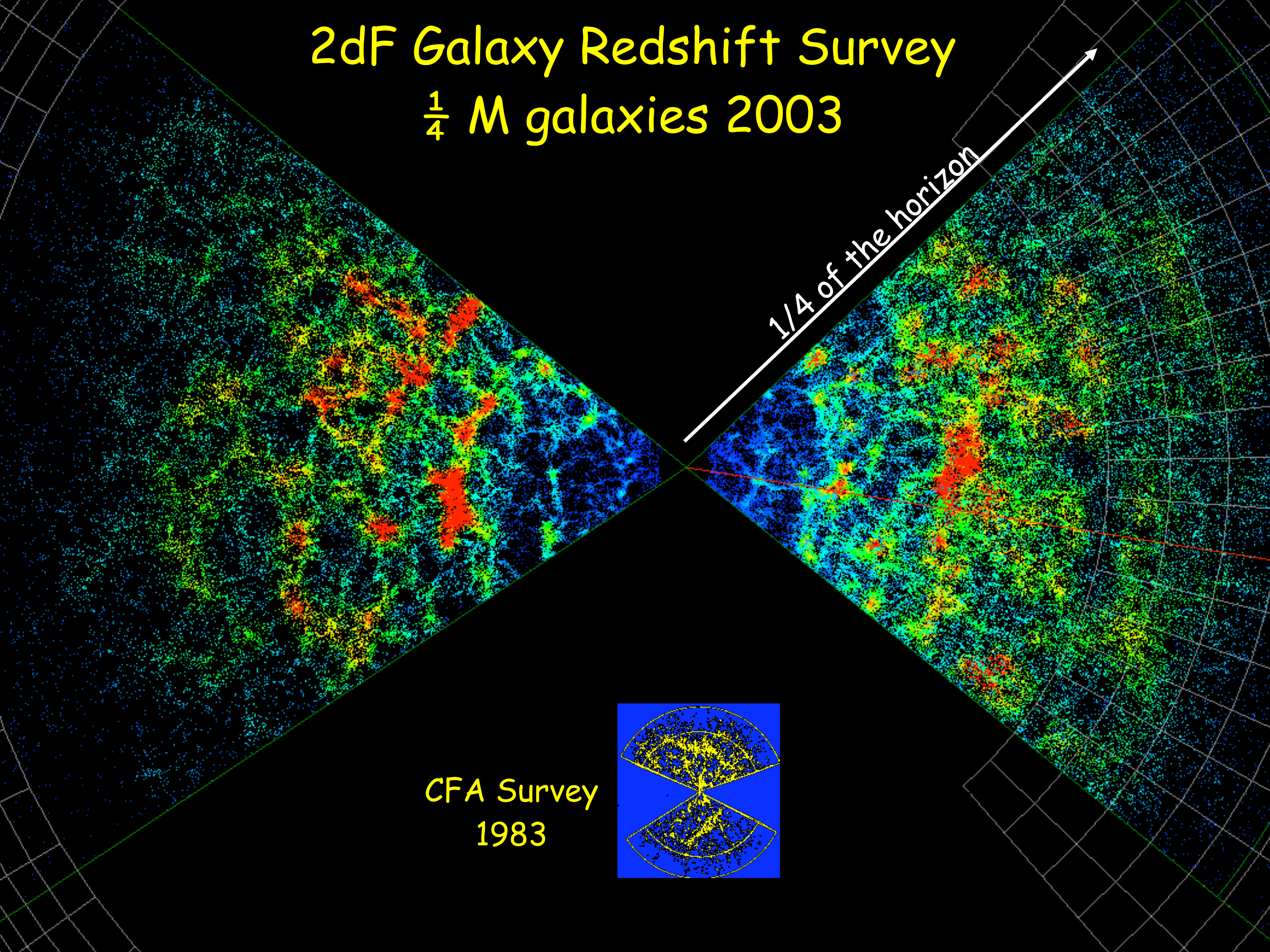

Lick Survey
1M galaxies
North Galactic

CfA survey: Great Wall



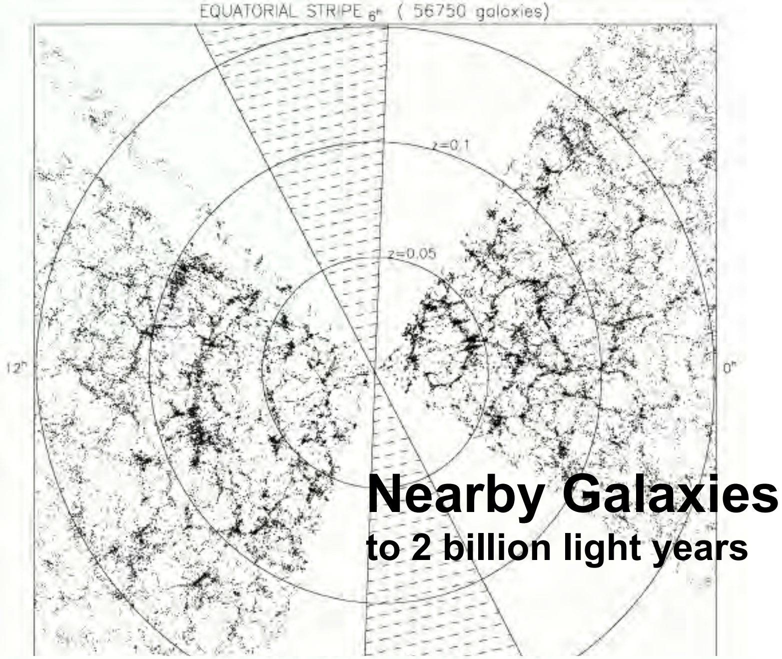

The APM Galaxy Survey
Maddox et al

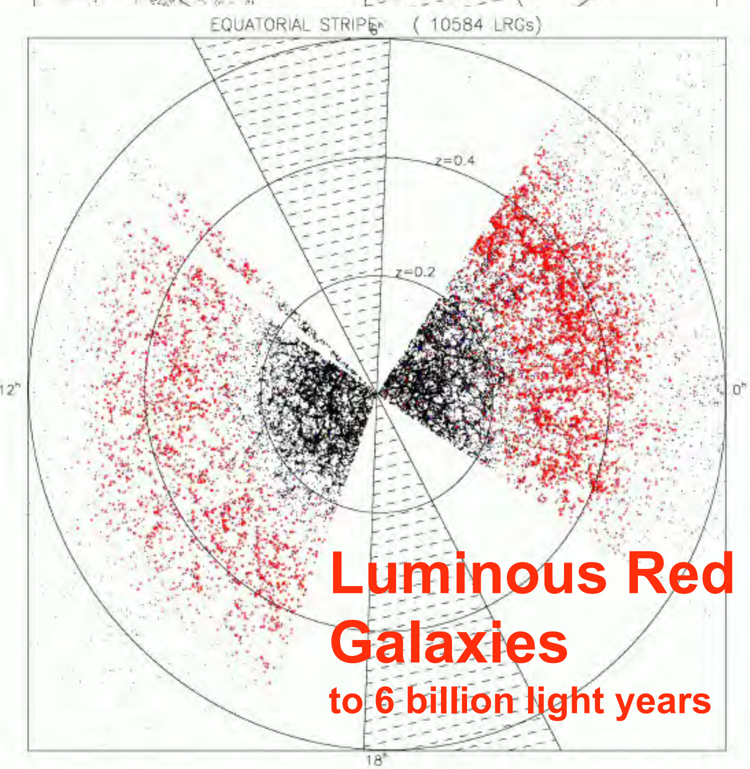
2dF Galaxy Redshift Survey
¼ M galaxies 2003

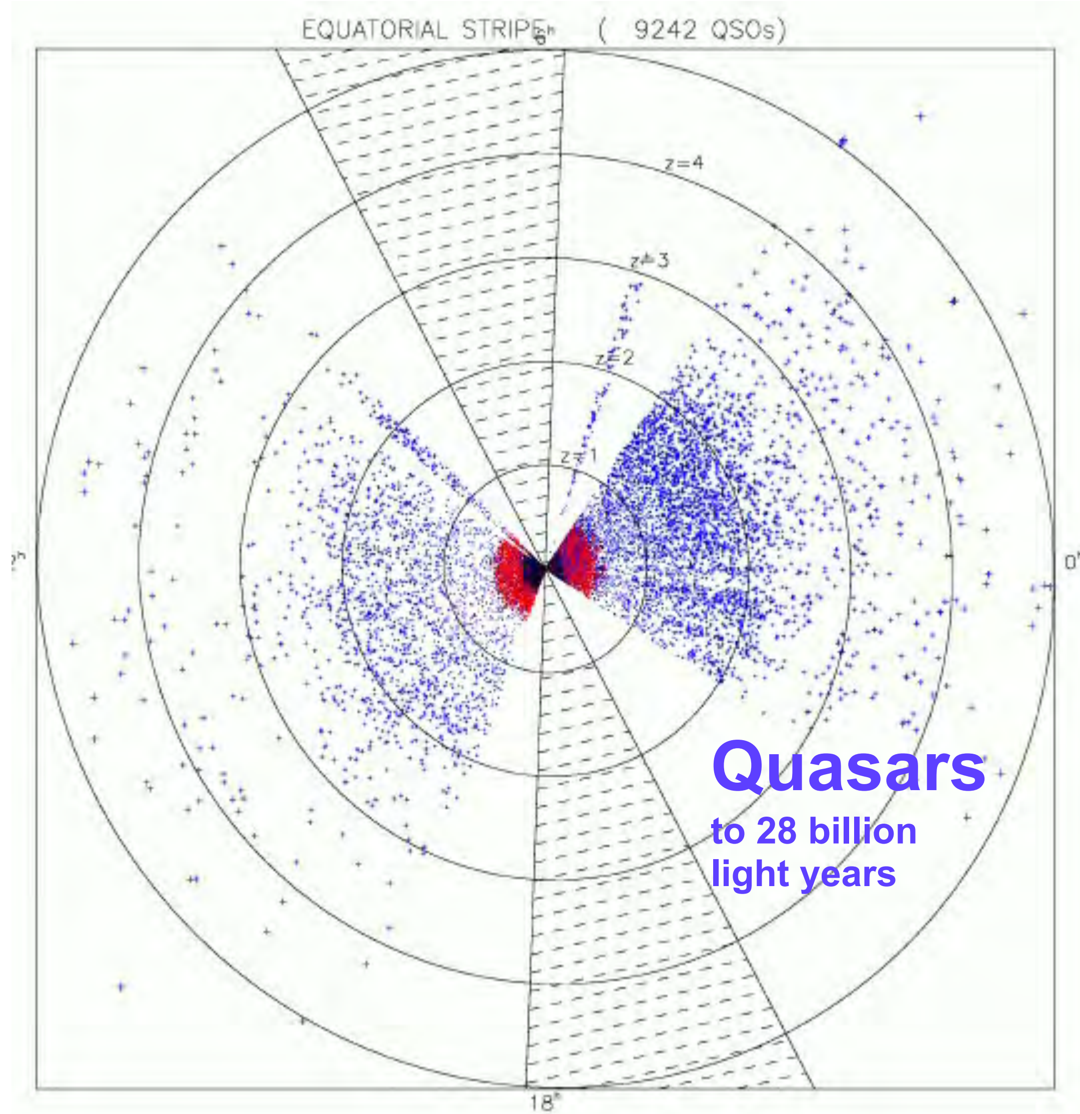


CFA Survey
1983

Mapping the Galaxies
Sloan Digital Sky Survey



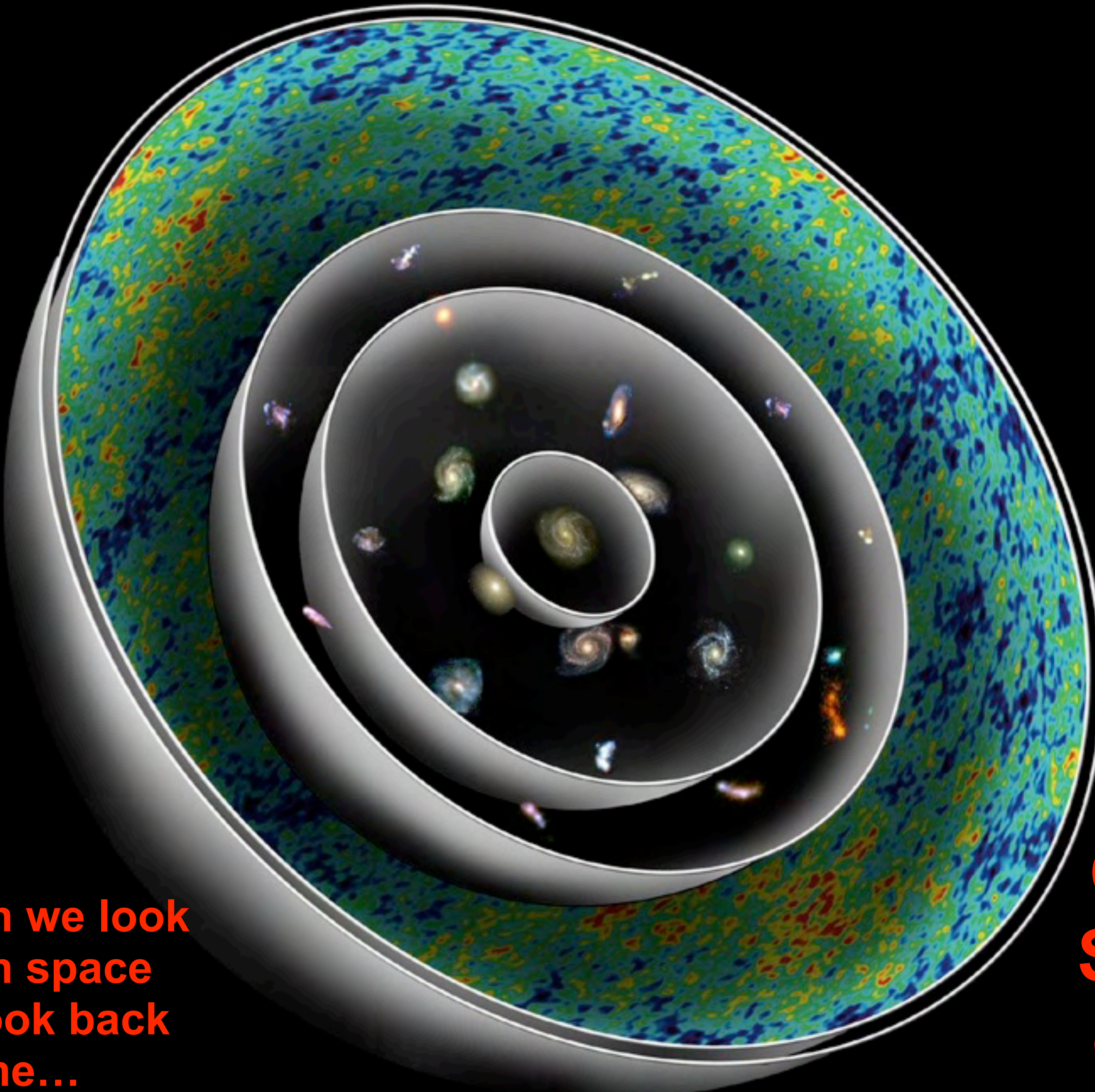




GALAXIES MAPPED BY THE SLOAN SURVEY

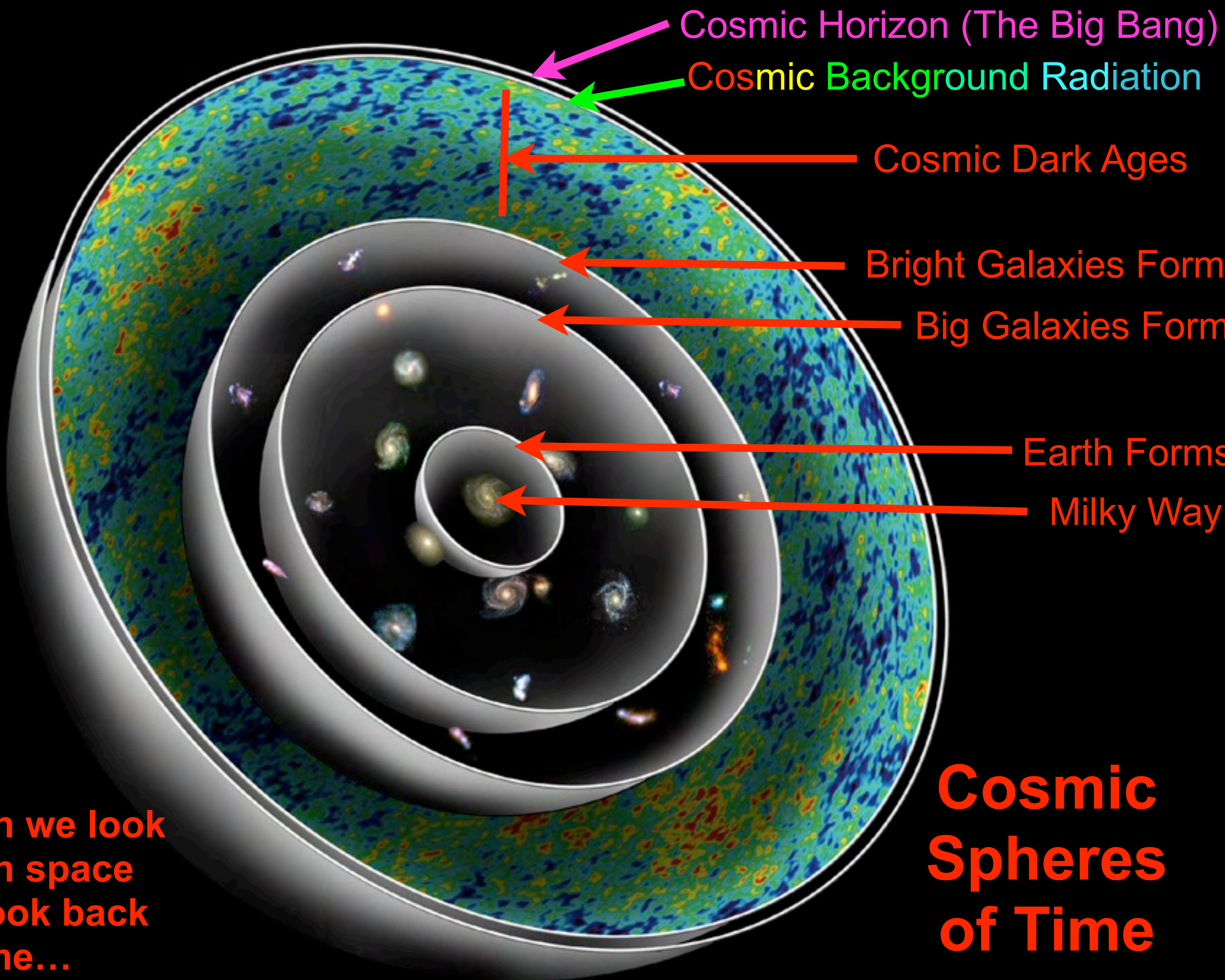
Data Release 4:
565,715 Galaxies & 76,403 Quasars

Cosmic Spheres of Time



When we look out in space we look back in time…

Cosmic Spheres of Time



When we look out in space we look back in time…

Medieval Universe

The geocentric pre-Copernican Universe in Christian Europe. At center, Earth is divided into Heaven (tan) and Hell (brown). The elements water (green), air (blue) and fire (red) surround the Earth. Moving outward, concentrically, are the spheres containing the seven planets, the Moon and the Sun, as well as the "Twelve Orders of the Blessed Spirits," the Cherubim and the Seraphim. German manuscript, c. 1450.