Astro/Physics 224 Winter 2008

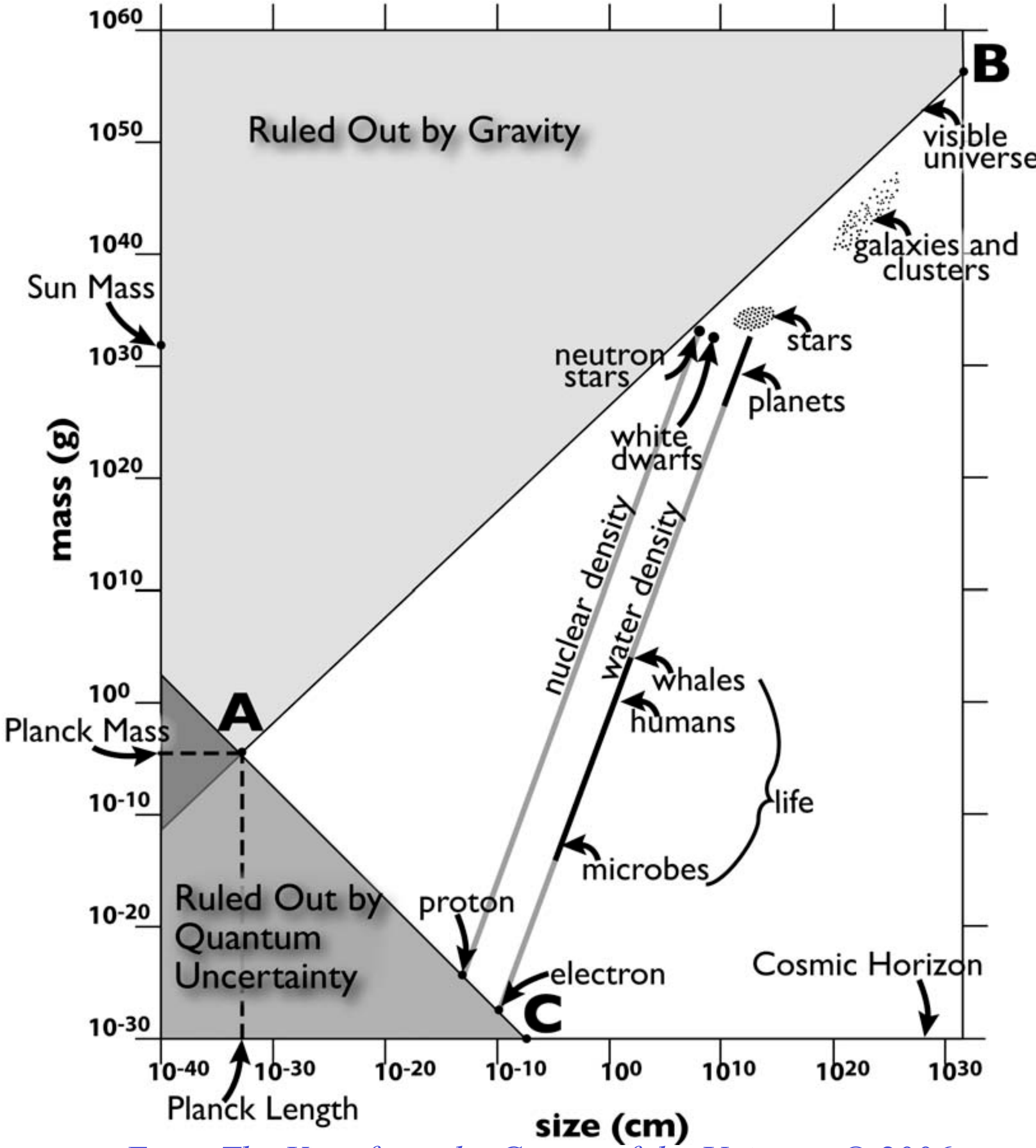
Origin and Evolution of the Universe

Dark Matter I

Friday, February 1

Joel Primack
University of California, Santa Cruz

The Wedge of Material Reality



From *The View from the Center of the Universe* © 2006

The **Planck Length**

$$l_{Pl} = \sqrt{\frac{hG}{2\pi c^3}} = 1.6 \times 10^{-33}\ \text{cm}$$

is the smallest possible length. Here h is Planck's constant

$$h = 6.626068 \times 10^{-34}\ \text{m}^2\ \text{kg} / \text{s}$$

The Planck Mass is

$$m_{Pl} = \sqrt{\frac{hc}{2\pi G}} = 2.2 \times 10^{-5}\ \text{g}$$

The Compton (i.e. quantum) wavelength $l_C = \dfrac{h}{2\pi mc}$

equals the Schwarzschild radius $l_S \approx \dfrac{Gm}{c^2}$

when $m = m_{Pl}$

In addition to the textbooks listed on the Syllabus for this course, a good place to start looking for up-to-date information is the Particle Data Group website http://pdg.lbl.gov

For example, there are 2007 Mini-Reviews of

Big Bang Nucleosynthesis including a discussion of ^{7}Li
http://pdg.lbl.gov/2007/reviews/bigbangnucrpp.pdf

Big-Bang Cosmology
http://pdg.lbl.gov/2007/reviews/bigbangrpp.pdf

Cosmological Parameters
http://pdg.lbl.gov/2007/reviews/hubblerpp.pdf

CMB http://pdg.lbl.gov/2007/reviews/microwaverpp.pdf

and Dark Matter http://pdg.lbl.gov/2007/reviews/darkmatrpp.pdf

(Re)combination: $e^- + p \rightarrow H$

As long as e- + p ⇔ H remains in equilibrium, the condition

$$\left\{ \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right\} = 0$$ with 1 = e^-, 2 = p, 3 = H, ensures that $$\frac{n_e n_p}{n_H} = \frac{n_e^{(0)} n_p^{(0)}}{n_H^{(0)}}.$$

Neutrality ensures $n_p = n_e$. Defining the free electron fraction $X_e \equiv \frac{n_e}{n_e + n_H} = \frac{n_p}{n_p + n_H}$,

the equation above becomes $$\frac{X_e^2}{1 - X_e} = \frac{1}{n_e + n_H} \left[\left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-[m_e + m_p - m_H]/T} \right]$$, which

ε = 13.6 eV

is known as the *Saha equation*. When T ~ ε, the rhs ~ 10^{15}, so X_e is very close to 1 and very little recombination has yet occurred. As T drops, the free electron fraction also drops, and as it approaches 0 equilibrium cannot be maintained. To follow the freezeout of the electron fraction, it is necessary to use the Boltzmann equation

$$a^{-3} \frac{d(n_e a^3)}{dt} = n_e^{(0)} n_p^{(0)} \langle \sigma v \rangle \left\{ \frac{n_H}{n_H^{(0)}} - \frac{n_e^2}{n_e^{(0)} n_p^{(0)}} \right\}$$

$$= n_b \langle \sigma v \rangle \left\{ (1 - X_e) \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-\epsilon_0/T} - X_e^2 n_b \right\}$$

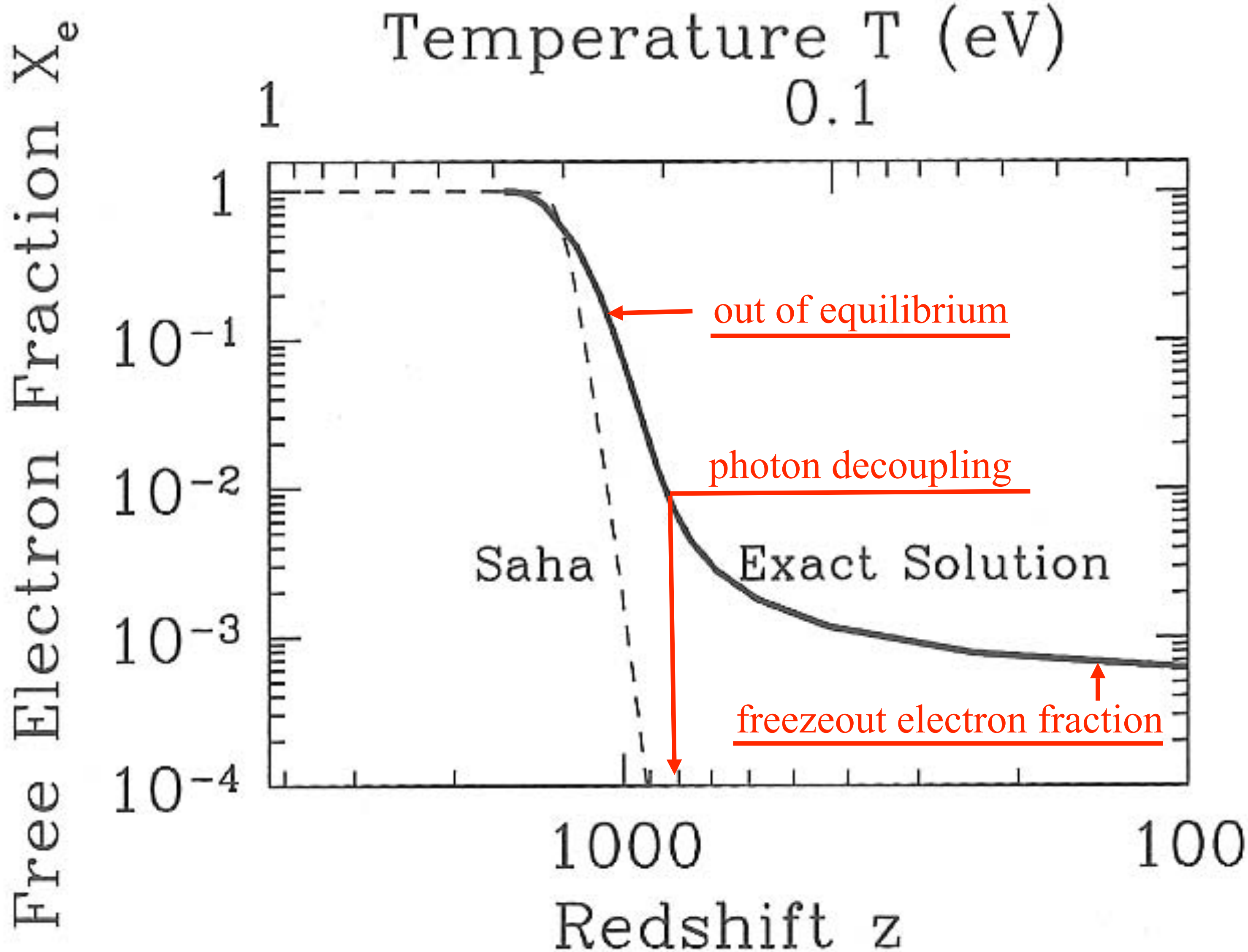
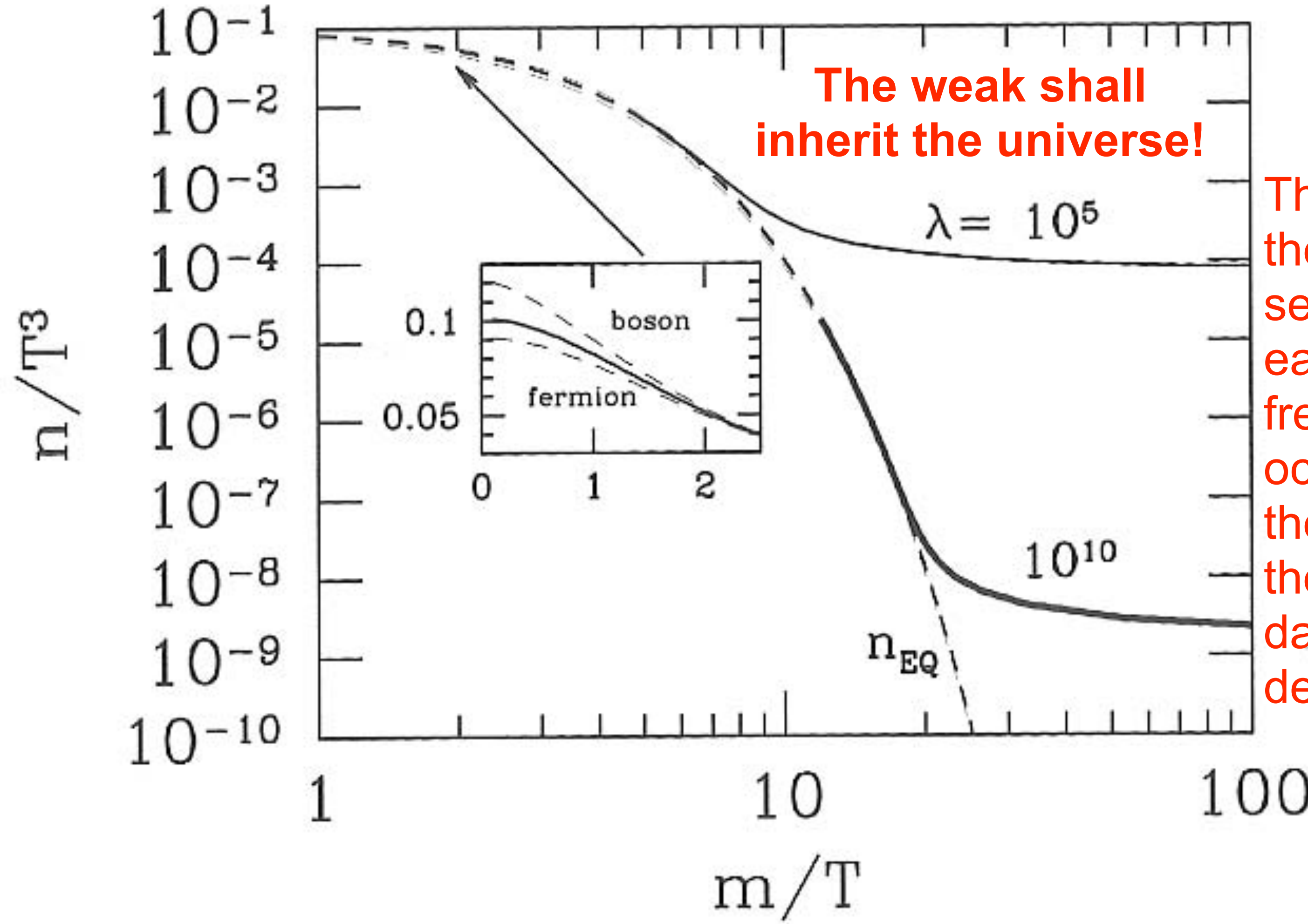


Figure 3.4. Free electron fraction as a function of redshift. Recombination takes place suddenly at $z \sim 1000$ corresponding to $T \sim 1/4$ eV. The Saha approximation, Eq. (3.37), holds in equilibrium and correctly identifies the redshift of recombination, but not the detailed evolution of X_e. Here $\Omega_b = 0.06, \Omega_m = 1, h = 0.5$.

Dodelson, *Modern Cosmology*, p. 72

Dark Matter Annihilation



The weaker the cross section, the earlier freezeout occurs, and the larger the resulting dark matter density.

Figure 3.5. Abundance of heavy stable particle as the temperature drops beneath its mass. Dashed line is equilibrium abundance. Two different solid curves show heavy particle abundance for two different values of λ, the ratio of the annihilation rate to the Hubble rate. Inset shows that the difference between quantum statistics and Boltzmann statistics is important only at temperatures larger than the mass.

Dodelson, *Modern Cosmology*, p. 76

Dark Matter Annihilation

The abundance today of dark matter particles X of the WIMP variety is determined by their survival of annihilation in the early universe. Supersymmetric neutralinos can annihilate with each other (and sometimes with other particles: "co-annihilation"). Dark matter annihilation follows the same pattern as the previous discussions: initially the abundance of dark matter particles X is given by the equilibrium Boltzmann exponential $\exp(-m_X/T)$, but as they start to disappear they have trouble finding each other and eventually their number density freezes out. The freezeout process can be followed using the Boltzmann equation, as discussed in Kolb and Turner, Dodelson, Mukhanov, and other textbooks. For a detailed discussion of Susy WIMPs, see the review article by Jungman, Kamionkowski, and Griest (1996). The result is that the abundance today of WIMPs X is given in most cases by (Dodelson's Eqs. 3.59-60)

$$\Omega_X = \left[\frac{4\pi^3 G g_*(m)}{45}\right]^{1/2} \frac{x_f T_0^3}{30\langle\sigma v\rangle\rho_{cr}} = 0.3h^{-2}\left(\frac{x_f}{10}\right)\left(\frac{g_*(m)}{100}\right)^{1/2}\frac{10^{-39}\mathrm{cm}^2}{\langle\sigma v\rangle}.$$

Here $x_f \approx 10$ is the ratio of m_X to the freezeout temperature T_f, and $g_*(m_X) \approx 100$ is the density of states factor in the expression for the energy density of the universe when the temperature equals m_X

$$\rho = \frac{\pi^2}{30}T^4\left[\sum_{i=\mathrm{bosons}} g_i + \frac{7}{8}\sum_{i=\mathrm{fermions}} g_i\right] \equiv g_*\frac{\pi^2}{30}T^4.$$

The sum is over relativistic species *i* (see the graph of *g*(*T*) on the next slide). Note that more X's survive, the weaker the cross section σ. For Susy WIMPs the natural values are $\sigma \sim 10^{-39}$ cm^2, so $\Omega_X \approx 1$ naturally.

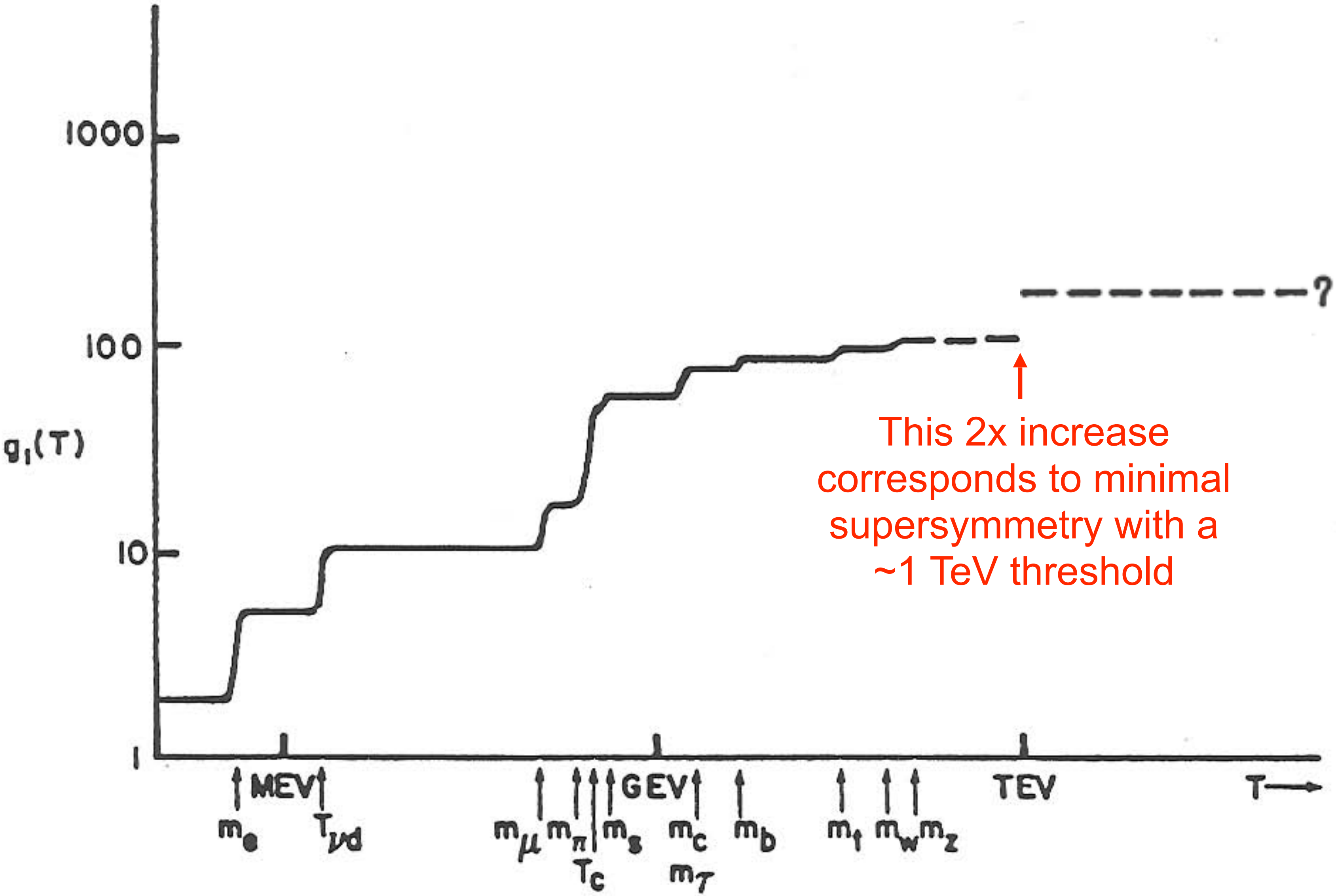


Fig. 1 The effective number of degrees of freedom of thermally interacting relativistic particles as a function of temperature.

Supersymmetry is the basis of most attempts, such as superstring theory, to go beyond the current “Standard Model” of particle physics. Heinz Pagels and Joel Primack pointed out in a 1982 paper that the lightest supersymmetric partner particle is stable because of R-parity, and is thus a good candidate for the dark matter particles – weakly interacting massive particles (**WIMP**s).

Michael Dine and others pointed out that the **axion**, a particle needed to save the strong interactions from violating CP symmetry, could also be the dark matter particle. Searches for both are underway.

Supersymmetric WIMPs

When the British physicist Paul Dirac first combined Special Relativity with quantum mechanics, he found that this predicted that for every ordinary particle like the electron, there must be another particle with the opposite electric charge – the anti-electron (positron). Similarly, corresponding to the proton there must be an anti-proton. Supersymmetry appears to be required to combine General Relativity (our modern theory of space, time, and gravity) with the other forces of nature (the electromagnetic, weak, and strong interactions). The consequence is another doubling of the number of particles, since supersymmetry predicts that for every particle that we now know, including the antiparticles, there must be another, thus far undiscovered particle with the same electric charge but with *spin* differing by half a unit.

Spin	Matter (fermions)	Forces (bosons)
2		graviton
1		photon, $W^{\pm}, Z^0$ gluons
1/2	quarks u,d,... leptons $e, \nu_e, \ldots$	
0		Higgs bosons axion

Supersymmetric WIMPs

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after doubling

Spin	**Matter** (fermions)	**Forces** (bosons)	**Hypothetical Superpartners**	**Spin**
2		graviton	gravitino	3/2
1		photon, $W^{\pm}$, Z^0 gluons	photino, winos, zino, gluinos	1/2
1/2	quarks u,d,... leptons $e, \nu_e, \ldots$		squarks $\tilde{u}, \tilde{d}, \ldots$ sleptons $\tilde{e}, \tilde{\nu}_e, \ldots$	0
0		Higgs bosons axion	Higgsinos axinos	1/2

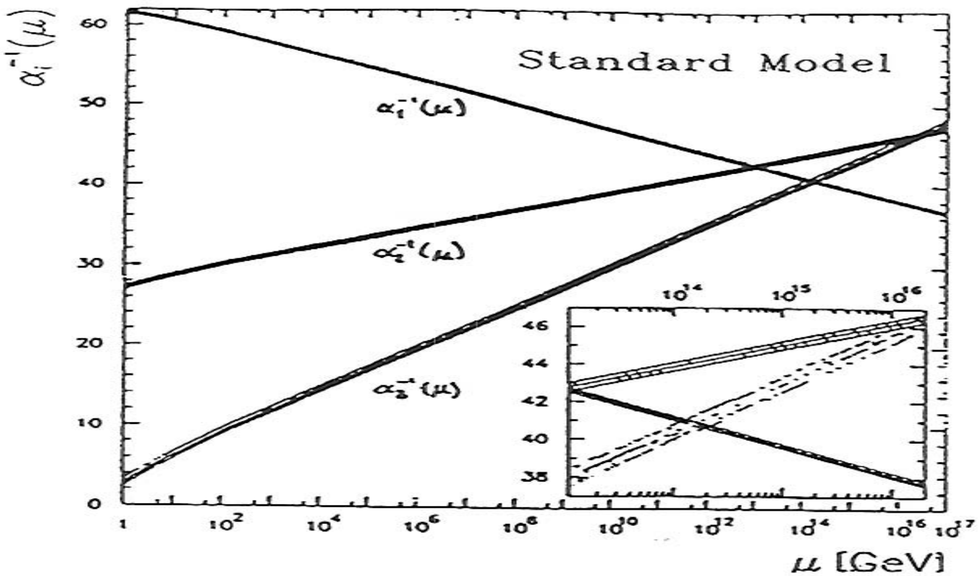
Note: Supersymmetric cold dark matter candidate particles are underlined.

Supersymmetric WIMPs, continued

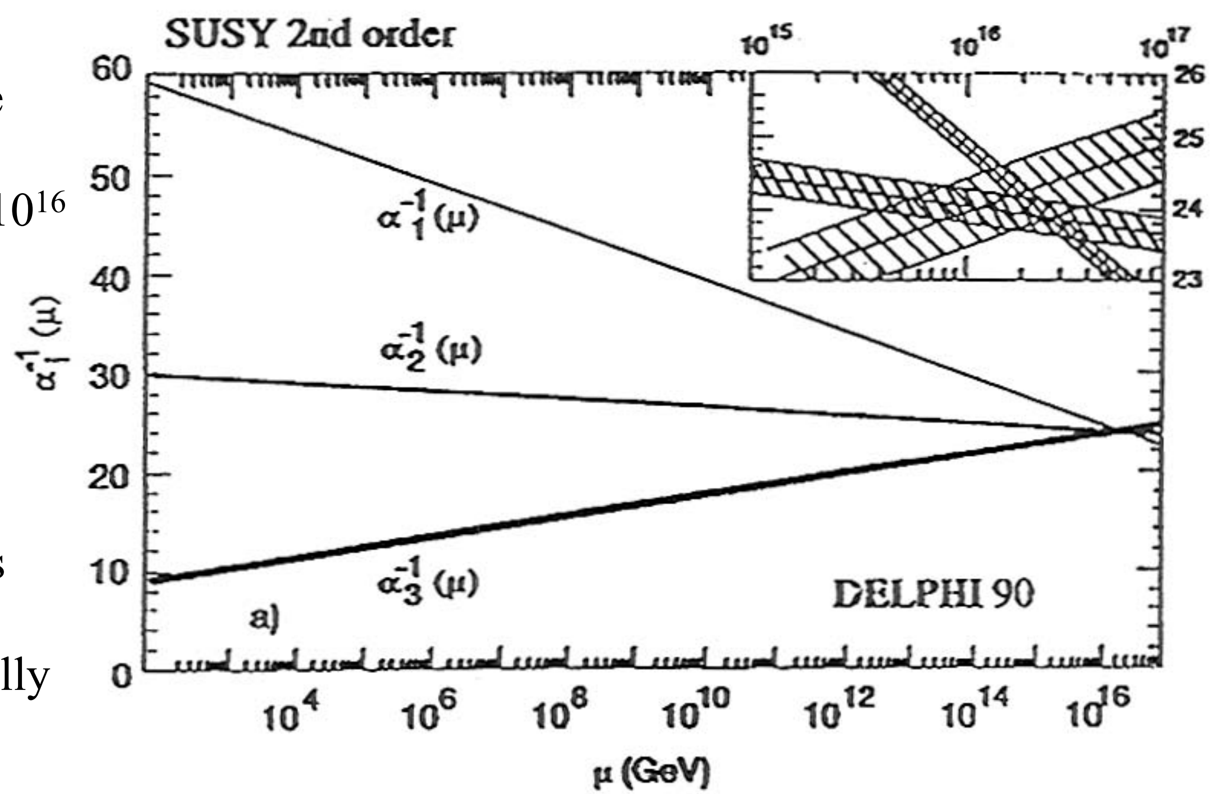
Spin is a fundamental property of elementary particles. Matter particles like electrons and quarks (protons and neutrons are each made up of three quarks) have spin ½, while force particles like photons, W,Z, and gluons have spin 1. The supersymmetric partners of electrons and quarks are called selectrons and squarks, and they have spin 0. The supersymmetric partners of the force particles are called the photino, Winos, Zino, and gluinos, and they have spin ½, so they might be matter particles. The lightest of these particles might be the photino. Whichever is lightest should be stable, so it is a natural candidate to be the dark matter WIMP. Supersymmetry does not predict its mass, but it must be more than 50 times as massive as the proton since it has not yet been produced at accelerators. But it will be produced soon at the LHC, if it exists and its mass is not above ~1 TeV!

SUPERSYMMETRY

The only experimental evidence for supersymmetry is that running of coupling constants in the Standard Model does not lead to Grand Unification (of the weak, electromagnetic, and strong interactions)



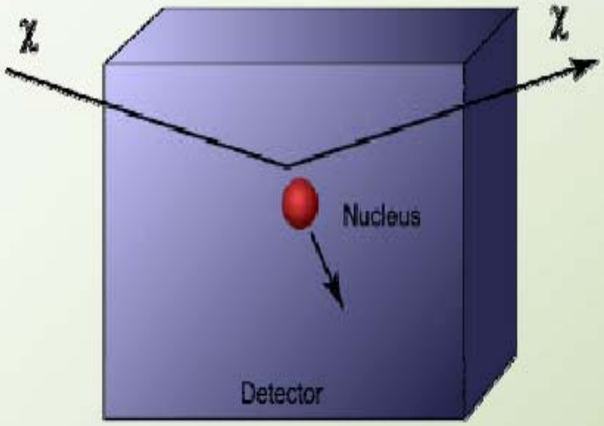
while with supersymmetry the three couplings all do come together at a scale just above 10^{16} GeV.



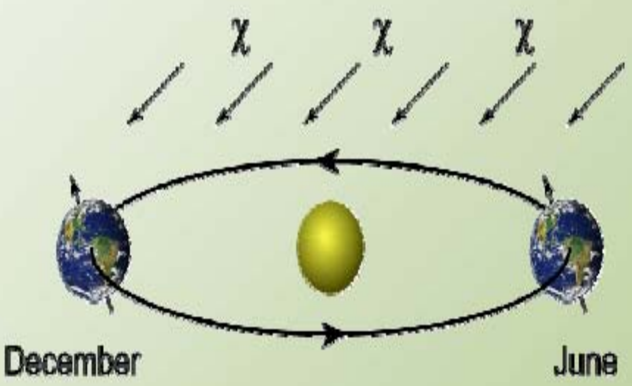
Other arguments for SUSY include: helps unification of gravity since it controls the vacuum energy and moderates loop divergences, solves the hierarchy problem, and naturally leads to DM with Ω≈1.

Experiments are Underway for Detection of WIMPs

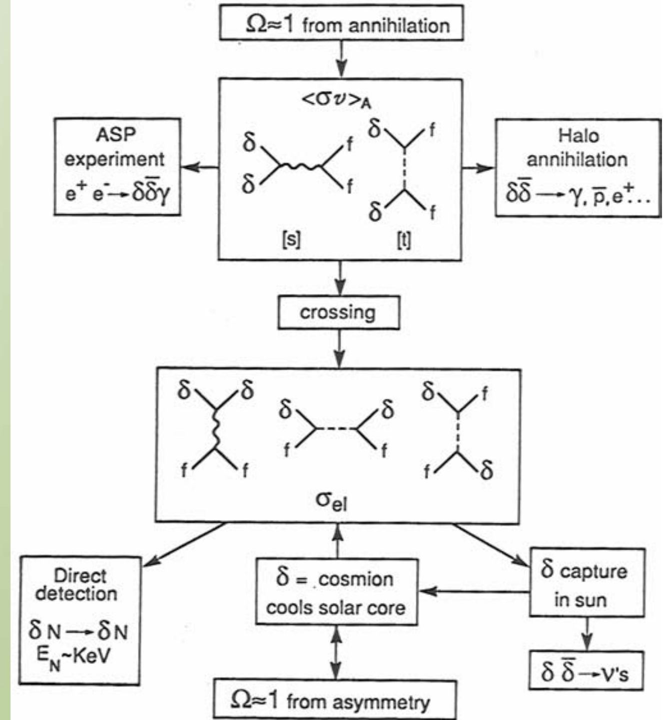
Direct detection - general principles

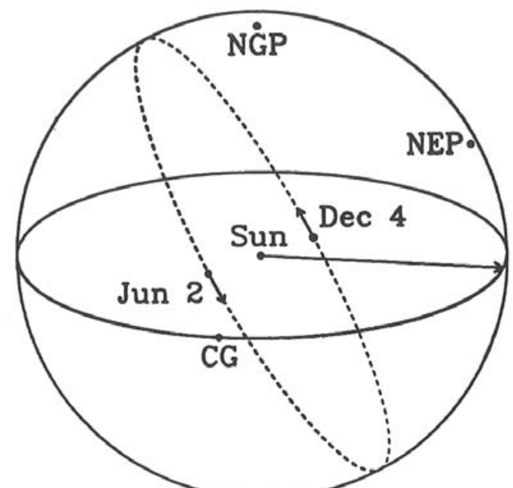


- WIMP + nucleus → WIMP + nucleus
- Measure the nuclear recoil energy
- Suppress backgrounds enough to be sensitive to a signal, or...



- Search for an annual modulation due to the Earth's motion around the Sun

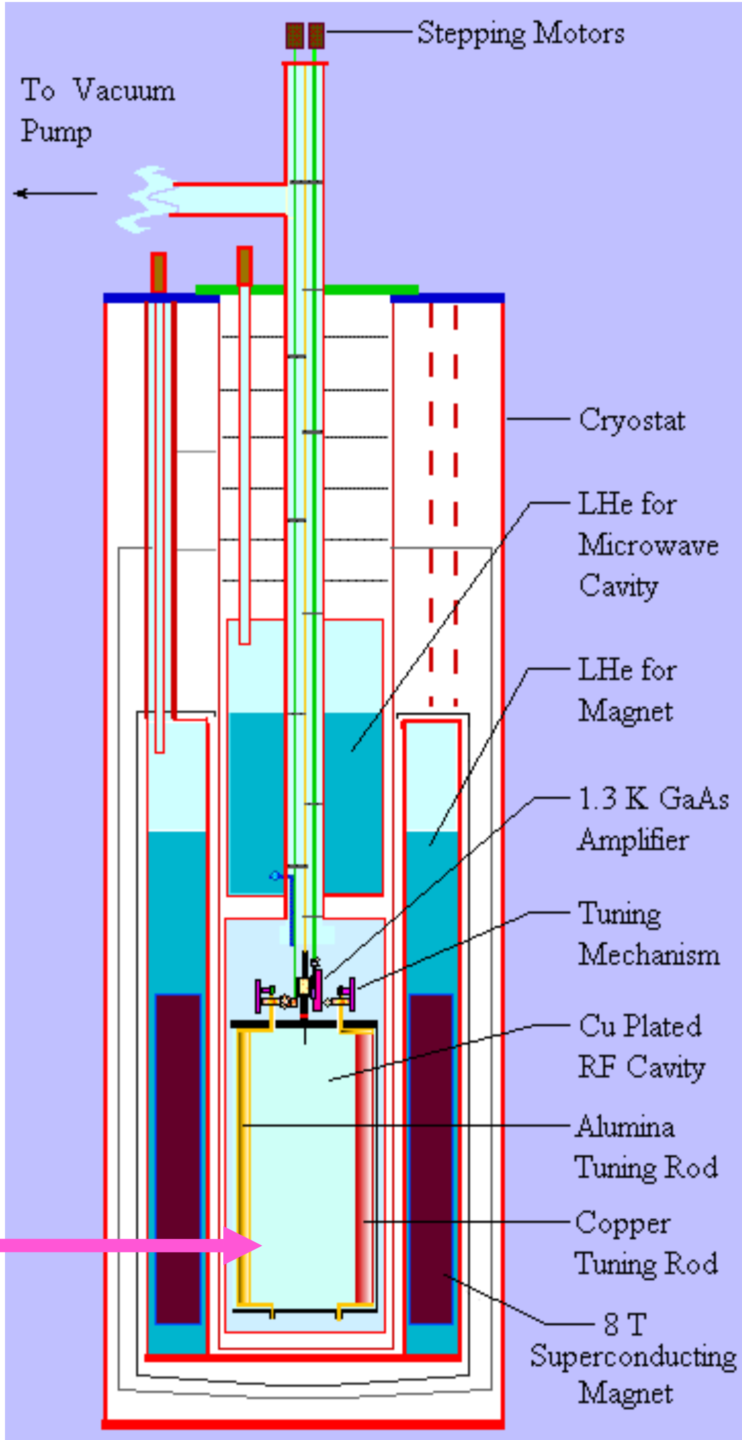




Primack, Seckel, & Sadoulet (1987)

and also AXIONs

The diagram at right shows the layout of the axion search experiment now underway at the Lawrence Livermore National Laboratory. Axions would be detected as extra photons in the Microwave Cavity.



Types of Dark Matter

Ω_i represents the fraction of the critical density $\rho_c = 10.54\ h^2$ keV/cm^3 needed to close the Universe, where h is the Hubble constant H_0 divided by 100 km/s/*Mpc*.

Dark Matter Type	Fraction of Critical Density	Comment
Baryonic	$\Omega_b \sim 0.04$	about 10 times the visible matter
Hot	$\Omega_\nu \sim 0.001$–0.1	light neutrinos
Cold	$\Omega_c \sim 0.3$	most of the dark matter in galaxy halos

Dark Matter and Associated Cosmological Models

Ω_m represents the fraction of the critical density in all types of matter.
Ω_Λ is the fraction contributed by some form of "dark energy."

Acronym	Cosmological Model	Flourished
HDM	hot dark matter with $\Omega_m = 1$	1978–1984
SCDM	standard cold dark matter with $\Omega_m = 1$	1982–1992
CHDM	cold + hot dark matter with $\Omega_c \sim 0.7$ and $\Omega_\nu = 0.2$–0.3	1994–1998
ΛCDM	cold dark matter $\Omega_c \sim 1/3$ and $\Omega_\Lambda \sim 2/3$	1996–today

Joel Primack, *Beam Line*, Fall 2001

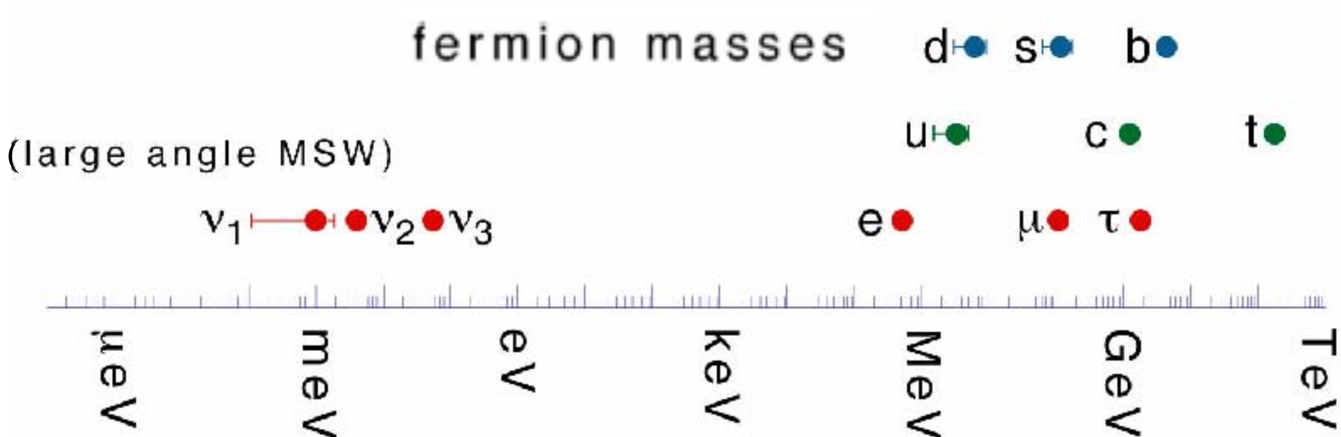
THE ATMOSPHERIC-NEUTRINO DATA from the Super-Kamiokande underground neutrino detector in Japan provide strong evidence of muon to tau neutrino oscillations, and therefore that these neutrinos have nonzero mass (see the article by John Learned in the Winter 1999 *Beam Line*, Vol. 29, No. 3). This result is now being confirmed by results from the K2K experiment, in which a muon neutrino beam from the KEK accelerator is directed toward Super-Kamiokande and the number of muon neutrinos detected is about as expected from the atmospheric-neutrino data (see article by Jeffrey Wilkes and Koichiro Nishikawa, this issue).

But oscillation experiments cannot measure neutrino masses directly, only the squared mass difference $\Delta m^2_{ij} = |m_i^2 - m_j^2|$ between the oscillating species. The Super-Kamiokande atmospheric neutrino data imply that $1.7\times10^{-4} < \Delta m^2_{\tau\mu} < 4\times10^{-3}$ eV² (90 percent confidence), with a central value $\Delta m^2_{\tau\mu} = 2.5\times10^{-3}$ eV². If the neutrinos have a hierarchical mass pattern $m_{\nu_e} << m_{\nu_\mu} << m_{\nu_\tau}$ like the quarks and charged leptons, then this implies that $\Delta m^2_{\tau\mu} \cong m^2_{\nu_\tau}$ so $m_{\nu_\tau} \sim 0.05$ eV.

These data then imply a lower limit on the HDM (or light neutrino) contribution to the cosmological matter density of $\Omega_\nu > 0.001$—almost as much as that of all the stars in the disks of galaxies. There is a connection between neutrino mass and the corresponding contribution to the cosmological density, because the thermodynamics of the early Universe specifies the abundance of neutrinos to be about 112 per cubic centimeter for each of the three species (including both neutrinos and antineutrinos). It follows that the density Ω_ν contributed by neutrinos is $\Omega_\nu = m(\nu)/(93\ h^2$ eV), where $m(\nu)$ is the sum of the masses of all three neutrinos. Since $h^2 \sim 0.5$, $m_{\nu_\tau} \sim 0.05$ eV corresponds to $\Omega_\nu \sim 10^{-3}$.

This is however a lower limit, since in the alternative case where the oscillating neutrino species have nearly equal masses, the values of the individual masses could be much larger. The only other laboratory approaches to measuring neutrino masses are attempts to detect neutrinoless double beta decay, which are sensitive to a possible Majorana component of the electron neutrino mass, and measurements of the endpoint of the tritium beta-decay spectrum. The latter gives an upper limit on the electron neutrino mass, currently taken to be 3 eV. Because of the small values of both squared-mass differences, this tritium limit becomes an upper limit on all three neutrino masses, corresponding to $m(\nu) < 9$ eV. A bit surprisingly, cosmology already provides a stronger constraint on neutrino mass than laboratory measurements, based on the effects of neutrinos on large-scale structure formation.

Joel Primack, *Beam Line*, Fall 2001



Hitoshi Murayama

Neutrino Properties

See the note on "Neutrino properties listings" in the Particle Listings.

Mass $m < 2$ eV (tritium decay)

Mean life/mass, $\tau/m > 300$ s/eV, CL = 90% (reactor)

Mean life/mass, $\tau/m > 7 \times 10^9$ s/eV (solar)

Mean life/mass, $\tau/m > 15.4$ s/eV, CL = 90% (accelerator)

Magnetic moment $\mu < 0.9 \times 10^{-10}\ \mu_B$, CL = 90% (reactor)

Number of Neutrino Types

Number $N = 2.994 \pm 0.012$ (Standard Model fits to LEP data)

Number $N = 2.93 \pm 0.05$ (S = 1.2) (Direct measurement of invisible Z width)

Neutrino Mixing

The following values are obtained through data analyses based on the 3-neutrino mixing scheme described in the review "Neutrino mass, mixing, and flavor change" by B. Kayser in this *Review*.

$\sin^2(2\theta_{12}) = 0.86^{+0.03}_{-0.04}$

$\Delta m^2_{21} = (8.0^{+0.4}_{-0.3}) \times 10^{-5}$ eV^2

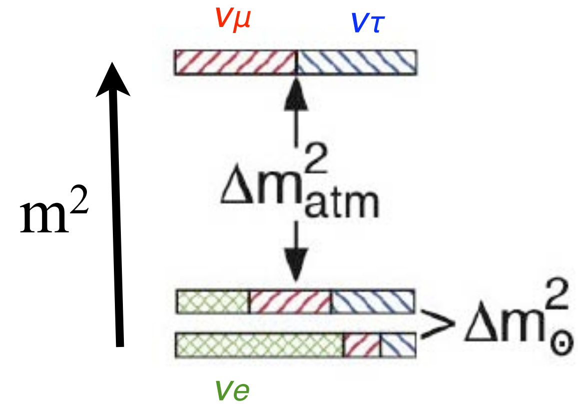
The ranges below for $\sin^2(2\theta_{23})$ and Δm^2_{32} correspond to the projections onto the appropriate axes of the 90% CL contours in the $\sin^2(2\theta_{23})$-Δm^2_{32} plane.

$\sin^2(2\theta_{23}) > 0.92$

$\Delta m^2_{32} = 1.9$ to 3.0×10^{-3} eV^2 [i]

$\sin^2(2\theta_{13}) < 0.19$, CL = 90%

Citation: W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006) (URL: http://pdg.lbl.gov)

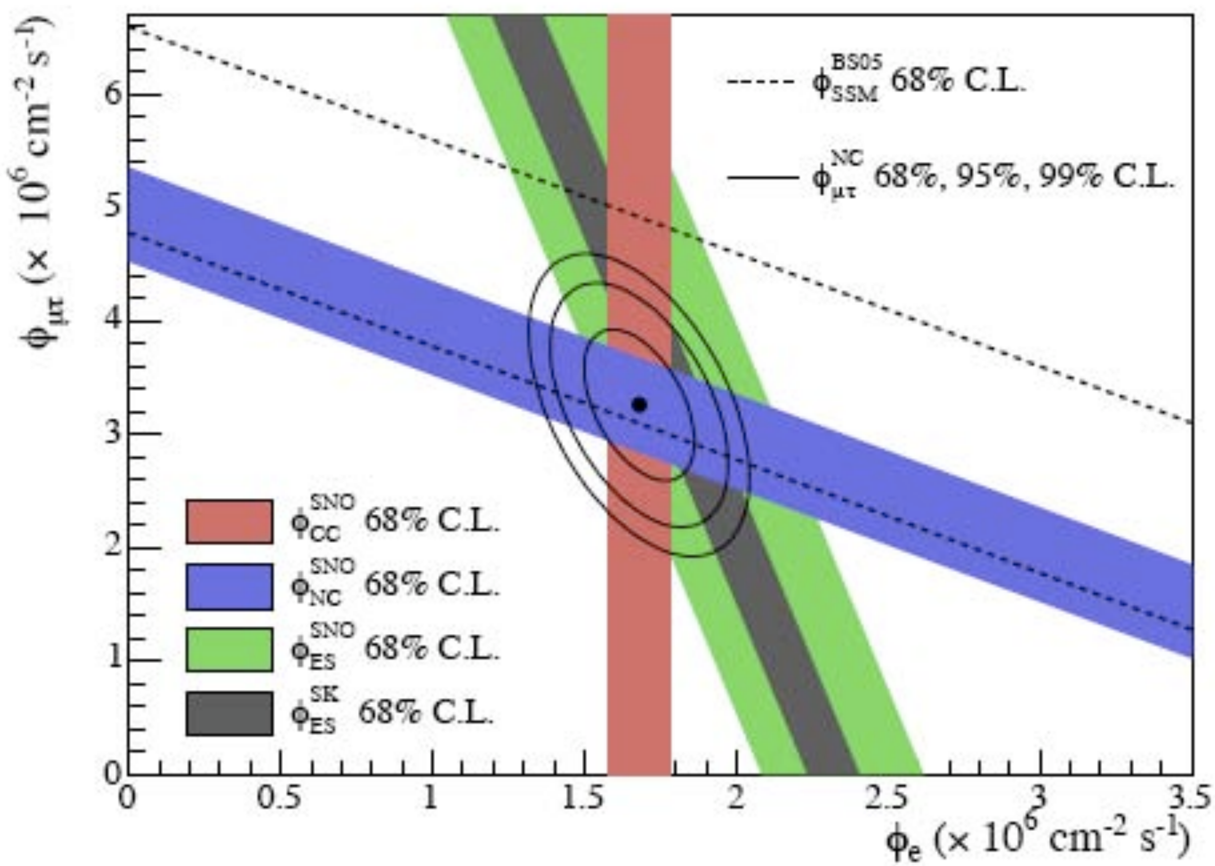


A three-neutrino squared-mass spectrum that accounts for the observed flavor changes of solar, reactor, atmospheric, and long-baseline accelerator neutrinos. The νe fraction of each mass eigenstate is crosshatched, the νμ fraction is indicated by right-leaning hatching, and the ντ fraction by left-leaning hatching. From B. Kaiser, http://pdg.lbl.gov/2007/reviews/

numixrpp.pdf

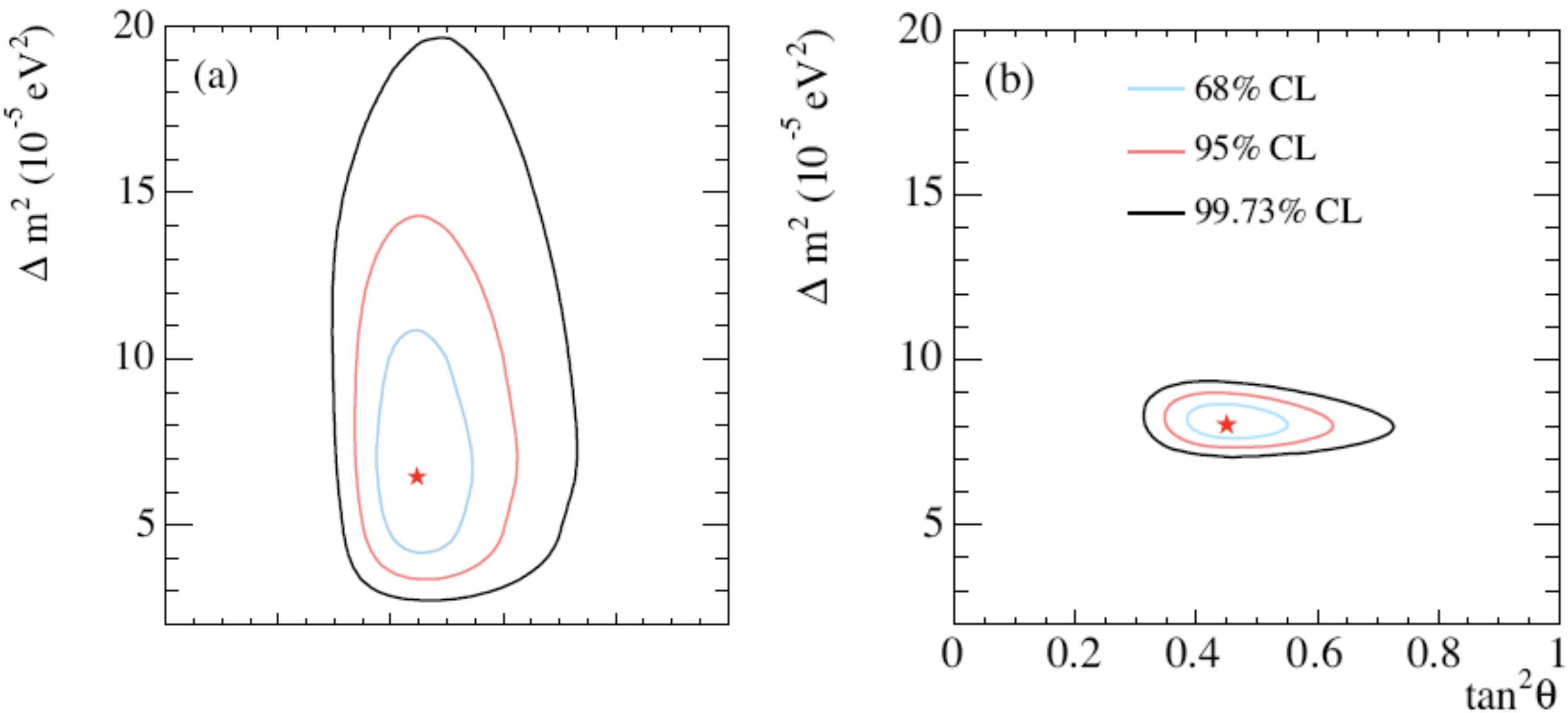
Sudbury Neutrino Observatory Confirms Solar Neutrinos Oscillate

$n \rightarrow p\, e^- \bar{\nu}_e$ must happen twice per ^4He, and then ~1/3 of the electron antineutrinos oscillate to mu or tau neutrinos



Fluxes of ^8B solar neutrinos, $\phi(\nu_e)$,and $\phi(\nu_{\mu \text{ or } \tau})$, deduced from the SNO's charged current (CC), ν_e elastic scattering (ES), and neutral-current (NC) results for the salt phase measurement. The Super-Kamiokande ES flux and the BS05(OP) standard solar model prediction are also shown. The bands represent the 1σ error. The contours show the 68%, 95%, and 99% joint probability for $\phi(\nu_e)$ and $\phi(\nu_{\mu \text{ or } \tau})$.

[From PDG 2005 review by K. Nakamura.]

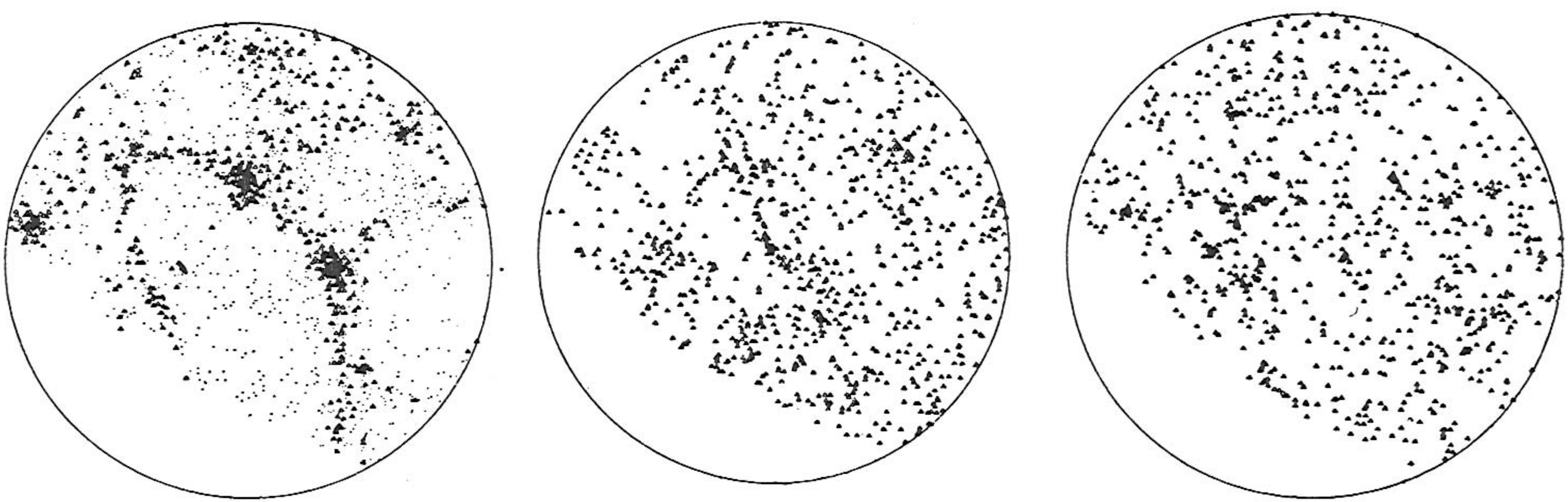


Update of the global neutrino oscillation contours given by the SNO Collaboration assuming that the ^{8}B neutrino flux is free and the *hep* neutrino flux is fixed. (a) Solar global analysis. (b) Solar global + KamLAND. [From PDG 2005 review by K. Nakamura.]

$$\Delta m_{12}{}^2 = 8 \text{x} 10^{-5}\ \text{eV}^2 \Rightarrow m_2 \geq 9 \text{x} 10^{-3}\ \text{eV}$$

Whatever Happened to Hot Dark Matter?

In ~1980, when purely baryonic adiabatic fluctuations were ruled out by the improving upper limits on CMB anisotropies, theorists led by Zel'dovich turned to what we now call the HDM scenario, with light neutrinos making up most of the dark matter. However, in this scheme the fluctuations on small scales are damped by relativistic motion ("free streaming") of the neutrinos until T becomes less than m_ν, which occurs when the mass entering the horizon is about 10^{15} solar masses, the supercluster mass scale. Thus superclusters would form first, and galaxies later by fragmentation. This predicted a galaxy distribution much more inhomogeneous than observed.



HDM Observed Galaxy Distribution CDM

Since 1984, the most successful structure formation scenarios have been those in which most of the matter is CDM. With the COBE CMB data in 1992, two CDM variants appeared to be viable: ΛCDM with $\Omega_m \approx 0.3$, and $\Omega_m = 1$Cold+Hot DM with $\Omega_\nu \approx 0.2$. A potential problem with CHDM was that, like all $\Omega_m = 1$ theories, it predicted rather late structure formation. A potential problem with ΛCDM was that the correlation function of the dark matter was higher around 1 Mpc than the power-law $\xi_{gg}(r) = (r/r_0)^{-1.8}$ observed for galaxies, so "scale-dependent anti-biasing" was required (Klypin, Primack, & Holtzman 1996, Jenkins et al. 1998). When better ΛCDM simulations could resolve halos that could host galaxies, they were found to have the same correlations as observed for galaxies.

By 1998, the evidence of early galaxy and cluster formation and the increasing evidence that $\Omega_m \approx 0.3$ had doomed CHDM. But now we also know from neutrino oscillations that neutrinos have mass. The upper limit from cosmology is $\Omega_\nu h^2 < 0.002$, corresponding to $m_\nu < 0.17$ eV (95% CL) for the most massive neutrino (Seljak et al. 2006).