Cosmology and Culture

Physics 80C Co-sponsored by Crown College Spring 2007

Practice Problems on Numbers

Express all your answers in scientific notation, for example 3×10^8. One digit of accuracy is adequate. (ANSWERS are on the next page.)

1. Work out how many meters there are in a light year.

   (a) What is the speed of light, in meters per second? _____

   (b) How many seconds are there in a year? _____

   (c) Multiply to get the answer: _____ meters

   (d) Convert your answer above to miles, using 1.61 km = 1 mile.

$$\text{\_\_\_\_\_ meters} \times 10^{-3} \frac{\text{km}}{\text{meter}} \times \frac{\text{mile}}{1.61\ \text{km}} = \text{\_\_\_\_\_ miles}$$

2. Ratios of big numbers. To find out how much bigger the cosmic horizon (10^{28} cm) is than the earth (10^7 cm), divide: 10^{28} cm / 10^7 cm = $10^{28-7} = 10^{21}$ times bigger.

   (a) How much bigger is a galaxy (10^{23} cm) than a person (1 m)? _____

   (b) How much bigger is a person than an atom (10^{-8} cm)? _____

3. (a) Multiply 5×10^{28} times 2×10^7 _____

   (b) Divide 6×10^8 by 10^7 _____

4. The amount of energy E in a kilogram of matter is given by Einstein's famous formula $E = mc^2$, where m is the mass in kilograms and $c = 3 \times 10^8$ m/s is the speed of light (in meters per second) and E is the energy in Joules.

   (a) How much energy is in a kilogram of matter?

   (b) You are billed for electric power at around 10 cents per kilowatt-hour (kwh), and 1 kwh = 3.6×10^6 Joules. How much is the energy in a kilogram of matter worth at that rate?

ANSWERS

1. Work out how many meters there are in a light year.

   (a) What is the speed of light, in meters per second? 3×10^8

   (b) How many seconds are there in a year? 3×10^7

   (c) Multiply to get the answer: 9×10^{15} meters

   (d) Convert your answer above to miles, using 1.61 km = 1 mile.

$$9 \times 10^{15} \text{ meters} \times 10^{-3} \frac{\text{km}}{\text{meter}} \times \frac{\text{mile}}{1.61 \text{ km}} = 6 \times 10^{12} \text{ miles}$$

2. Ratios of big numbers. To find out how much bigger the cosmic horizon (10^{28} cm) is than the earth (10^7 cm), divide: 10^{28} cm / 10^7 cm = $10^{28-7} = 10^{21}$ times bigger.

   (a) How much bigger is a galaxy (10^{23} cm) than a person (1 m)? 10^{21}

   (b) How much bigger is a person than an atom (10^{-8} cm)? 10^{10}

3. (a) Multiply 5×10^{28} times 2×10^7 10^{36}

   (b) Divide 6×10^8 by 10^7 $6 \times 10^1 = 60$

4. The amount of energy E in a kilogram of matter is given by Einstein's famous formula $E = mc^2$, where m is the mass in kilograms and $c = 3 \times 10^8$ m/s is the speed of light (in meters per second) and E is the energy in Joules.

   (a) How much energy is in a kilogram of matter? 9×10^{16} Joules

   (b) You are billed for electric power at around 10 cents per kilowatt-hour (kwh), and 1 kwh = 3.6×10^6 Joules. How much is the energy in a kilogram of matter worth at that rate? 3×10^9 = 3 billion dollars

   In more detail, using the same approach as for problem 1,

$$1 \text{ kg} = (9 \times 10^{16} \text{ Joules}) \left(\frac{1 \text{ kwh}}{3.6 \times 10^6 \text{ Joules}} \right) \left(\frac{\$0.10}{1 \text{ kwh}} \right) = \$2.5 \times 10^9$$

   and I rounded up 2.5 to 3.