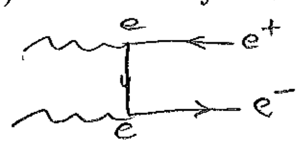
Note: one-digit accuracy is acceptable.

Physics 129 Nuclear and Particle Astrophysics: Final Exam Winter 2014

8-11 am Tuesday March 18 – Open Book and Notes – Attach Extra Pages If Needed

PRINT YOUR NAME: ANSWERS

1. (a) Draw the Feynman diagram for the process photon + photon → $e^+ + e^-$



(b) Give an estimate of the cross section for this process at center of mass energy E_{cm} and briefly explain the basis for your estimate.

Amplitude $T \sim e^2 \sim \alpha = e^2/\hbar c$, so $\sigma \propto |T|^2 \propto \alpha^2$

Cross section $\underline{\sigma \sim \frac{\alpha^2}{s}}$ ← since σ has units of $\text{length}^2 \sim \frac{1}{\text{energy}^2}$

Perkins (1.26b) says $\sigma(\gamma\gamma \to e^+e^-) = \frac{2\pi\alpha^2}{s}\left[\ln\left(\frac{s}{m_e^2}\right) - 1\right]$, consistent with this.

(c) What is the minimum value of E_{cm} for this process to occur? If one of the photons has energy 1 eV, what is the minimum energy of the other photon?

$$E_{cm} \geq 2m_e = 2(0.511\ \text{MeV}) = \underline{1.22\ \text{MeV}}$$

$$(2m_e)^2 \leq E_{cm}^2 = (E_1+E_2)^2 - (E_1 + E_2\cos\theta)^2 = 2E_1E_2(1-\cos\theta) \leq 4E_1E_2$$

$$\text{so } E_1 \geq \frac{m_e^2}{E_2} \geq \frac{(0.511\ \text{MeV})^2}{1\text{eV}} = \underline{0.26\ \text{TeV}}$$

$\underline{E_1 \gtrsim 1\text{TeV}}$ is an acceptable rough answer

(d) Explain briefly how this process can be used to measure the light emitted at various wavelengths by all the galaxies in the lifetime of the universe.

All the light emitted by galaxies is called the extragalactic background light (EBL). The EBL is very hard to measure directly because the solar system (zodaical light) and galaxy are so bright. But by comparing the gamma rays emitted by distant blazars with the gamma rays detected on earth by atmospheric Cherenkov Telescopes (ACTs), we can measure the attenuation by the $\gamma_{TeV}\gamma_{eV} \to e^+e^-$ process and thus deduce the abundance of γ_{eV}, the EBL. By measuring the attenuation of gamma rays from blazars at different distances we can measure how the EBL varies with redshift.

2. (a) Suppose that neutrons of energy 10^{21} eV are produced in a distant galaxy. Calculate roughly how far away this galaxy could be, if the neutrons are to reach earth?

10^{21} eV = 10^{12} GeV so $\gamma = 10^{12}$ and $\tau_n \gamma = (10^3 s)10^{12} = 10^{15} s$

↖ lifetime of neutron

$10^{15} s / (3\times10^7 s/yr) = 3\times10^7$ yr

So the neutrons can go about 3×10^7 lyr $\simeq$ 10 Mpc before 1/e have not decayed, ~20 Mpc before only $1/e^2$ are left, etc.

(b) Suppose that protons of energy 10^{21} eV are produced in a distant galaxy. Calculate roughly how far away this galaxy could be, if the protons are to reach earth?

The energy of these charged particles will be degraded by interaction with the cosmic microwave background — the GZK effect — with a mean free path $\lambda = 1/\rho\sigma$ $\approx$ 4 Mpc (Perkins, p. 253)

3. (a) Calculate how many neutrinos are emitted per second by the sun, and briefly explain your calculation.

About 24 MeV is released per ^{4}He formed, and each ^{4}He requires two $p \to n + e^+ + \nu_e$; thus the energy released per neutrino is about 12 MeV $= (1.2\times10^7)(1.6\times10^{-19} J) = 1.9\times10^{-12} J$.

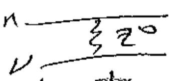
The luminosity of the sun is $L_\odot = 4\times10^{33}$ erg/s $= 4\times10^{26}$ J/s.

Thus the number of neutrinos per second is

$$N_\nu/s = L_\odot/(12\ \text{MeV}/\nu) = (4\times10^{26}\ \text{J/s})/(1.9\times10^{-12}\ \text{J}) = 2.1\times10^{38}/s$$

(b) What fraction of these neutrinos reach earth as electron-type neutrinos, and what happens to the rest of them? Explain briefly the evidence that supports this.

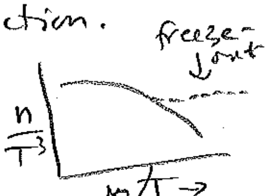
About 1/3 of the solar ν_e reach earth as ν_e. The rest oscillate to ν_μ or ν_τ.

The evidence is that the Sudbury neutrino observatory saw the other 2/3 that converted to ν_μ or ν_τ using the elastic scattering $\nu + n \to \nu + n$ via Z^0 exchange  with heavy water D_2O, and the energy transferred to the deuterium released the neutron which was detected.

Also, neutrino oscillations were seen in reactor and accelerator experiments, and in atmospheric neutrino oscillations.

4. (a) What is the "WIMP miracle" and how is it related to "freezeout"?

When the temperature in the early universe dropped below m_{WIMP}, the abundance of WIMPs dropped because their annihilation was no longer balanced by production. As long as the WIMPs are in thermal equilibrium their abundance will be given by the Boltzmann factor $e^{-m_{WIMP}/T}$. But when the WIMP mean free path $\lambda_{WIMP} = \frac{1}{\sigma\rho}$ exceeds the Hubble radius c/H, they "freeze out" and stop annihilating because they no longer find each other. The typical supersymmetric cross section σ_{WIMP} gives an abundance Ω_m approximately equal to observation. This is known as the WIMP miracle.



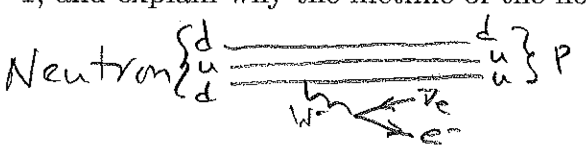
(b) Summarize the evidence that a WIMP of about 35 GeV has been detected indirectly.

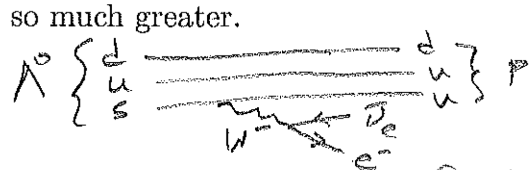
The Daylan et al. paper analyses Fermi γ-ray data from the direction of the center of the Milky Way. After subtracting backgrounds they find a signal of ~5 GeV that is spherically symmetric and falls off as ρ_{DM}^2 where $\rho_{DM} \propto r^{-1.25}$, consistent with $\rho_{NFW} \sim 1/r$ with a bit more central concentration presumably due to the concentration of ordinary matter in the galaxy center. The cross section for WIMP annihilation in the early universe gives the flux of observed gamma rays; that is, $\sigma v \sim (1-2) \times 10^{-26}\ cm^3/s$ is required for both $\Omega_m \approx 0.3$ and the observed gamma signal. WIMP annihilation to quarks ($b\bar{b}$, etc.) results in the ~5 GeV signal for ~35 GeV WIMPs.

5. (a) There are no known mesons of strangeness greater than -1, but there are baryons with strangeness -2 and -3. Give a simple explanation for this and identify the lightest of these mesons and baryons.

The strange quark s has strangeness -1, and since mesons have only one quark and one antiquark, mesons can have only $S=-1$ (one s quark), $S=+1$ (one $\bar{s}$ quark), or $S=0$ (no s quarks). Baryons have 3 quarks, so can have $S=-2$ ($\Xi^0 = ssu$ and $\Xi^- = ssd$) or $S=-3$ ($\Omega^- = sss$).

(b) Draw the Feynman diagrams for the decay of a neutron and of a baryon of strangeness -1, and explain why the lifetime of the neutron is so much greater.

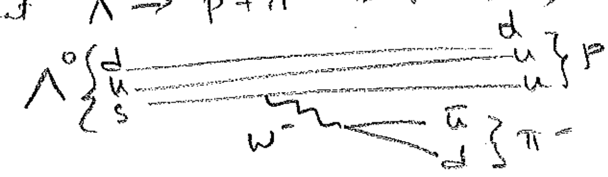




$\tau_n \sim G_F^2 Q^5$ where $Q \sim 1\,\text{MeV}$

$\tau_{\Lambda,\text{weak}} \sim G_F^2 Q^5$ where $Q \sim 100\,\text{MeV}$

Also, the Λ is massive enough that $\Lambda^0 \to p + \pi^-$ is possible, again with $Q \sim 100$ MeV.



Thus we expect $\tau_\Lambda / \tau_n \sim 100^5$
$\sim 10^{10}$

6. Consider two particles of charge +e and mass m. For what value of m is the magnitude of the electric force equal to that of the gravitational force? Compare this value of m to the Planck mass, and discuss the implications.

Electric Force $\dfrac{e^2}{r^2} = \dfrac{Gm^2}{r^2}$ = Gravitational Force

$$\Rightarrow m^2 = e^2/G = \frac{\alpha \hbar c}{G} = \frac{1}{137}\frac{\hbar c}{G}, \quad m = \frac{1}{\sqrt{137}}\underbrace{\sqrt{\frac{\hbar c}{G}}}_{= M_{PL}} = \frac{M_{PL}}{11.7}$$

$\simeq 0.1\, M_{PL}$

Implications: for very large mass or energy, the gravitational force becomes comparable to the electroweak force, suggesting that gravity might unify with the other forces.

7. Explain briefly how the GPS system works, and describe the role of relativity.

There are enough GPS satellites that four or more are visible everywhere on earth. The GPS satellites carry atomic clocks, and they send time signals corrected for the special relativity slowing of clocks with speed and the general relativity speeding of clocks in lower gravitational potentials, as if the satellites were at rest in Euclidean space. GPS receivers use these signals to correct their clocks and use triangulation to determine their location.

8. The blue hydrogen Balmer line has a wavelength of 435 nm. In the spectrum of a certain galaxy that line has an observed wavelength of 455 nm. (a) What is the redshift of that galaxy?

$$z = \frac{\delta\lambda}{\lambda} = \frac{455-435}{435} = \frac{20}{435} = \underline{0.046}$$

(b) With what speed is it moving away from us?

$v \simeq cz$ for low z, so $v = (3 \times 10^5 \text{ km/s})(0.046)$
$= \underline{1.38 \times 10^4 \text{ km/s}}$

(c) What is its approximate distance, in light years? $v = H_0 d = (70 \text{ km/s/Mpc})\, h_{70} d$

$$d = \frac{1.38 \times 10^4 \text{ km/s}}{(70 \text{ km/s/Mpc})\, h_{70}} = 197\, h_{70} \text{ Mpc} = \underline{6.4 \times 10^7\, h_{70} \text{ lyr}}$$

since $pc = 3.26$ lyr

(Just using $H_0 = 70$ km/s/Mpc without including h_{70} is ok.)

9. (a) The average matter density of the universe is $\Omega_m = 0.31$ in units of critical density. What is the mass of a sphere of radius equal to that of the visible Milky Way, about 50,000 light years?

$$\rho_c = 2.8h^2 \times 10^{11}\, M_\odot/Mpc^3 = 1.4\times 10^{11}\, M_\odot/Mpc^3$$

$$\bar\rho = 0.31\,\rho_c = 4.3\times 10^{10}\, M_\odot/Mpc^3 = 4.3\times10^{10}\, M_\odot / \underbrace{(3.26\times 10^6\, lyr)^3}_{3.46\times 10^{19}\, lyr^3}$$

$$= 1.24\times 10^{-9}\, M_\odot/lyr^3$$

$$= 1.24\times 10^{9} M_\odot/(Mlyr)^3$$

$$\bar M = \frac{4}{3}\pi R^3 \bar\rho = \frac{4}{3}\pi\,(0.05 Mlyr)^3 (1.24\times10^9 M_\odot)(\cancel{Mlyr})^{-3}$$

$$= \frac{4}{3}\pi \underbrace{(0.05)^3}_{0.125} (1.24\times 10^9\, M_\odot) = \underline{6.5\times 10^5\, M_\odot}$$

(b) The rotational velocity of stars around the center of the Milky Way is about 220 km/s. Assuming that the mass is spherically distributed, estimate the total mass of the Milky Way out to a radius of 50,000 light years. By comparing it with the mass calculated in part (a), estimate the "over density" of this inner, visible part of the Milky Way.

Since the earth's speed around the sun is 30 km/s at $r_\oplus = 1.5\times 10^8$ km,

$\frac{GM_\odot}{r_\oplus} = (30\,km/s)^2$. Divide $\frac{GM_{gal}}{(0.5 Mlyr)} = (220\,km/s)^2$ by that, $\Rightarrow$

$$M_{gal}/M_\odot = \left(\frac{220}{30}\right)^2 \frac{5\times10^4\, lyr}{1.5\times 10^8\, km} \underbrace{(3\times10^5\, km/s)(3.16\times10^7\, s/yr)}_{0.95\times10^{13}\, km/lyr}$$

$$= \left(\frac{220}{30}\right)^2 3.16\times 10^9$$

$$= 1.69\times10^{11}, \text{ or } M_{gal} = 1.7\times10^{11}\, M_\odot$$

Thus "overdensity" $= M_{gal}/\bar M = \frac{1.7\times10^{11} M_\odot}{6.5\times10^5 M_\odot} = \underline{2.6\times10^5}$

10. The total mass of the Milky Way, mostly in the form of dark matter, is about $10^{12} M_\odot$.
(a) Suppose that the Milky Way's dark matter halo has a radius such that the average density is 200 times the average density of the universe. What is this radius?

$$\frac{4}{3}\pi R_{halo}^3\, 200\bar\rho = 10^{12} M_\odot \Rightarrow R_{halo}^3 = \frac{10^{12} M_\odot}{\frac{4}{3}\pi\, 200\bar\rho}$$

$$R_{halo}^3 = \frac{10^{12} M_\odot\, Mlyr^3}{(\frac{4}{3}\pi)(200)(1.24\times10^9 M_\odot)} = 0.96\, Mlyr, \text{ so } R_{halo} = \underline{0.98\, Mlyr} = \underline{300\, kpc}$$

(b) The fraction of ordinary matter compared to total matter in the universe is $\Omega_b/\Omega_m = 0.044/0.31 = 0.14$. The total mass of the stars in the Milky Way is about $7 \times 10^{10} M_\odot$. By comparing this to the total amount of ordinary matter in a mass of cosmic matter equal to the mass of the Milky Way, determine the fraction of ordinary matter that has become stars in the Milky Way.

$$M_{\text{ordinary matter}} = 0.14 \times 10^{12} M_\odot = 1.4 \times 10^{11} M_\odot$$

$$\text{Stellar fraction} = \frac{M_*}{M_{\text{ordinary matter}}} = \frac{7 \times 10^{10} M_\odot}{14 \times 10^{10} M_\odot} = \underline{0.5}$$

11. In the later stages of evolution of a massive star, it will begin fusing carbon nuclei to form, for example, magnesium: $^{12}_{6}\text{C} + ^{12}_{6}\text{C} \rightarrow ^{24}_{1} 2\text{Mg} + \gamma$. The mass of $^{12}_{6}C$ is 12 u (exactly) and that of $^{24}_{1}2\text{Mg}$ is 23.985042 u, where the atomic mass unit $u = 1.660 \times 10^{-27}$ kg. (a) How much energy in MeV is released in this reaction.

$$Q = 2 m_C c^2 - m_{Mg} c^2 = [\underbrace{2(12u) - 23.985042u}_{0.014958}] c^2 \frac{931.5 \text{ MeV}}{c^2}$$

$$= 13.93 \text{ MeV}$$

(b) How much kinetic energy must each carbon nucleus have (assumed equal) in a head-on collision if they are to just touch so that the strong interaction can lead to this fusion reaction? Assume that the radius of a nucleus of atomic number A is given by $R = (1.2 \times 10^{-15}\text{m})A^{1/3}$.

$$R = (1.2 \times 10^{-15} \text{m})\, 12^{1/3} = (1.2)(2.29) \times 10^{-15} \text{m} = 2.75 \times 10^{-15} \text{m}$$

The KE = PE when the nuclei just touch, at $2R$


R R

Each nucleus has half the KE, so

$$KE = \frac{1}{2} PE = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{(6e)^2}{2R} = \frac{1}{2} \left(9.0 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \frac{(6^2)(1.60 \times 10^{-19} C)}{2(2.75 \times 10^{-15} m)} \frac{\text{MeV}}{1.6 \times 10^{-13} J}$$

$$= \underline{4.7 \text{ MeV}}$$

(c) What temperature (in K) does this kinetic energy correspond to?

$1\ \text{eV} = 1.16\times10^{4}\ \text{K}$ so

$4.7\ \text{MeV} = (4.7\times10^{6})(1.16\times10^{4}\ \text{K}) = \underline{5.5\times10^{10}\ \text{K}}$

(Of course, the reaction can proceed by quantum tunneling at $\sim 10^{10}$ K = 10 billion degrees, still pretty hot.)

12. (a) What is the missing particle in the reaction $p + ? \to n + \mu^+$

Muon anti-neutrino $\bar{\nu}_\mu$

(b) How many fundamental fermions are there in a water molecule?

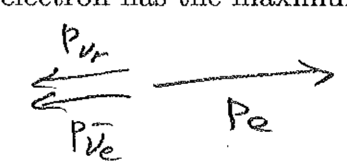
There are $\underline{10}$ electrons in the H_2O $(2+8)$

Each of the 18 nucleons has 3 quarks, so $\underline{54}$ quarks

The total is $\underline{64}$ fundamental fermions.

13. What is the maximum kinetic energy of the electron in the decay $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$? (The mass of the electron is 0.511 MeV/c^2 and that of the muon is 106 MeV/c^2. *Hint:* in what direction do the two neutrinos move in order that the electron has the maximum possible kinetic energy?)

The electron KE will be maximized if the neutrinos both move in the opposite direction. Then



$$p_e = p_{\bar{\nu}_e} + p_{\nu_\mu}$$

$$m_\mu c^2 = E_e + E_{\bar{\nu}_e} + E_{\nu_\mu} = E_e + (p_{\bar{\nu}_e} + p_{\nu_\mu})c = E_e + p_e c$$

$$\Rightarrow m_\mu c^2 - E_e = p_e c \Rightarrow (m_\mu c^2 - E_e)^2 = p_e^2 c^2 = E_e^2 - m_e^2 c^4$$

$$\Rightarrow m_\mu^2 c^4 - 2m_\mu c^2 E_e + \cancel{E_e^2} = \cancel{E_e^2} - m_e^2 c^4$$

$$\Rightarrow E_e = \frac{m_\mu^2 c^4 + m_e^2 c^4}{2m_\mu c^2} = K_e + m_e c^2$$

$$\Rightarrow K_e = \frac{1}{2}\left(m_\mu + \frac{m_e^2}{m_\mu}\right)c^2 - m_e c^2 \approx \frac{1}{2}m_\mu c^2 - m_e c^2$$

$$= 53\ \text{MeV} - 0.5\ \text{MeV} = \underline{52.5\ \text{MeV}}$$

14. (a) The galaxy cluster A496 has a recession velocity of 9885 km/s. What is its distance, as a function of h_{70}?

$V = H_0 d$ so $d = V/H_0 = (9885 \text{ km/s})/(70\, h_{70} \text{ km/s/Mpc})$

$= 141\, h_{70}^{-1}$ Mpc

(b) The X-ray luminosity and temperature of A496 imply that a total mass within $0.7h_{70}^{-1}$ Mpc of the cluster center is about $1.7 \times 10^{14} M_{\odot}$, and the total gas mass within this same radius is $1.4 \times 10^{13} M_{\odot}$. The mass of all the stars in the cluster is less than 10% that of the gas. Assuming from Big Bang nucleosynthesis that $\Omega_b = 0.044$ and that the gas in A496 is a fair sample of the universal baryon density, determine Ω_m. (Applying this method to a number of clusters was one of the first ways that Ω_m was reliably determined.)

$$\frac{\Omega_b}{\Omega_m} = \frac{0.044}{\Omega_m} = \frac{1.4 \times 10^{13} M_{\odot}}{1.7 \times 10^{14} M_{\odot}} = 0.082 \Rightarrow \Omega_m = \frac{0.044}{0.082}$$

$$\Omega_m = 0.53$$

This doesn't agree very well with the modern value $\Omega_m = 0.31$, but such arguments showed that $\Omega_m \neq 1$, ruling out the Einstein-de Sitter cosmology.

15. Explain briefly why the strength of the electric interaction increases as energy increases while the strength of the strong interaction decreases as energy increases, and briefly discuss the implications for unification of the coupling constants and for the quark model of hadrons.

The electric charge is shielded by e^+e^- pairs in the Feynman diagram, but at high energy the shielding decreases.

The strong interaction is carried by gluons and while there is shielding there is also gluon-gluon interactions, for example, and the net force decreases with increasing energy.

Implications: Since the electric interaction increases with energy while the strong interaction decreases with energy, they can cross, or unify. The strong interaction confines quarks at low energy, inside hadrons, but it becomes asymptotically free at high energy.