Physics 129 LECTURE 12 February 18, 2014

More Early Universe Cosmology

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Big Bang Nucleosynthesis Review

Before freezeout of n ↔ p conversion by neutrinos,

$$\frac{N_{\mathrm{n}}}{N_{\mathrm{p}}} = \exp\left(\frac{-Q}{kT}\right); \quad Q = \left(M_{\mathrm{n}} - M_{\mathrm{p}}\right)c^2 = 1.293 \text{ MeV}$$

At freezeout T_f = 0.80 MeV; substituting T_f into the above equation gives N_n/N_p = 0.20. Because the deuterium binding energy is only 2.22 MeV and there are about 109 photons for every nucleon, deuterium nuclei are photodissociated as fast as the form until T = 0.05 MeV, which for N_ν = 3 corresponds to t = 300 s = 5 min. This is when Big Bang nucleosynthesis begins. At that time, because of neutron decay with a lifetime of 885 s, r = N_n/N_p = 0.135. Almost all the neutrons are bound into 4He, which gives a promordial helium abundance by mass of

$$Y = \frac{4N_{\mathrm{He}}}{(4N_{\mathrm{He}} + N_{\mathrm{H}})} = \frac{2r}{(1+r)} \approx 0.24$$

All the nucleosynthesis in stars adds only about 0.01 to Y (Perkins problem 6.4).

The two most accurate ways of measuring the primordial abundance of baryons are the relative heights of the first two peaks in the CMB angular power spectrum and the D/H ratio in near-primordial hydrogen; both give Ω_b = 0.044. Compared with the number density of photons, this gives the baryon/photon ratio of

$$\frac{N_{\mathrm{B}}}{N_\gamma} \approx \frac{\left(N_{\mathrm{B}} - N_{\overline{\mathrm{B}}}\right)}{N_\gamma} = (6.1 \pm 0.6) \times 10^{-10}$$

Baryogenesis Generates Matter - Antimatter Asymmetry

Perkins Example 6.1 calculates the baryon-antibaryon ratio after annihilation freezes out if there were equal amounts initially of baryons and antibaryons, and gets the answer $N_B/N_\gamma = N_{\overline{B}}/N_\gamma = 0.72 \times 10^{-18}$ instead of the observed value $N_B/N_\gamma = 0.6 \times 10^{-9}$. It follows that there must have been an initial asymmetry between matter and antimatter. Since Cosmic Inflation ended with matter-antimatter symmetry, something must have happened after the end of Inflation to generate the asymmetry.

The Three Sakharov Requirements

As we discussed, CP violation in the Weak interactions was discovered in 1964, and in 1967 Andrei Sakharov showed that the following conditions are necessary in order to generate baryon-antibaryon asymmetry:

- Baryon-number violation
- Out of thermal equilibrium
- C and CP violation

The first requirement is obvious, and it occurs naturally in Grand Unified Theories (GUTs). The second condition follows from the fact that CPT requires equal masses for particles and their antiparticles, and then standard statistical mechanics implies that in thermal equilibrium particle and antiparticle abundances are equal. The third condition follows from the requirement that particles and antiparticles must behave differently.

Scenarios for the Three Sakharov Requirements

There are many ways that the matter-antimatter asymmetry could have been generated. The main ideas for such “baryogenesis” that have been investigated are

- GUT baryogenesis
- Electroweak baryogenesis
- Leptogenesis converted into baryogenesis
- Coherent motion of supersymmetric scalar fields

The first two are disfavored. CP-violating decays of the GUT X and Y leptoquark bosons can certainly generate the needed ~10^{-9} baryon-antibaryon asymmetry, but it probably would be wiped out by subsequent “instanton” processes associated with the Electroweak interactions. Electroweak baryogenesis also doesn’t appear to work.

The massive right-handed neutrinos introduced in the Seesaw Mechanism for the light left-handed neutrino masses could decay out of equilibrium into more antineutrinos than neutrinos. “Sphaleron” processes in the lower-energy-scale Electroweak interactions preserve the difference $B - L$ between baryon and lepton numbers, but allow conversion for example of

$$\bar{\nu}_e + \bar{\nu}_\mu + \bar{\nu}_\tau \longrightarrow (u + d + d) + (c + s + s) + (t + b + b)$$

The same heavy right-handed neutrino masses needed to generate light left-handed neutrino masses appear to be able to generate the needed baryon asymmetry.

What's this about Sphalerons?

Below the electroweak scale of ~ 100 GeV, the **sphaleron** quantum tunneling process that violates B and L conservation (but preserves B - L) in the Standard Model is greatly suppressed, by ~ exp(-2π/α_W) ~ 10^{-65}. But at T ~ 100 GeV this process can occur. It can satisfy all three Sakharov conditions, but it cannot produce a large enough B and L. However, it can easily convert L into a mixture of B and L (Leptogenesis).

When one quantizes the Standard Model, one finds that the baryon number current is not exactly conserved, but rather satisfies

$$\partial_\mu j_B^\mu = \frac{3}{16\pi^2} F^a_{\mu\nu} \tilde{F}^a_{\mu\nu} = \frac{3}{8\pi^2} \mathrm{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu}.$$

The same parity-violating term occurs in the divergence of the lepton number current, so the difference (the B - L current) is exactly conserved. The parity-violating term is a total divergence

$\mathrm{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu} = \partial_\mu K^\mu$ where $K^\mu = \epsilon^{\mu\nu\rho\sigma} tr[F_{\nu\rho} A_\sigma + \frac{2}{3} A_\nu A_\rho A_\sigma]$, so

$\tilde{j} = j_B^\mu - \frac{3g^2}{8\pi^2} K^\mu$ is conserved. In perturbation theory (i.e. Feynman diagrams)

K^μ falls to zero rapidly at infinity, so B and L are conserved.

In abelian -- i.e. U(1) -- gauge theories, this is the end of the story. In non-abelian theories, however, there are non-perturbative field configurations, called instantons, which lead to violations of B and L. They correspond to calculation of a tunneling amplitude. To understand what the tunneling process is, one must consider more carefully the ground state of the field theory. Classically, the ground states are field configurations for which the energy vanishes. The trivial solution of this condition is A = 0, where A is the vector potential, which is the only possibility in U(1). But a “pure gauge” is also a solution, where

$$\vec{A} = \frac{1}{i} g^{-1} \vec{\nabla} g,$$

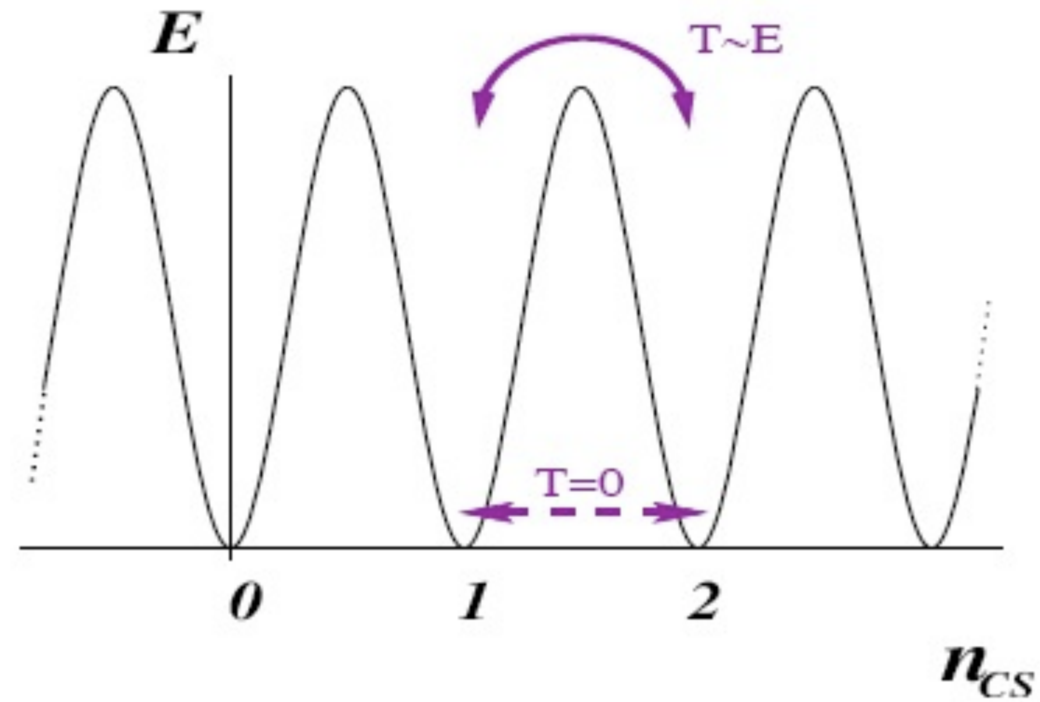
where g is a gauge transformation matrix. There is a class of gauge transformations g, labeled by a discrete index n, which must also be considered. These have the form

$$g_n(\vec{x}) = e^{i n f(\vec{x}) \hat{x}\cdot\tau/2} \text{ where } f(x) \to 2\pi \text{ as } \vec{x} \to \infty, \text{ and } f(\vec{x}) \to 0 \text{ as } \vec{x} \to 0.$$

The ground states are labeled by the index n. If we evaluate the integral of the current we obta K^{μ} quantity known as the Chern-Simons number

$$n_{CS} = \frac{1}{16\pi^2} \int d^3x K^o = \frac{2/3}{16\pi^2} \int d^3x \epsilon_{ijk} Tr(g^{-1}\partial_i g g^{-1}\partial_j g g^{-1}\partial_k g). \text{ For } g = g_n, n_{CS} = n.$$

Schematic Yang-Mills vacuum structure. At zero temperature, the instanton transitions between vacua with different Chern-Simons numbers are suppressed. At finite temperature, these transitions can proceed via sphalerons.



In tunneling processes which change the Chern-Simons number, because of the anomaly, the baryon and lepton numbers will change. The exponential suppression found in the instanton calculation is typical of tunneling processes, and in fact the instanton calculation is nothing but a field-theoretic WKB calculation. The probability that a single proton has decayed through this process in the history of the universe is infinitesimal. But this picture suggests that, at finite temperature, the rate should be larger. One can determine the height of the barrier separating configurations of different n_{CS} by looking for the field configuration which corresponds to sitting on top of the barrier. This is a solution of the static equations of motion with finite energy. It is known as a "sphaleron". It follows that when the temperature is of order the ElectroWeak scale ~ 100 GeV, B and L violating (but B - L conserving) processes can proceed rapidly.

This result leads to three remarks:

1. If in the early universe, one creates baryon and lepton number, but no net $B - L$, B and L will subsequently be lost through sphaleron processes.

2. If one creates a net $B - L$ (e.g. creates a lepton number) the sphaleron process will leave both baryon and lepton numbers comparable to the original $B - L$. This realization is crucial to the idea of Leptogenesis.

3. The Standard Model satisfies, by itself, all of the conditions for baryogenesis. However, detailed calculations show that in the Standard Model the size of the baryon and lepton numbers produced are much too small to be relevant for cosmology, both because the Higgs boson is more massive than ~ 80 GeV and because the CKM CP violation is much too small. In supersymmetric extensions of the Standard Model it is possible that a large enough matter-antimatter asymmetry might be generated, but the parameter space for this is extremely small. (See Dine and Kusenko for details and refereneces.)

This leaves Leptogenesis (and Affleck-Dine baryogenesis, to be discussed next) as the two most promising possibilities. What is exciting about each of these is that, if they are operative, they have consequences for experiments which will be performed at accelerators over the next few years.

Baryogenesis by coherent motion of scalar fields (the Affleck-Dine mechanism)

The formation of an AD condensate can occur quite generically in cosmological models. Also, the AD scenario potentially can give rise simultaneously to the ordinary matter and the dark matter in the universe. This can explain why the amounts of luminous and dark matter are surprisingly close to each other, within one order of magnitude. If the two entities formed in completely unrelated processes (for example, the baryon asymmetry from leptogenesis, while the dark matter from freeze-out of neutralinos), the observed relation $\Omega_{DARK} \sim \Omega_{baryon}$ is fortuitous.

In supersymmetric theories, the ordinary quarks and leptons are accompanied by scalar fields. These scalar fields carry baryon and lepton number. A coherent field, i.e., a large classical value of such a field, can in principle carry a large amount of baryon number. As we will see, it is quite plausible that such fields were excited in the early universe. To understand the basics of the mechanism, consider first a model with a single complex scalar field. Take the Lagrangian to be

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2$$

This Lagrangian has a symmetry, $\varphi \rightarrow e^{i\alpha\varphi}$, and a corresponding conserved current, which we will refer to as baryon current:

$$j^\mu_B = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*).$$

It also possesses a “CP” symmetry: $\varphi \leftrightarrow \varphi_*$. With supersymmetry in mind, we will think of m as of order M_W.

Let us add interactions in the following way, which will closely parallel what happens in the supersymmetric case. Include a set of quartic couplings:

$$\mathcal{L}_I = \lambda|\phi|^4 + \epsilon\phi^3\phi^* + \delta\phi^4 + c.c.$$

These interactions clearly violate B. For general complex ε and δ, they also violate CP. In supersymmetric theories, as we will shortly see, the couplings will be extremely small. In order that these tiny couplings lead to an appreciable baryon number, it is necessary that the fields, at some stage, were very large.

To see how the cosmic evolution of this system can lead to a non-zero baryon number, first note that at very early times, when the Hubble constant, $H \gg m$, the mass of the field is irrelevant. It is thus reasonable to suppose that at this early time $\varphi = \varphi_o \gg 0$. How does the field then evolve? First ignore the quartic interactions. In the expanding universe, the equation of motion for the field is as usual

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0.$$

At very early times, $H \gg m$, and so the system is highly overdamped and essentially frozen at φo. At this point, $B = 0$.

Once the universe has aged enough that H ≪ m, φ begins to oscillate. Substituting H = 1/2t or H = 2/3t for the radia pectively, one finds that

$$\phi = \begin{cases} \frac{\phi_o}{(mt)^{3/2}} \sin(mt) \quad (\text{radiation}) \\ \frac{\phi_o}{(mt)} \sin(mt) \quad (\text{matter}). \end{cases}$$

In either case, the energy behaves, in terms of the scale factor, R(t), as

$$E \approx m^2 \phi_o^2 (\frac{R_o}{R})^3$$

Now let's consider the effects of the quartic couplings. Since the field amplitude damps with time, their significance will decrease with time. Suppose, initially, that $\varphi = \varphi_o$ is real. Then the imaginary part of φ satisfies, in the approximation that ε and δ are small,

$$\ddot{\phi}_i + 3H\dot{\phi}_i + m^2\phi_i \approx \text{Im}(\epsilon + \delta)\phi_r^3.$$

For large times, the right hand falls as $t^{-9/2}$, whereas the left hand side falls off only as $t^{-3/2}$. As a result, baryon number violation becomes negligible. The equation goes over to the free equation, with a solution of the form

$$\phi_i = a_r \frac{\text{Im}(\epsilon + \delta)\phi_o^3}{m^2 (mt)^{3/4}} \sin(mt + \delta_r) \quad (\text{radiation}), \qquad \phi_i = a_m \frac{\text{Im}(\epsilon + \delta)\phi_o^3}{m^3 t} \sin(mt + \delta_m) \quad (\text{matter}),$$

The constants can be obtained numerically, and are of order unity

$$a_r = 0.85 \qquad a_m = 0.85 \qquad \delta_r = -0.91 \qquad \delta_m = 1.54.$$

But now we have a non-zero baryon number; substituting in the expression for the current,

$$n_B = 2a_r \mathrm{Im}(\epsilon+\delta)\frac{\phi_o^2}{m(mt)^2}\sin(\delta_r+\pi/8) \quad \text{(radiation)}$$

$$n_B = 2a_m \mathrm{Im}(\epsilon+\delta)\frac{\phi_o^2}{m(mt)^2}\sin(\delta_m) \quad \text{(matter)}.$$

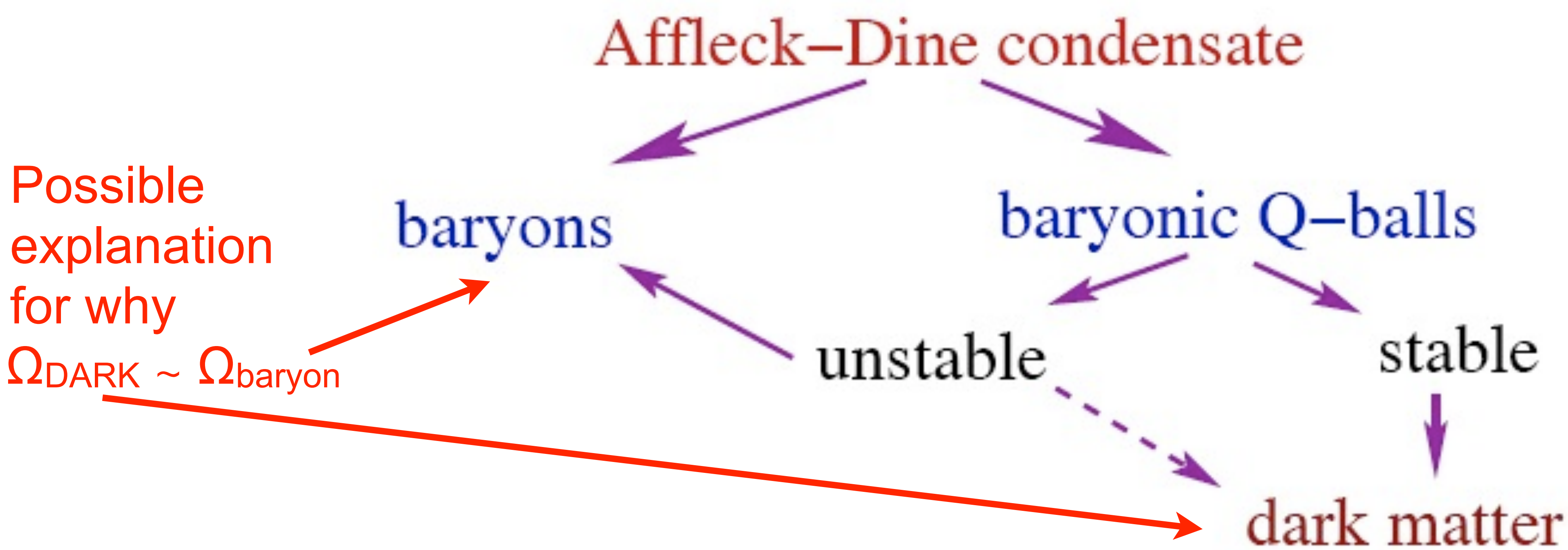
Two features of these results should be noted. First, if ε and δ vanish, n_B vanishes. If they are real, and φ_o is real, n_B vanishes. It is remarkable that the Lagrangian parameters can be real, and yet φ_o can be complex, still giving rise to a net baryon number. Supersymmetry breaking in the early universe can naturally lead to a very large value for a scalar field carrying B or L. Finally, as expected, n_B is conserved at late times.

This mechanism for generating baryon number could be considered without supersymmetry. In that case, it begs several questions:

- What are the scalar fields carrying baryon number?
- Why are the φ^4 terms so small?
- How are the scalars in the condensate converted to more familiar particles?

In the context of supersymmetry, there is a natural answer to each of these questions. First, there are scalar fields (squarks and sleptons) carrying baryon and lepton number. Second, in the limit that supersymmetry is unbroken, there are typically directions in the field space in which the quartic terms in the potential vanish. Finally, the scalar quarks and leptons will be able to decay (in a baryon and lepton number conserving fashion) to ordinary quarks.

In addition to topologically stable solutions to the field equations such as strings or monopoles, it is sometimes also possible to find non-topological solutions, called Q-balls, which can form as part of the Affleck-Dine condensate. These are usually unstable and could decay to the dark matter, but in some theories they are stable and could be the dark matter. The various possibilities are summarized as follows:



The parameter space of the MSSM consistent with LSP dark matter is very different, depending on whether the LSPs froze out of equilibrium or were produced from the evaporation of AD baryonic Q-balls. If supersymmetry is discovered, one will be able to determine the properties of the LSP experimentally. This will, in turn, provide some information on the how the dark-matter SUSY particles could be produced. The discovery of a Higgsino-like LSP would be a evidence in favor of Affleck–Dine baryogenesis. This is a way in which we might be able to establish the origin of matter-antimatter asymmetry.

Review of Baryogenesis Scenarios

1. **GUT Baryogenesis**. Grand Unified Theories unify the gauge interactions of the strong, weak and electromagnetic interactions in a single gauge group. They inevitably violate baryon number, and they have heavy particles, with mass of order $M_{GUT} \approx 10^{16}$ GeV, whose decays can provide a departure from equilibrium. The main objections to this possibility come from issues associated with inflation. While there does not exist a compelling microphysical model for inflation, in most models, the temperature of the universe after reheating is well below M_{GUT}. But even if it were very large, there would be another problem. Successful unification requires supersymmetry, which implies that the graviton has a spin-3/2 partner, called the gravitino. In most models for supersymmetry breaking, these particles have masses of order TeV, and are very long lived. Even though these particles are weakly interacting, too many gravitinos are produced unless the reheating temperature is well below the unification scale -- too low for GUT baryogenesis to occur.

2. **Electroweak baryogenesis**. The Standard Model satisfies all of the conditions for baryogenesis, but any baryon asymmetry produced is far too small to account for observations. In certain extensions of the Standard Model, it is possible to obtain an adequate asymmetry, but in most cases the allowed region of parameter space is very small.

3. **Leptogenesis**. The possibility that the weak interactions will convert some lepton number to baryon number means that if one produces a large lepton number at some stage, this will be processed into a net baryon and lepton number at the electroweak phase transition. The observation of neutrino masses makes this idea highly plausible. Many but not all of the relevant parameters can be directly measured.

4. **Production by coherent motion of scalar fields (the Affleck-Dine mechanism)**, which can be highly efficient, might well be operative if nature is supersymmetric.

Types of Dark Matter

Ω_i represents the fraction of the critical density $\rho_c = 10.54\ h^2$ keV/cm^3 needed to close the Universe, where h is the Hubble constant H_0 divided by 100 km/s/*Mpc*.

Dark Matter Type	Fraction of Critical Density	Comment
Baryonic	$\Omega_b \sim 0.04$	about 10 times the visible matter
Hot	$\Omega_\nu \sim 0.001–0.1$	light neutrinos
Cold	$\Omega_c \sim 0.3$	most of the dark matter in galaxy halos

Dark Matter and Associated Cosmological Models

Ω_m represents the fraction of the critical density in all types of matter.
Ω_Λ is the fraction contributed by some form of "dark energy."

Acronym	Cosmological Model	Flourished
HDM	hot dark matter with $\Omega_m = 1$	1978–1984
SCDM	standard cold dark matter with $\Omega_m = 1$	1982–1992
CHDM	cold + hot dark matter with $\Omega_c \sim 0.7$ and $\Omega_\nu = 0.2–0.3$	1994–1998
ΛCDM	cold dark matter $\Omega_c \sim 1/3$ and $\Omega_\Lambda \sim 2/3$	1996–today

Joel Primack, *Beam Line*, Fall 2001

Cold Dark Matter Candidates

The ΛCDM cosmological model is in very good agreement with detailed observations of the cosmic background radiation and the large scale distribution of galaxies, and it appears to give a very good description of galaxy formation. But the CDM could be many alternative sorts of particles, of which the following three types have been investigated in detail:

- MAssive Compact Halo Objects (MACHOs)
- Axions
- Weakly Interacting Massive Particles (WIMPS)

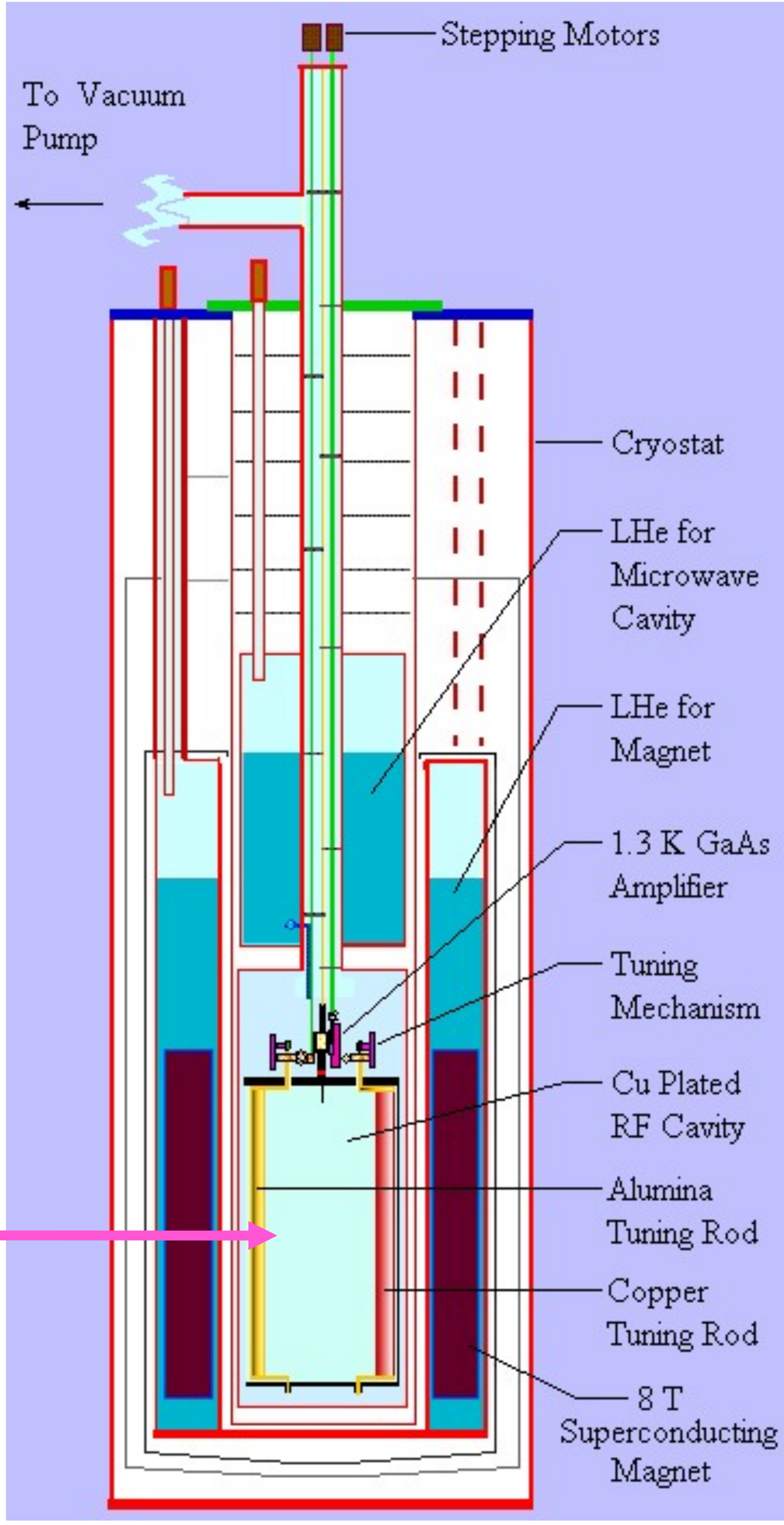
There is strong evidence (some of which is summarized in Perkins section 7.7) that the dark matter is not made of massive compact halo objects (MACHOs). The MACHO and EROS experiments looked for gravitational lensing of stars in the Large Magellanic Cloud, a satellite galaxy of the Milky Way, by MACHOs in the Milky Way's halo, and showed that at most a tiny fraction of the halo mass could come from MACHOs. The strongest such evidence comes from the lack of gravitational lensing of Type 1a supernovas, which rules out (Metcalf & Silk 07) the wide range

$$10^{-2} < M_{MACHO}/M_{\odot} < 10^{8} .$$

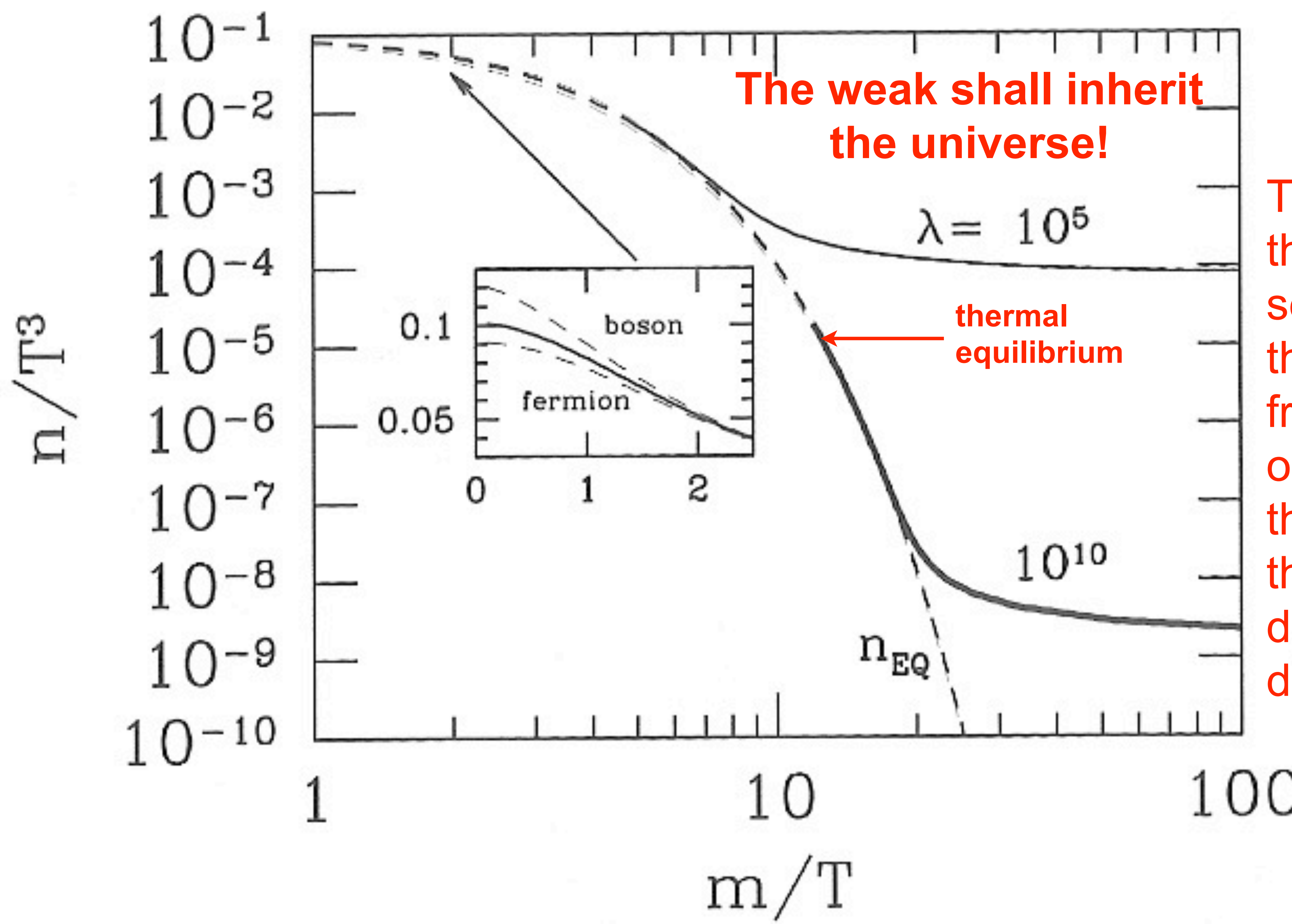
Axions are the best solution known to the strong CP problem, and Michael Dine and otehrs pointed out that axions could be cold dark matter if the axion mass lies in the range 10^{-3} - 10^{-5} eV. The lower part of this mass range is being probed by an experiment at the University of Washington. Axions, like π^0, couple to two photons, so an axion can convert to a photon in a microwave cavity.

AXION search

The diagram at right shows the layout of the axion search experiment now underway at the University of Washington. Axions would be detected as extra photons in the Microwave Cavity.



WIMP Dark Matter Annihilation



The weaker the cross section, the earlier freezeout occurs, and the larger the resulting dark matter density.

Figure 3.5. Abundance of heavy stable particle as the temperature drops beneath its mass. Dashed line is equilibrium abundance. Two different solid curves show heavy particle abundance for two different values of λ, the ratio of the annihilation rate to the Hubble rate. Inset shows that the difference between quantum statistics and Boltzmann statistics is important only at temperatures larger than the mass.

Dodelson, *Modern Cosmology*, p. 76

WIMP Dark Matter Annihilation

The abundance today of dark matter particles X of the WIMP variety is determined by their survival of annihilation in the early universe. Supersymmetric neutralinos can annihilate with each other (and sometimes with other particles: "co-annihilation"). Dark matter annihilation follows the same pattern as the previous discussions: initially the abundance of dark matter particles X is given by the equilibrium Boltzmann exponential exp($-m_X/T$), but as they start to disappear they have trouble finding each other and eventually their number density freezes out. The freezeout process can be followed using the Boltzmann equation, as discussed in Kolb and Turner, Dodelson, Mukhanov, and other textbooks. For a detailed discussion of Susy WIMPs, see the review article by Jungman, Kamionkowski, and Griest (1996). The result is that the abundance today of WIMPs X is given in most cases by (Dodelson's Eqs. 3.59-60; see also Perkins Eq. 7.18)

$$\Omega_X = \left[\frac{4\pi^3 G g_*(m)}{45}\right]^{1/2} \frac{x_f T_0^3}{30\langle\sigma v\rangle\rho_{\rm cr}} = 0.3h^{-2}\left(\frac{x_f}{10}\right)\left(\frac{g_*(m)}{100}\right)^{1/2} \frac{10^{-39}\,{\rm cm}^2}{\langle\sigma v\rangle}.$$

Here $x_f \approx 10$ is the ratio of m_X to the freezeout temperature T_f, and $g_*(m_X) \approx 100$ is the density of states factor in the expression for the energy density of the universe when the temperature equals m_X

$$\rho = \frac{\pi^2}{30}T^4\left[\sum_{i=\rm bosons} g_i + \frac{7}{8}\sum_{i=\rm fermions} g_i\right] \equiv g_*\frac{\pi^2}{30}T^4.$$

The sum is over relativistic species *i* (see the graph of *g*(*T*) on the next slide). Note that more X's survive, the weaker the cross section σ. For Susy WIMPs the natural values are $\sigma \sim 10^{-39}$ cm^2, so $\Omega_X \sim 1$ naturally.

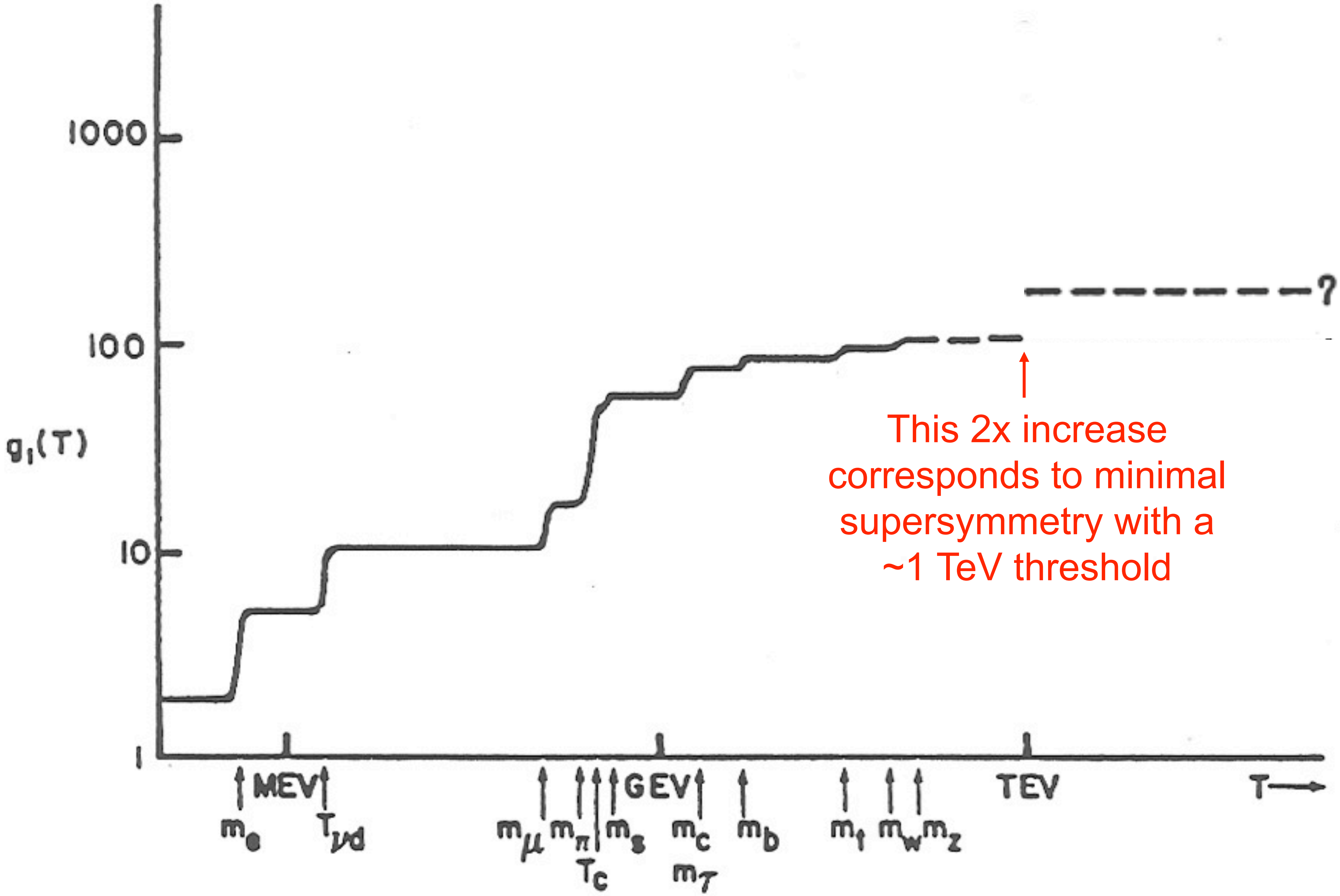


Fig. 1 The effective number of degrees of freedom of thermally interacting relativistic particles as a function of temperature.