Physics 129 LECTURE 16 March 6, 2014

Cosmic Inflation and After

- Topological Defects: Strings and Monopoles
- GUT Monopoles
- Motivations for Inflation: Monopoles, Flatness, Horizon
- Inflation Basics
- Inflationary Fluctions
- Eternal Inflation
- Supersymmetric Inflation
- Comparison with Observations
- Post-Inflation

Topological Defects

These arise when some n-component scalar field $\phi_i(\mathbf{x}) = 0$ because of topological trapping that occurs as a result of a phase transition in the early universe (as I will explain shortly).

If the ϕ field is complex then n=2, and $\phi_i(\mathbf{x}) = 0$ occurs along a linear locus of points, a **string,** in three dimensional space. This corresponds to a 2-dimensional world-sheet in the 3+1 dimensions of spacetime.

If the ϕ field has three components, then $\phi_i(\mathbf{x}) = 0$ occurs at a point in three dimensional space, a **monopole**. This corresponds to a 1-dimensional world-line in the 3+1 dimensions of spacetime.

If the ϕ field has four components, then $\phi_i(\mathbf{x}) = 0$ occurs at a point in space-time, an **instanton**. A related concept is **texture**.

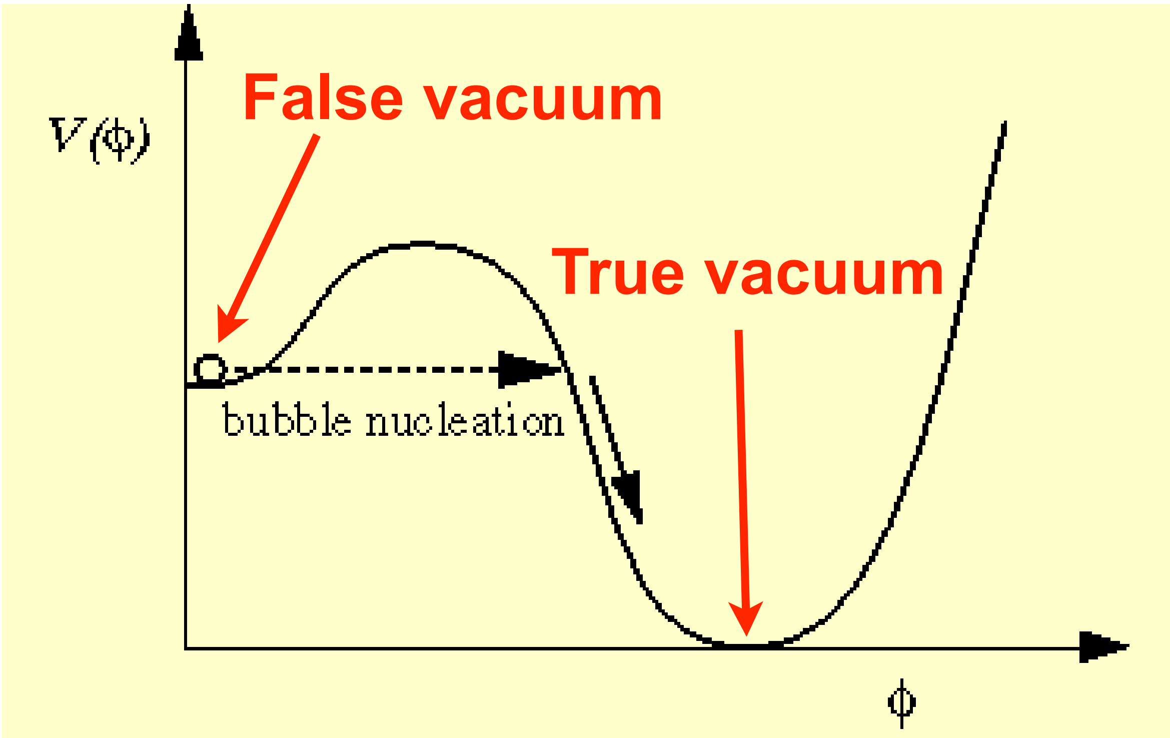
Phase transitions

The cosmological significance of symmetry breaking is due to the fact that symmetries are restored at high temperature (just as it is for liquid water when ice melts). For extremely high temperatures in the early universe, we might even achieve a grand unified state G. Viewed from the moment of creation forward, the universe will pass through a sucession of phase transitions at which the strong nuclear force will become differentiated and then the weak nuclear force and electromagnetism.

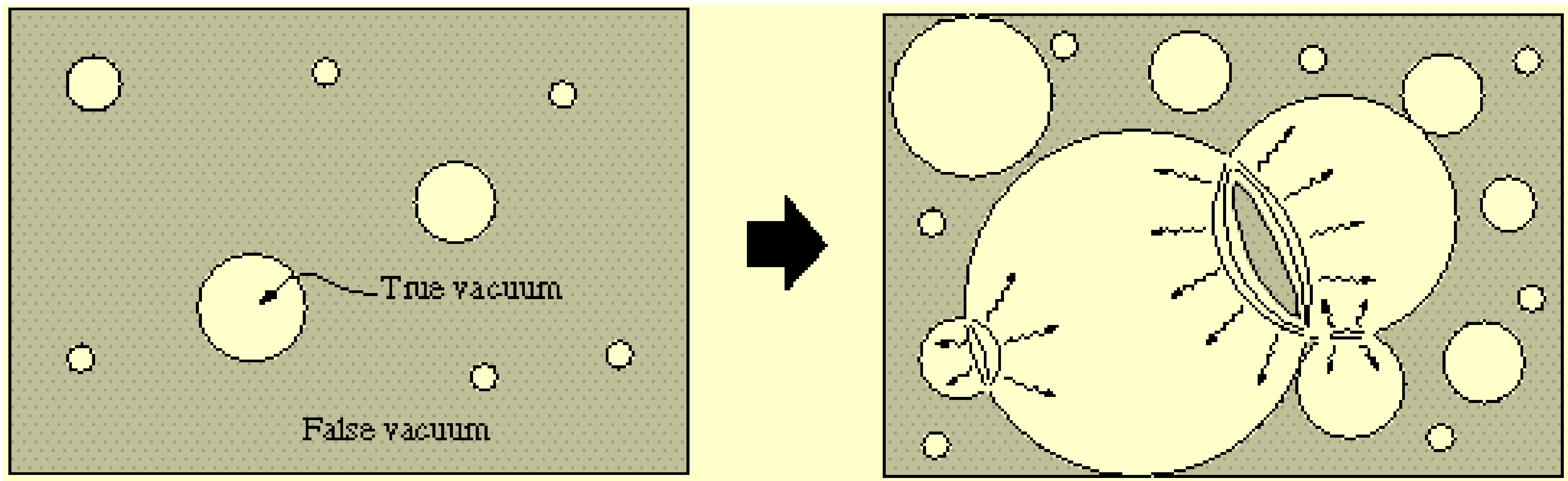
Phase transitions can have a wide variety of important implications including the formation of topological defects - cosmic strings, domain walls, monopoles and textures, or it may even trigger a period of exponential expansion (inflation).

Phase transitions can be either dramatic - first order, or smooth - second order.

During a first-order phase transition, the matter fields get trapped in a `false vacuum' state from which they can only escape by nucleating bubbles of the new phase, that is, the `true vacuum' state.



First-order phase transitions (illustrated below) occur through the formation of bubbles of the new phase in the middle of the old phase; these bubbles then expand and collide until the old phase disappears completely and the phase transition is complete.



First-order phase transitions proceed by bubble nucleation. A bubble of the new phase (the true vacuum) forms and then expands until the old phase (the false vacuum) disappears. A useful analogue is boiling water in which bubbles of steam form and expand as they rise to the surface.

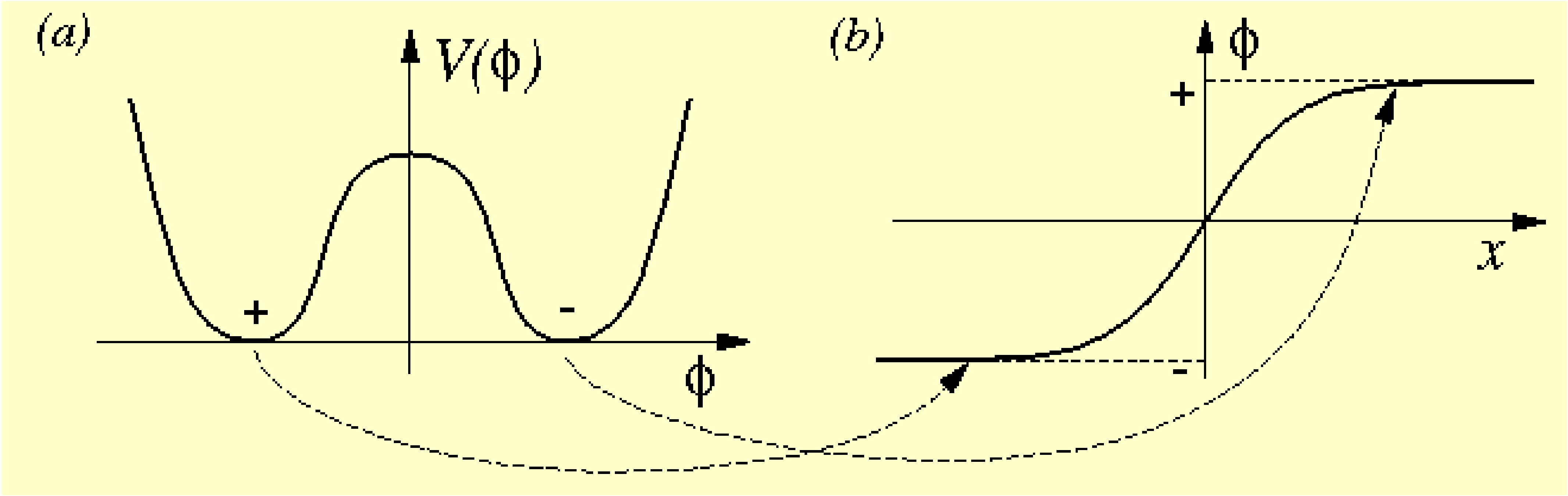
Second-order phase transitions, on the other hand, proceed smoothly. The old phase transforms itself into the new phase in a continuous manner. There is energy (specific heat of vaporization, for example) associated with a first order phase transition.

Either type of phase transition can produce stable configurations called “topological defects.”

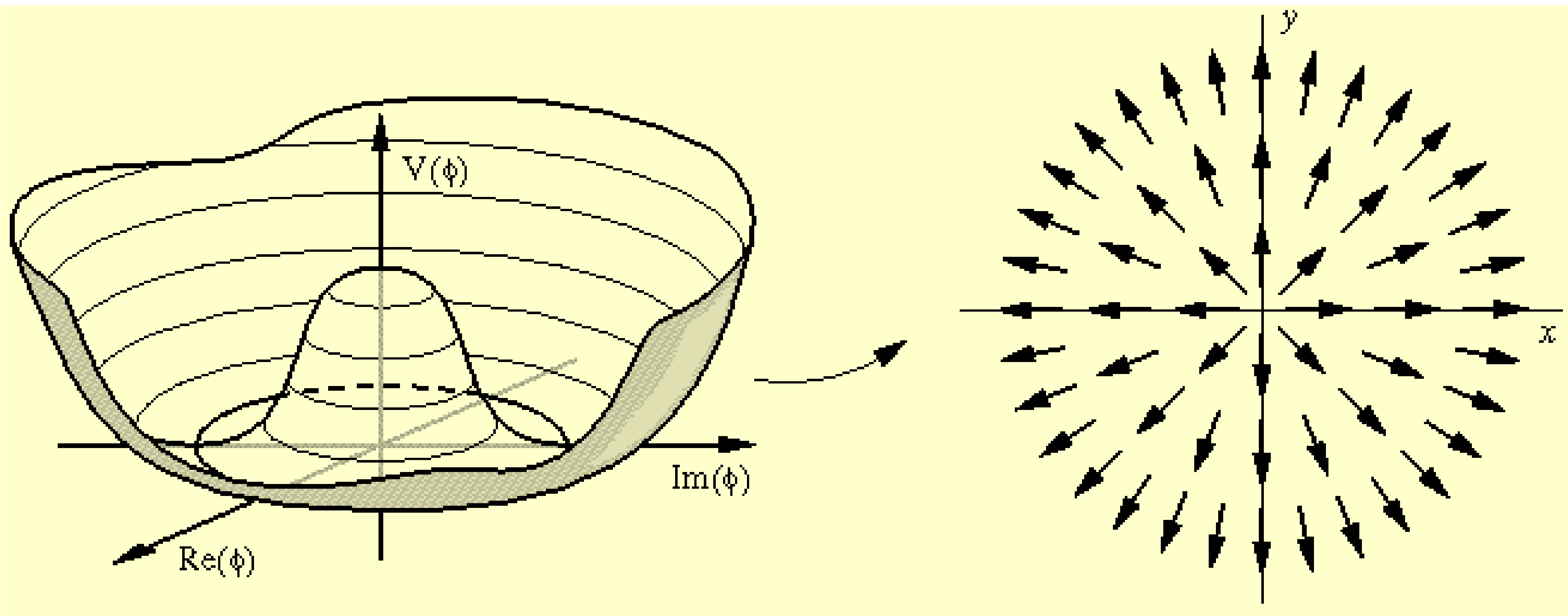
Cosmic Strings & Other Topological Defects

Topological defects are stable configurations that are in the original, symmetric or old phase, but nevertheless for topological reasons they persist after a phase transition to the asymmetric or new phase is completed - because to unwind them would require a great deal of energy. There are a number of possible types of defects, such as domain walls, cosmic strings, monopoles, and textures. The type of defect is determined by the symmetry properties of the matter and the nature of the phase transition.

Domain walls: These are two-dimensional objects that form when a discrete symmetry is broken at a phase transition. A network of domain walls effectively partitions the universe into various `cells'. Domain walls have some rather peculiar properties. For example, the gravitational field of a domain wall is repulsive rather than attractive.

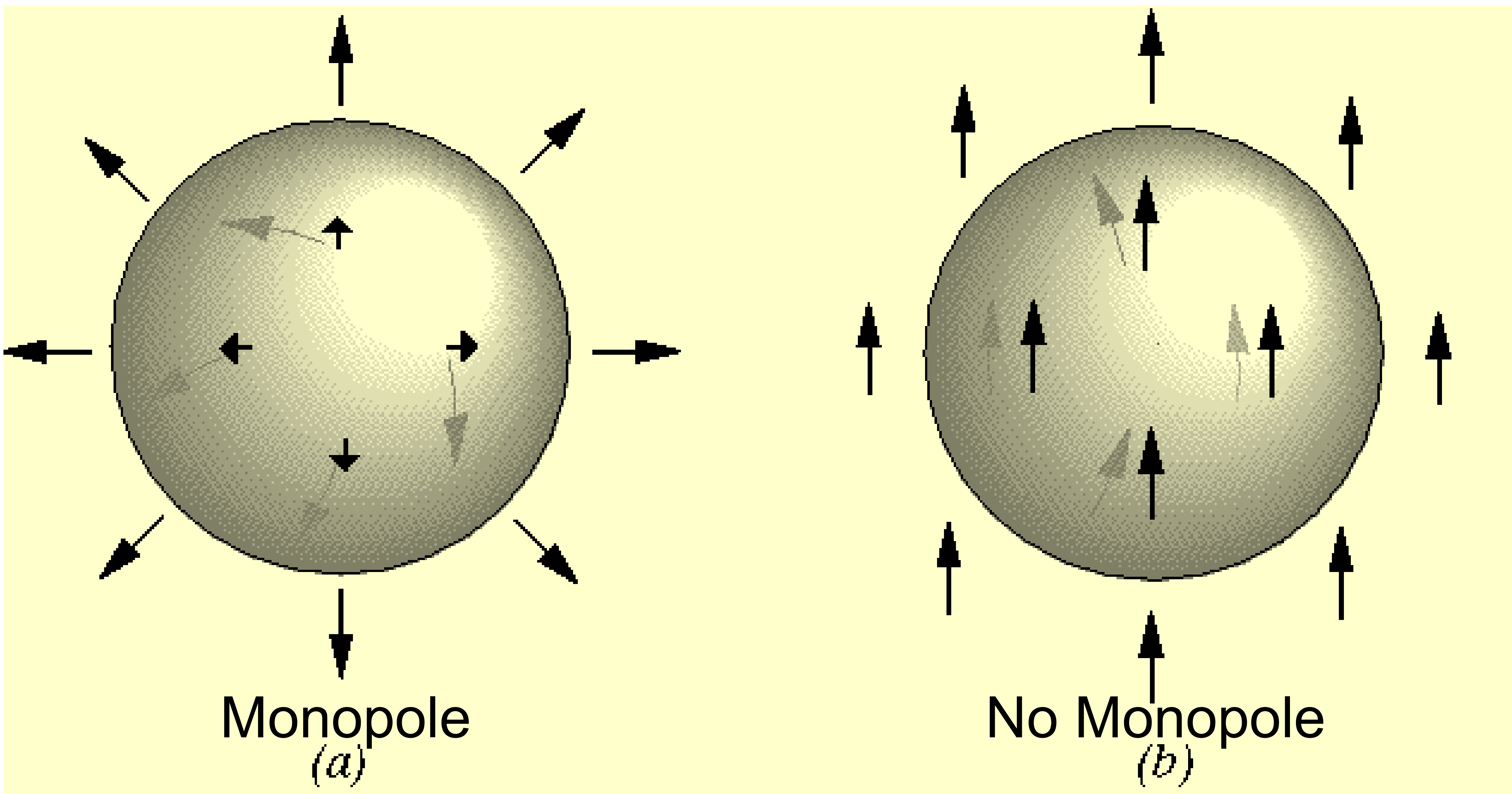


Cosmic strings: These are one-dimensional (that is, line-like) objects which form when an axial or cylindrical symmetry is broken. Strings can be associated with grand unified particle physics models, or they can form at the electroweak scale. They are very thin and may stretch across the visible universe. A typical GUT string has a thickness that is less then a trillion times smaller that the radius of a hydrogen atom, but a 10 km length of one such string would weigh as much as the earth itself!



Cosmic strings are associated with models in which the set of minima are not simply-connected, that is, the vacuum manifold has `holes' in it. The minimum energy states on the left form a circle and the string corresponds to a non-trivial winding around this.

Monopoles: These are zero-dimensional (point-like) objects which form when a spherical symmetry is broken. Monopoles are predicted to be supermassive and carry magnetic charge. The existence of monopoles is an inevitable prediction of grand unified theories (GUTs - more on this shortly); why the universe isn't filled with them is one of the puzzles of the standard cosmology.

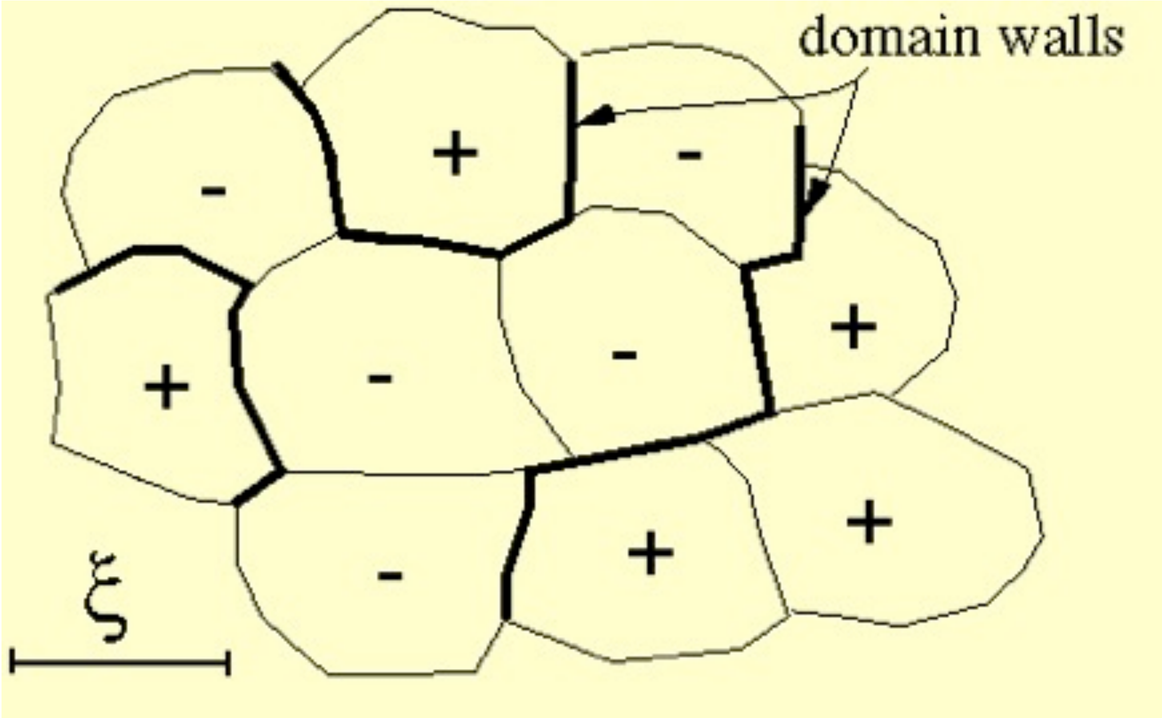


Why do cosmic topological defects form?

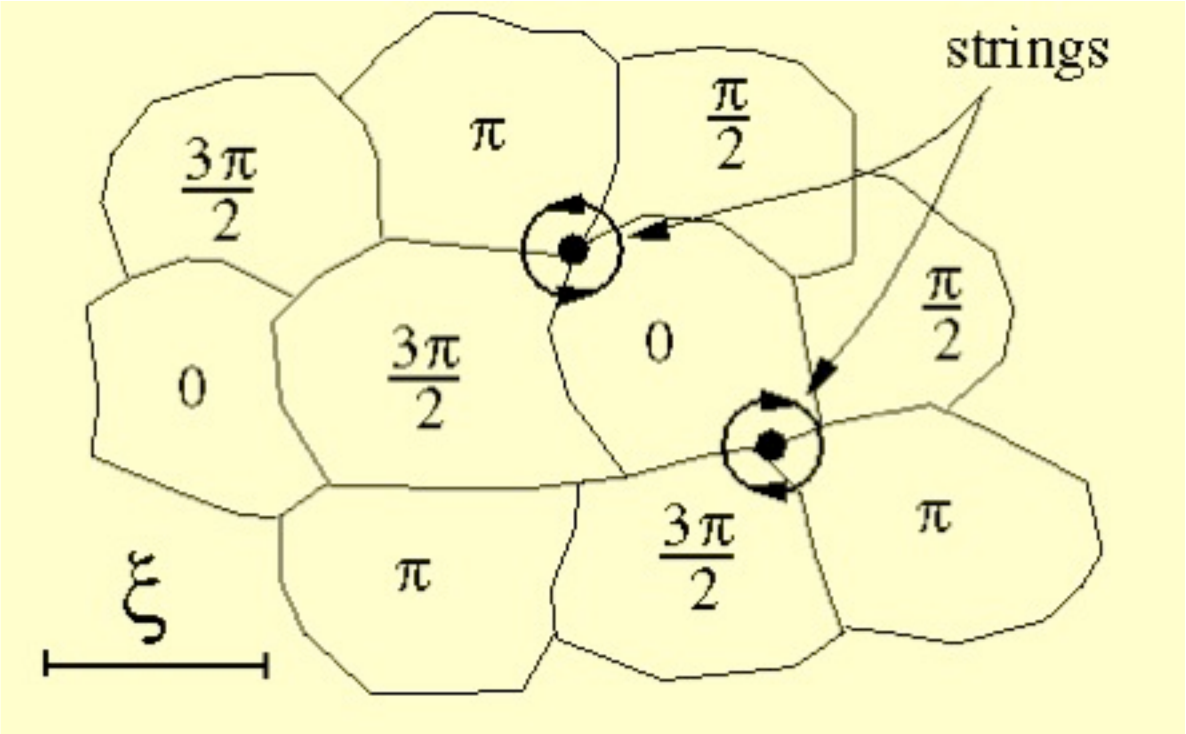
If cosmic strings or other topological defects *can* form at a cosmological phase transition, then they *will* form. This was first pointed out by Tom Kibble and, in a cosmological context, the defect formation process is known as the **Kibble mechanism**.

The simple fact is that causal effects in the early universe can only propagate (as at any time) at the speed of light c. This means that at a time t, regions of the universe separated by more than a distance $d=ct$ can know nothing about each other. In a symmetry breaking phase transition, different regions of the universe will choose to fall into different minima in the set of possible states (this set is known to mathematicians as the vacuum manifold). Topological defects are precisely the "boundaries" between these regions with different choices of minima, and their formation is therefore an inevitable consequence of the fact that different regions cannot agree on their choices.

For example, in a theory with two minima, plus + and minus -, then neighboring regions separated by more than ct will tend to fall randomly into the different states (as shown below). Interpolating between these different minima will be a **domain wall**.



Cosmic strings will arise in slightly more complicated theories in which the minimum energy states possess `holes'. The strings will simply correspond to non-trivial `windings' around these holes (as illustrated at right).



The Kibble mechanism for the formation of cosmic strings.

Topological defects can provide a unique link to the physics of the very early universe. Furthermore, they can crucially affect the evolution of the universe, so their study is an unavoidable part of any serious attempt to understand the early universe. The cosmological consequences vary with the type of defect considered. **Domain walls and monopoles are cosmologically catastrophic.** Any cosmological model in which they form will evolve in a way that contradicts the basic observational facts that we know about the universe. Such models must therefore be ruled out! **Cosmic inflation was invented to solve this problem.**

Cosmic strings and textures are (possibly) much more benign. Among other things, they were until recently thought to be a possible source of the fluctuations that led to the formation of the large-scale structures we observe today, as well as the anisotropies in the Cosmic Microwave Background. However, the CMB anisotropies have turned out not to agree with the predictions of this theory.

By 2000, it was clear that **cosmic defects are not the main source of the CMB anisotropies**.

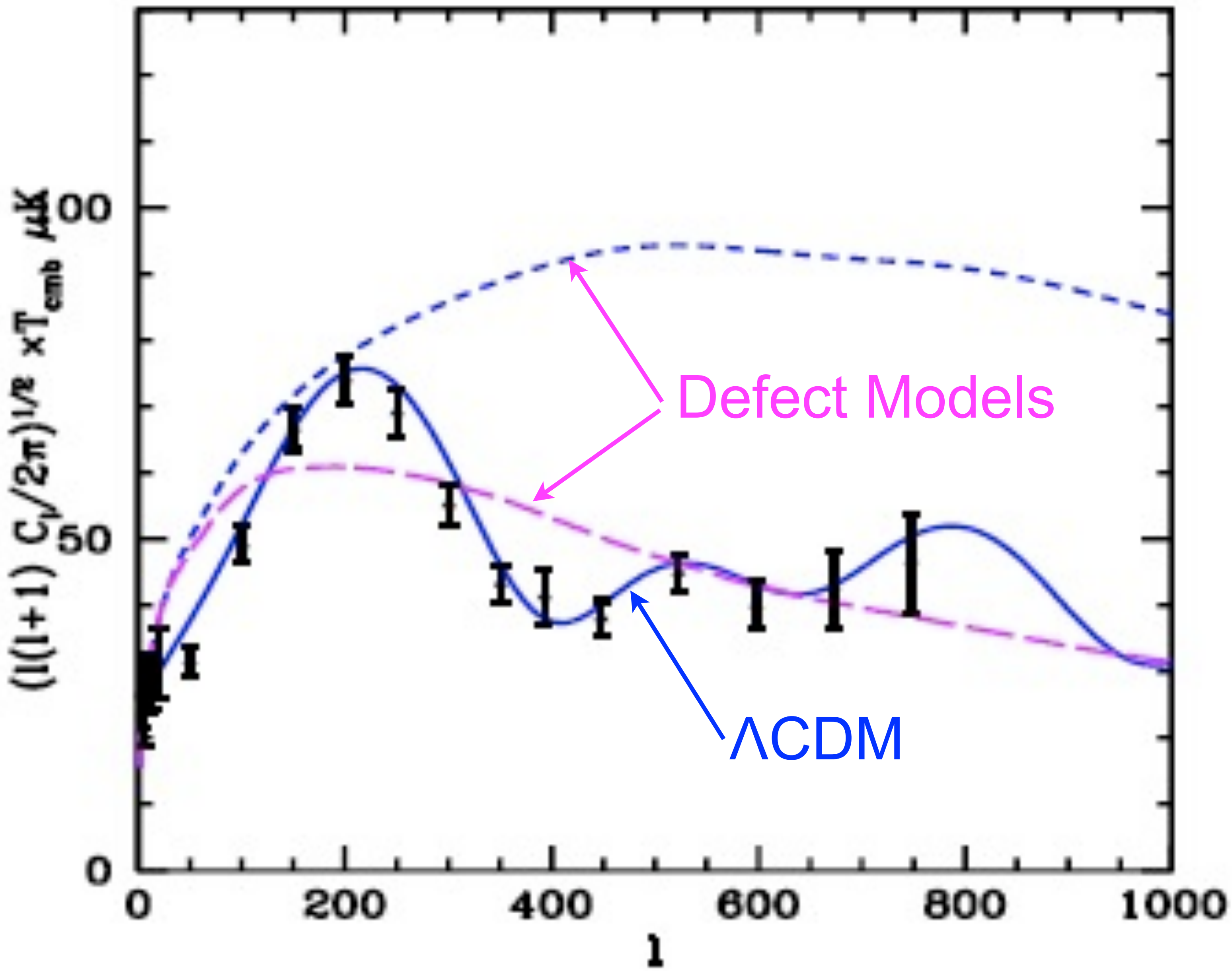


Figure 3: Current data (as complied by Knox[22]) with two defect models (dashed) and an inflation-based model (solid). The upper defect model has a standard ionization history and the lower model has an ionization history specifically designed to produce a sharper, shifted peak.

Andreas Albrecht, Defect models of cosmic structure in light of the new CMB data, XXXVth Rencontres de Moriond ``Energy Densities in the Universe'' (2000).

GUT Monopoles

A simple SO(3) GUT illustrates how nonsingular monopoles arise. The Lagragian is

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}D_\mu\Phi^a D^\mu\Phi^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{8}\lambda(\Phi^a\Phi^a - \sigma^2)^2, \\
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - e\varepsilon_{abc}A^b_\mu A^c_\nu, \\
D_\mu\Phi^a &= \partial_\mu\Phi^a - e\varepsilon_{abc}A^b_\mu\Phi^c.
\end{aligned}$$

The masses of the resulting charged vector and Higgs bosons after spontaneous symmetry breaking are

$$\begin{aligned}
M_V^2 &= e^2\sigma^2, \\
M_S^2 &= \lambda\sigma^2.
\end{aligned}$$

If the Higgs field Φ^a happens to rotate about a sphere in SO(3) space as one moves around a sphere about any particular point in $\mathbf{x}$-space, then it must vanish at that point. Remarkably, if we identify the massless vector field as the photon, this configuration corresponds to a nonsingular magnetic monopole, as was independently discovered by ‘tHooft and Polyakov. The monopole has magnetic charge twice the minimum Dirac value, $g = 2\pi/e = (4\pi/e^2)(e/2) \approx 67.5\ e$.

The singular magnetic field is cut off at scale σ, and as a result the GUT monopole has mass $M_{\text{monopole}} \approx M_V/\alpha \approx M_{\text{GUT}}/\alpha \approx 10^{18}$ GeV.

The first accurate calculation of the mass of the ‘t Hooft - Polyakov non-singular monopole was Bais & Primack (Phys. Rev. D13:819,1976).

GUT Monopole Problem

The Kibble mechanism produces ~ one GUT monopole per horizon volume when the GUT phase transition occurs. These GUT monopoles have a number density over entropy (using the old $T_{GUT} \sim M_{GUT} \sim 10^{14}$ GeV)

$$n_M/s \sim 10^2 (T_{GUT}/M_{Pl})^3 \sim 10^{-13}$$

(compared to $n_B/s \sim 10^{-9}$ for baryons) Their annihilation is inefficient since they are so massive, and as a result they are about as abundant as gold atoms but 10^{16} times more massive, so they “overclose” the universe. This catastrophe must be avoided! **This was Alan Guth’s initial motivation for inventing cosmic inflation.**

I will summarize the key ideas of inflation theory, following my lectures at the Jerusalem Winter School, published as the first chapter in Avishai Dekel & Jeremiah Ostriker, eds., *Formation of Structure in the Universe* (Cambridge University Press, 1999), and Dierck-Ekkehard Liebscher, *Cosmology* (Springer, 2005) (available online through the UCSC library).

Motivations for Inflation

	Problem Solved
Horizon	Homogeneity, Isotropy, Uniform T
Flatness/Age	Expansion and gravity balance
"Dragons"	Monopoles, doman walls,...banished
Structure	Small fluctuations to evolve into galaxies, clusters, voids

Cosmological constant $\Lambda > 0 \Rightarrow$ space repels space, so the more space the more repulsion, $\Rightarrow$ de Sitter exponential expansion $a \propto e^{\sqrt{\Lambda}t}$.

Inflation is exponentially accelerating expansion caused by effective cosmological constant ("false vacuum" energy) associated with hypothetical scalar field ("inflaton").

		Forces of Nature	**Spin**
Known	{	Gravity	2
		Strong, weak, and electromagnetic	1
Goal of LHC		Mass (Higgs Boson)	0
Early universe		Inflation (Inflaton)	0

Inflation lasting only $\sim 10^{-32}$s suffices to solve all the problems listed above. Universe must then convert to ordinary expansion through conversion of false to true vacuum ("re-"heating).

Joel Primack, in *Formation of Structure in the Universe*, ed. Dekel & Ostriker (Cambridge Univ Press, 1999)

Inflation Basics

The basic idea of inflation is that before the universe entered the present adiabatically expanding Friedmann era, it underwent a period of de Sitter exponential expansion of the scale factor, termed *inflation* (Guth 1981). Actually, inflation is never precisely de Sitter, and any superluminal (faster-than-light) expansion is now called inflation. Inflation was originally invented to solve the problem of too many GUT monopoles, which, as mentioned in the previous section, would otherwise be disastrous for cosmology.

The de Sitter cosmology corresponds to the solution of Friedmann's equation in an empty universe (i.e., with $\rho = 0$) with vanishing curvature ($k = 0$) and positive cosmological constant ($\Lambda > 0$). The solution is $a = a_o e^{Ht}$, with constant Hubble parameter $H = (\Lambda/3)^{1/2}$. There are analogous solutions for $k = +1$ and $k = -1$ with $a \propto \cosh Ht$ and $a \propto \sinh Ht$ respectively. The scale factor expands exponentially because the positive cosmological constant corresponds effectively to a negative pressure. de Sitter space is discussed in textbooks on general relativity (for example, Rindler 1977, Hawking & Ellis 1973) mainly for its geometrical interest. Until cosmological inflation was considered, the chief significance of the de Sitter solution in cosmology was that it is a limit to which all indefinitely expanding models with $\Lambda > 0$ must tend, since as $a \to \infty$, the cosmological constant term ultimately dominates the right hand side of the Friedmann equation.

Joel Primack, in *Formation of Structure in the Universe*, (Cambridge Univ Press, 1999)

As Guth (1981) emphasized, the de Sitter solution might also have been important in the very early universe because the vacuum energy that plays such an important role in spontaneously broken gauge theories also acts as an effective cosmological constant. A period of de Sitter inflation preceding ordinary radiation-dominated Friedmann expansion could explain several features of the observed universe that otherwise appear to require very special initial conditions: the horizon, flatness/age, monopole, and structure formation problems. (See Table 1.6.)

Let us illustrate how inflation can help with the horizon problem. At recombination ($p^+ + e^- \rightarrow H$), which occurs at $a/a_o \approx 10^{-3}$, the mass encompassed by the horizon was $M_H \approx 10^{18} M_\odot$, compared to $M_{H,o} \approx 10^{22} M_\odot$ today. Equivalently, the angular size today of the causally connected regions at recombination is only $\Delta\theta \sim 3°$. Yet the fluctuation in temperature of the cosmic background radiation from different regions is very small: $\Delta T/T \sim 10^{-5}$. How could regions far out of causal contact have come to temperatures that are so precisely equal? This is the "horizon problem". With inflation, it is no problem because the entire observable universe initially lay inside a single causally connected region that subsequently inflated to a gigantic scale. Similarly, inflation exponentially dilutes any preceeding density of monopoles or other unwanted relics (a modern version of the "dragons" that decorated the unexplored borders of old maps). Joel Primack, in *Formation of Structure in the Universe*, (Cambridge Univ Press, 1999)

In the first inflationary models, the dynamics of the very early universe was typically controlled by the self-energy of the Higgs field associated with the breaking of a Grand Unified Theory (GUT) into the standard 3-2-1 model: GUT$\rightarrow SU(3)_{color} \otimes [SU(2) \otimes U(1)]_{electroweak}$. This occurs when the cosmological temperature drops to the unification scale $T_{GUT} \sim 10^{14}$ GeV at about 10^{-35} s after the Big Bang. Guth (1981) initially considered a scheme in which inflation occurs while the universe is trapped in an unstable state (with the GUT unbroken) on the wrong side of a maximum in the Higgs potential. This turns out not to work: the transition from a de Sitter to a Friedmann universe never finishes (Guth & Weinberg 1981). The solution in the "new inflation" scheme (Linde 1982; Albrecht and Steinhardt 1982) is for inflation to occur *after* barrier penetration (if any). It is necessary that the potential of the scalar field controlling inflation ("*inflaton*") be nearly flat (i.e., decrease very slowly with increasing inflaton field) for the inflationary period to last long enough. This nearly flat part of the potential must then be followed by a very steep minimum, in order that the energy contained in the Higgs potential be rapidly shared with the other degrees of freedom ("reheating"). A more general approach, "chaotic" inflation, has been worked out by Linde (1983, 1990) and others; this works for a wide range of inflationary potentials, including simple power laws such as $\lambda\phi^4$. However, for the amplitude of the fluctuations to be small enough for consistency with observations, it is necessary that the inflaton self-coupling be very small, for example $\lambda \sim 10^{-14}$ for the ϕ^4 model. This requirement prevents a Higgs field from being the inflaton, since Higgs fields by definition have gauge couplings to the gauge field (which are expected to be of order unity), and these would generate self-couplings of similar magnitude even if none were present.

It turns out to be necessary to inflate by a factor $\gtrsim e^{66}$ in order to solve the flatness problem, i.e., that $\Omega_0 \sim 1$. (With $H^{-1} \sim 10^{-34}$ s during the de Sitter phase, this implies that the inflationary period needs to last for only a relatively small time $\tau \gtrsim 10^{-32}$ s.) The "flatness problem" is essentially the question why the universe did not become curvature dominated long ago. Neglecting the cosmological constant on the assumption that it is unimportant after the inflationary epoch, the Friedmann equation can be written

$e^{66}=$
4×10^{28}

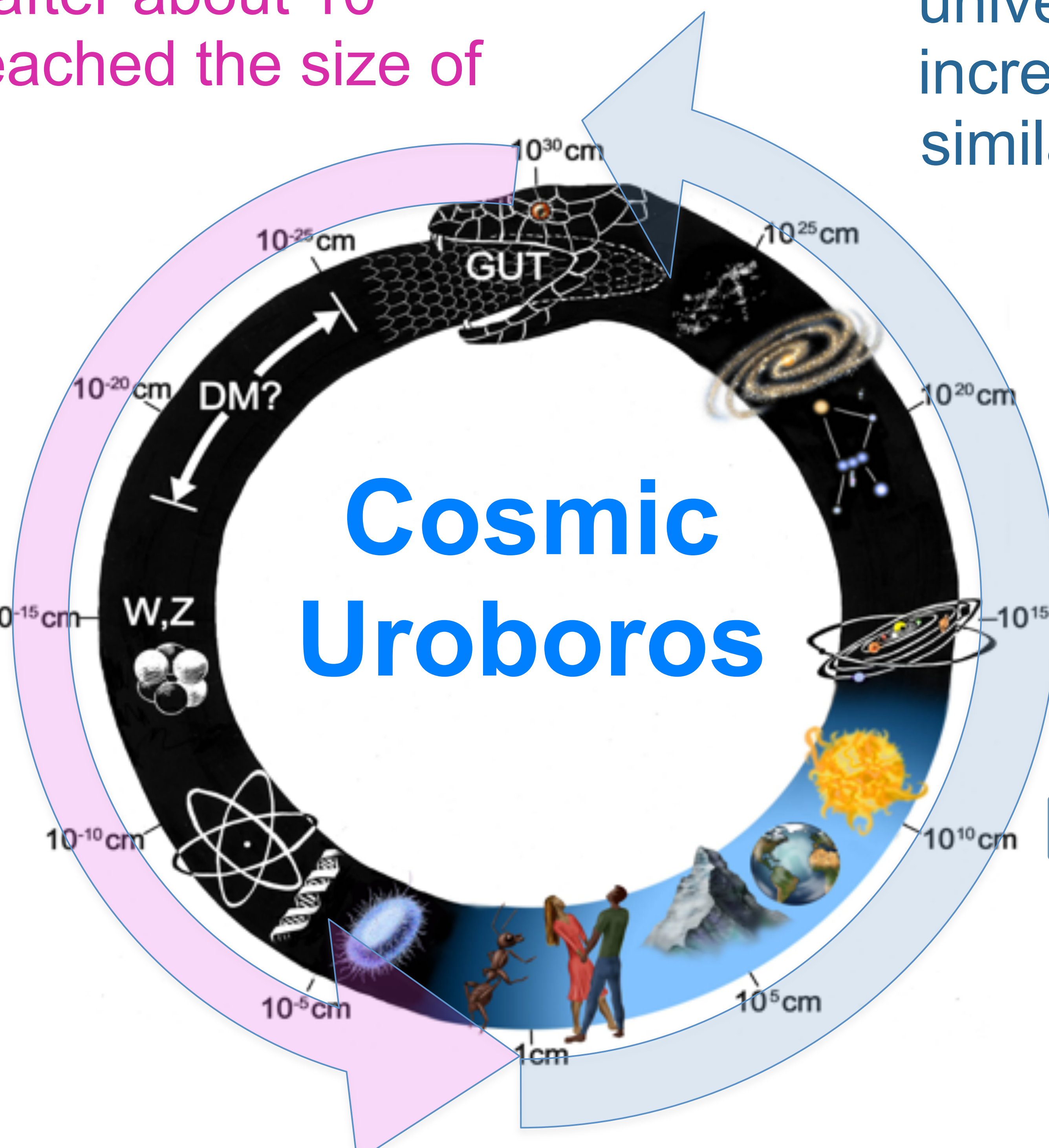
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\pi^2}{30}g(T)T^4 - \frac{kT^2}{(aT)^2}$$

where the first term on the right hand side is the contribution of the energy density in relativistic particles and $g(T)$ is the effective number of degrees of freedom. The second term on the right hand side is the curvature term. Since $aT \approx$ constant for adiabatic expansion, it is clear that as the temperature T drops, the curvature term becomes increasingly important. The quantity $K \equiv k/(aT)^2$ is a dimensionless measure of the curvature. Today, $|K| = |\Omega - 1|\, H_o^2/T_o^2 \leq 2 \times 10^{-58}$. Unless the curvature exactly vanishes, the most "natural" value for K is perhaps $K \sim 1$. Since inflation increases a by a tremendous factor $e^{H\tau}$ at essentially constant T (after reheating), it increases aT by the same tremendous factor and thereby decreases the curvature by that factor squared. Setting $e^{-2H\tau} \lesssim 2\times 10^{-58}$ gives the needed amount of inflation: $H\tau \gtrsim 66$. This much inflation turns out to be enough to take care of the other cosmological problems mentioned above as well.

According to Cosmic Inflation theory, the entire visible universe was once about 10^{-29} cm in size. Its size then inflated by a factor of about 10^{30} so that when Cosmic Inflation ended (after about 10^{-32} second) it had reached the size of a newborn baby.

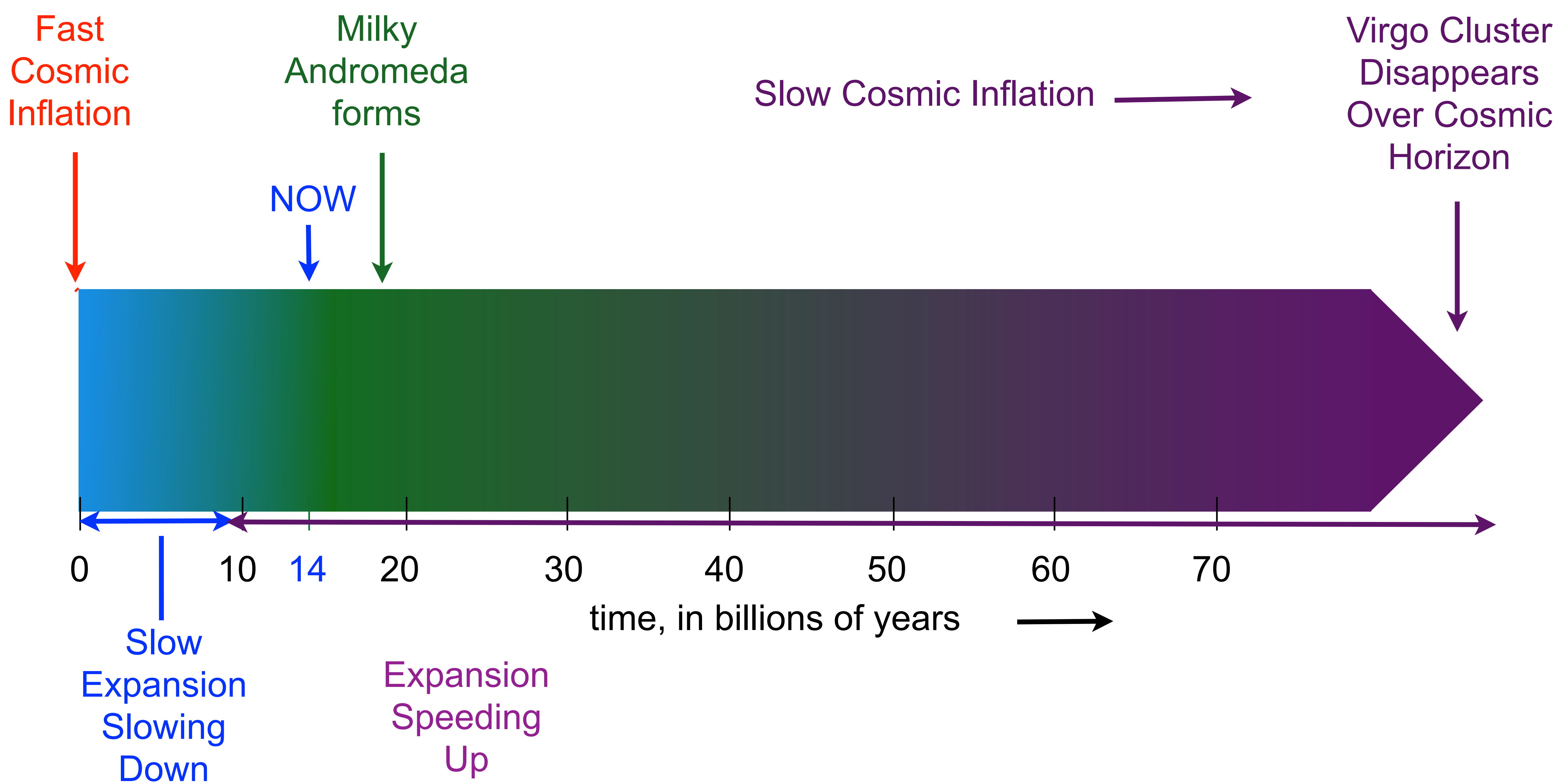
During its entire subsequent evolution, the size of the visible universe has increased by a similar factor of 10^{29}.

10^{-32} seconds of COSMIC INFLATION



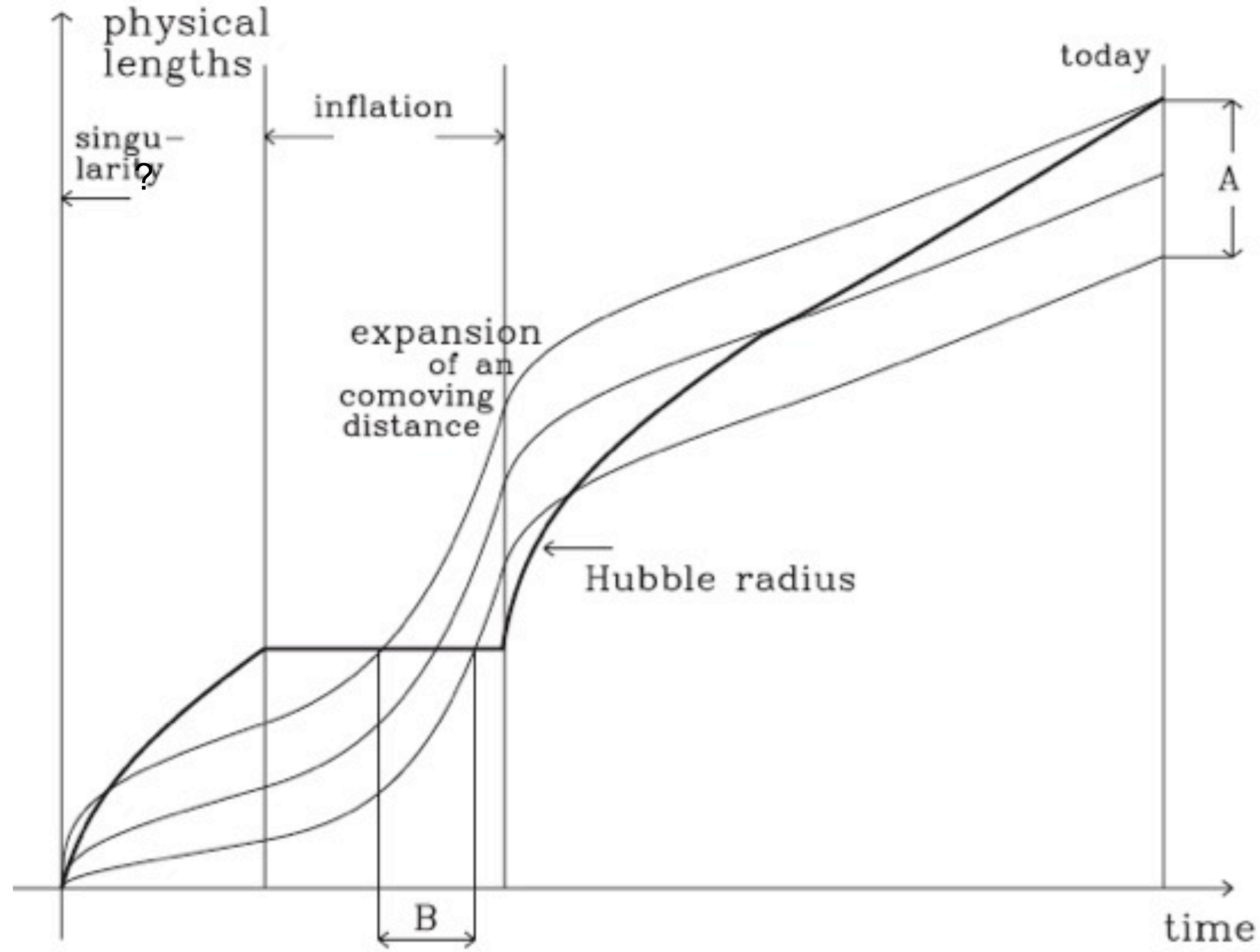
13.7 billion years of COSMIC EXPANSION

COSMIC TIME



Fast Cosmic Inflation
Milky Andromeda forms
Slow Cosmic Inflation
Virgo Cluster Disappears Over Cosmic Horizon
NOW
0
10
14
20
30
40
50
60
70
time, in billions of years
Slow Expansion Slowing Down
Expansion Speeding Up

Inflationary Fluctuations



Thus far, it has been sketched how inflation stretches, flattens, and smooths out the universe, thus greatly increasing the domain of initial conditions that could correspond to the universe that we observe today. But inflation also can explain the origin of the fluctuations necessary in the gravitional instability picture of galaxy and cluster formation. Recall that the very existence of these fluctuations is a problem in the standard Big Bang picture, since these fluctuations are much larger than the horizon at early times. How could they have arisen?

The answer in the inflationary universe scenario is that they arise from quantum fluctuations in the inflaton field ϕ whose vacuum energy drives inflation. The scalar fluctuations $\delta\phi$ during the de Sitter phase are of the order of the Hawking temperature $H/2\pi$. Because of these fluctuations, there is a time spread $\Delta t \approx \delta\phi/\dot{\phi}$ during which different regions of the same size complete the transition to the Friedmann phase. The result is that the density fluctuations when a region of a particular size re-enters the horizon are equal to (Guth & Pi 1982; see Linde 1990 for alternative approaches) $\delta_H \equiv (\delta\rho/\rho)_H \sim \Delta t/t_H = H\Delta t$. The time spread Δt can be estimated from the equation of motion of ϕ (the free Klein-Gordon equation in an expanding universe): $\ddot{\phi} + 3H\dot{\phi} = -(\partial V/\partial\phi)$. Neglecting the $\ddot{\phi}$ term, since the scalar potential V must be very flat in order for enough inflation to occur (this is called the "slow roll" approximation), $\dot{\phi} \approx -V'/(3H)$, so $\delta_H \sim H^3/V' \sim V^{3/2}/V'$. Unless there is a special feature in the potential $V(\phi)$ as ϕ rolls through the scales of importance in cosmology (producing such "designer inflation" features generally requires fine tuning — see e.g. Hodges et al. 1990), V and V' will hardly vary there and hence δ_H will be essentially constant. These are fluctuations of all the contents of the universe, so they are adiabatic fluctuations.

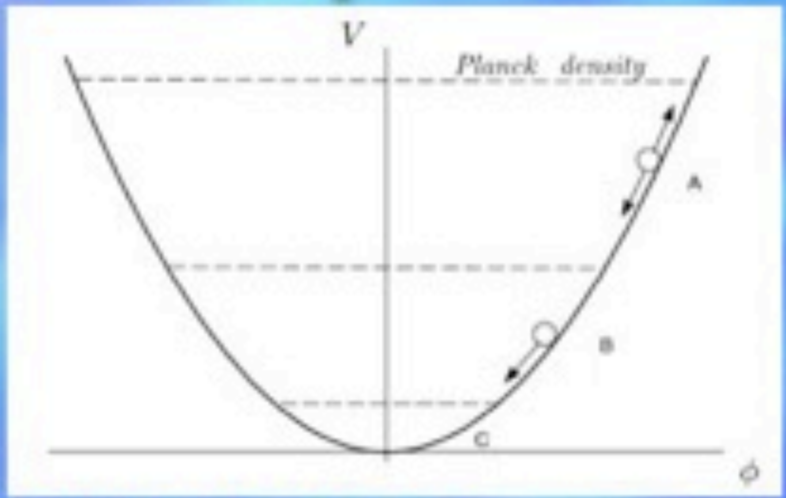
Joel Primack, in *Formation of Structure in the Universe*, (Cambridge Univ Press, 1999)

Thus *inflationary models typically predict a nearly constant curvature spectrum* $\delta_H =$ constant *of adiabatic fluctuations.* Some time ago Harrison (1970), Zel'dovich (1972), and others had emphasized that this is the only scale-invariant (i.e., power-law) fluctuation spectrum that avoids trouble at both large and small scales. If $\delta_H \propto M_H^{-\alpha}$, where M_H is the mass inside the horizon, then if $-\alpha$ is too large the universe will be less homogeneous on large than small scales, contrary to observation; and if α is too large, fluctuations on sufficiently small scales will enter the horizon with $\delta_H \gg 1$ and collapse to black holes (see e.g. Carr, Gilbert, & Lidsey 1995, Bullock & Primack 1996); thus $\alpha \approx 0$. The $\alpha = 0$ case has come to be known as the Zel'dovich spectrum.

Inflation predicts more: it allows the calculation of the value of the constant δ_H in terms of the properties of the scalar potential $V(\phi)$. Indeed, this proved to be embarrassing, at least initially, since the Coleman-Weinberg potential, the first potential studied in the context of the new inflation scenario, results in $\delta_H \sim 10^2$ (Guth & Pi 1982) some six orders of magnitude too large. But this does not seem to be an insurmountable difficulty; as was mentioned above, chaotic inflation works, with a sufficiently small self-coupling. Thus inflation at present appears to be a plausible solution to the problem of providing reasonable cosmological initial conditions (although it sheds no light at all on the fundamental question why the cosmological constant is so small now). Many variations of the basic idea of inflation have been worked out

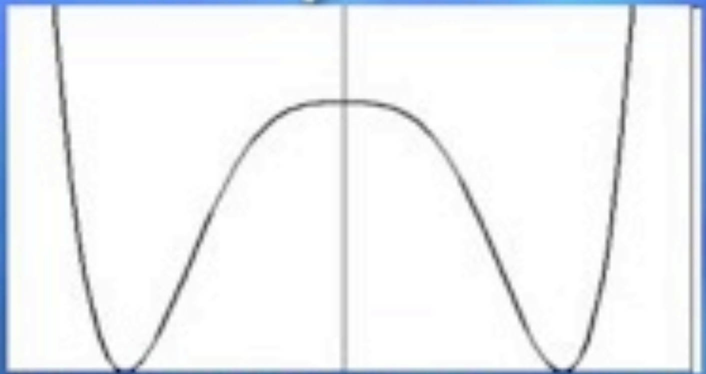
Comparing different inflationary models:

- **Chaotic inflation** can start in the smallest domain of size 10^{-33} cm with total mass ~ M_p (less than a milligram) and entropy O(1)



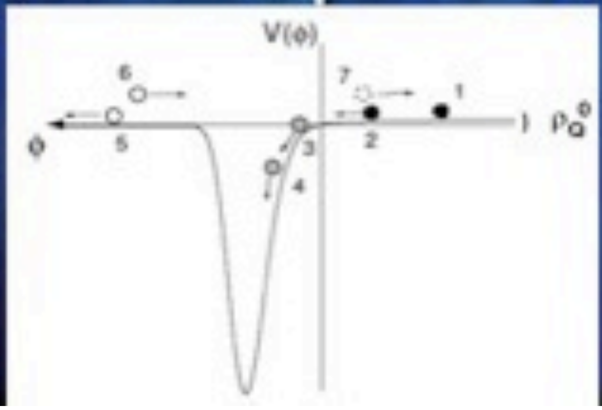
Solves flatness, mass and entropy problem

- **New inflation** can start only in a domain with mass 6 orders of magnitude greater than M_p and entropy greater than 10^9



Not very good with solving flatness, mass and entropy problem

- **Cyclic inflation** can occur only in the domain of size greater than the size of the observable part of the universe, with mass > 10^{55} g and entropy > 10^{87}



Does not solve flatness, mass and entropy problem

Andrei Linde

Eternal Inflation

Vilenkin (1983) and Linde (1986, 1990) pointed out that if one extrapolates inflation backward to try to imagine what might have preceeded it, in many versions of inflation the answer is "eternal inflation": in most of the volume of the universe inflation is still happening, and our part of the expanding universe (a region encompassing far more than our entire cosmic horizon) arose from a tiny part of such a region. To see how eternal inflation works, consider the simple chaotic model with $V(\phi) = (m^2/2)\phi^2$. During the de Sitter Hubble time H^{-1}, where as usual $H^2 = (8\pi G/3)V$, the slow rolling of ϕ down the potential will reduce it by

$$\Delta\phi = \dot{\phi}\Delta t = -\frac{V'}{3H}\Delta t = \frac{m_{pl}^2}{4\pi\phi}. \tag{1.7}$$

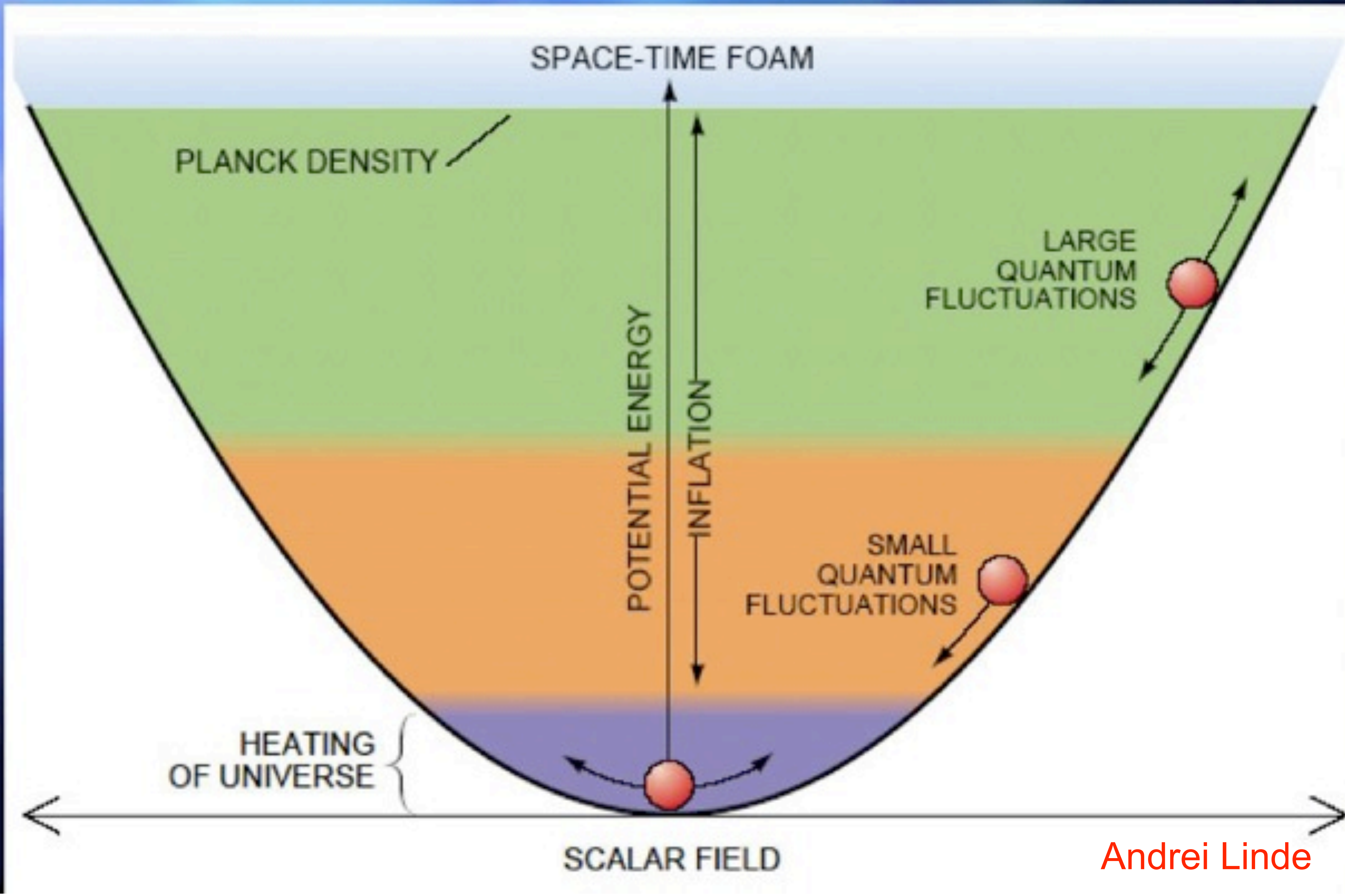
Here m_{Pl} is the Planck mass ($m_{Planck} = 1/G^{1/2}$). But there will also be quantum fluctuations that will change ϕ up or down by

$$\delta\phi = \frac{H}{2\pi} = \frac{m\phi}{\sqrt{3\pi}m_{Pl}} \tag{1.8}$$

These will be equal for $\phi_* = m_{pl}^{3/2}/2m^{1/2}$, $V(\phi_*) = (m/8m_{Pl})m_{Pl}^4$. If $\phi \gtrsim \phi_*$, *positive quantum fluctuations dominate* the evolution: after $\Delta t \sim H^{-1}$, an initial region becomes $\sim e^3$ regions of size $\sim H^{-1}$, in half of which ϕ increases to $\phi + \delta\phi$. Since $H \propto \phi$, this drives inflation faster in these regions.

Inflation as a theory of a harmonic oscillator

$$V(\phi) = \frac{m^2}{2}\phi^2$$

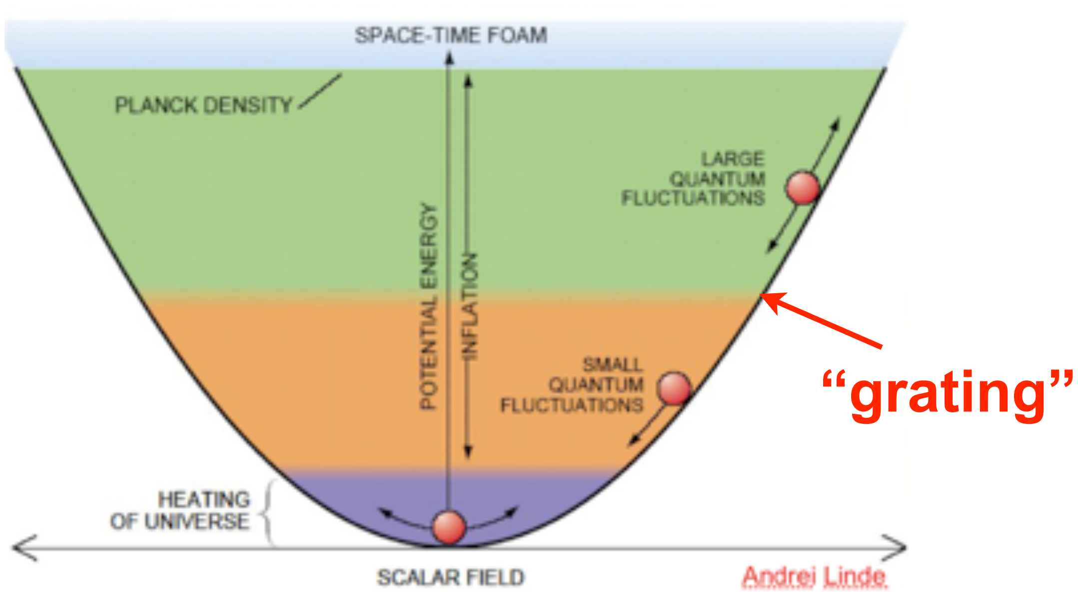


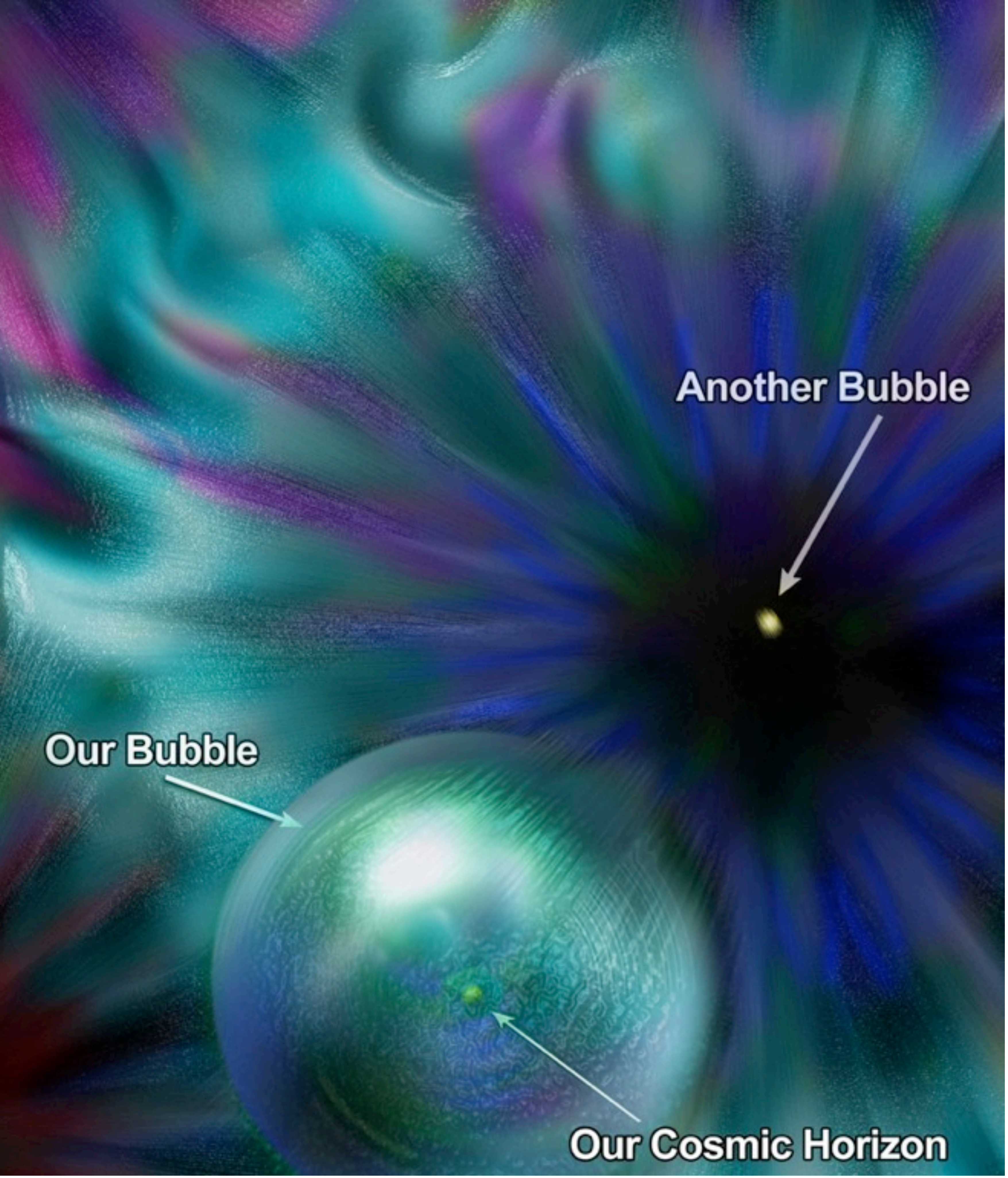
Andrei Linde

THE COSMIC LAS VEGAS

Coins constantly flip. Heads, and the coin is twice the size and there are two of them. Tails, and a coin is half the size.

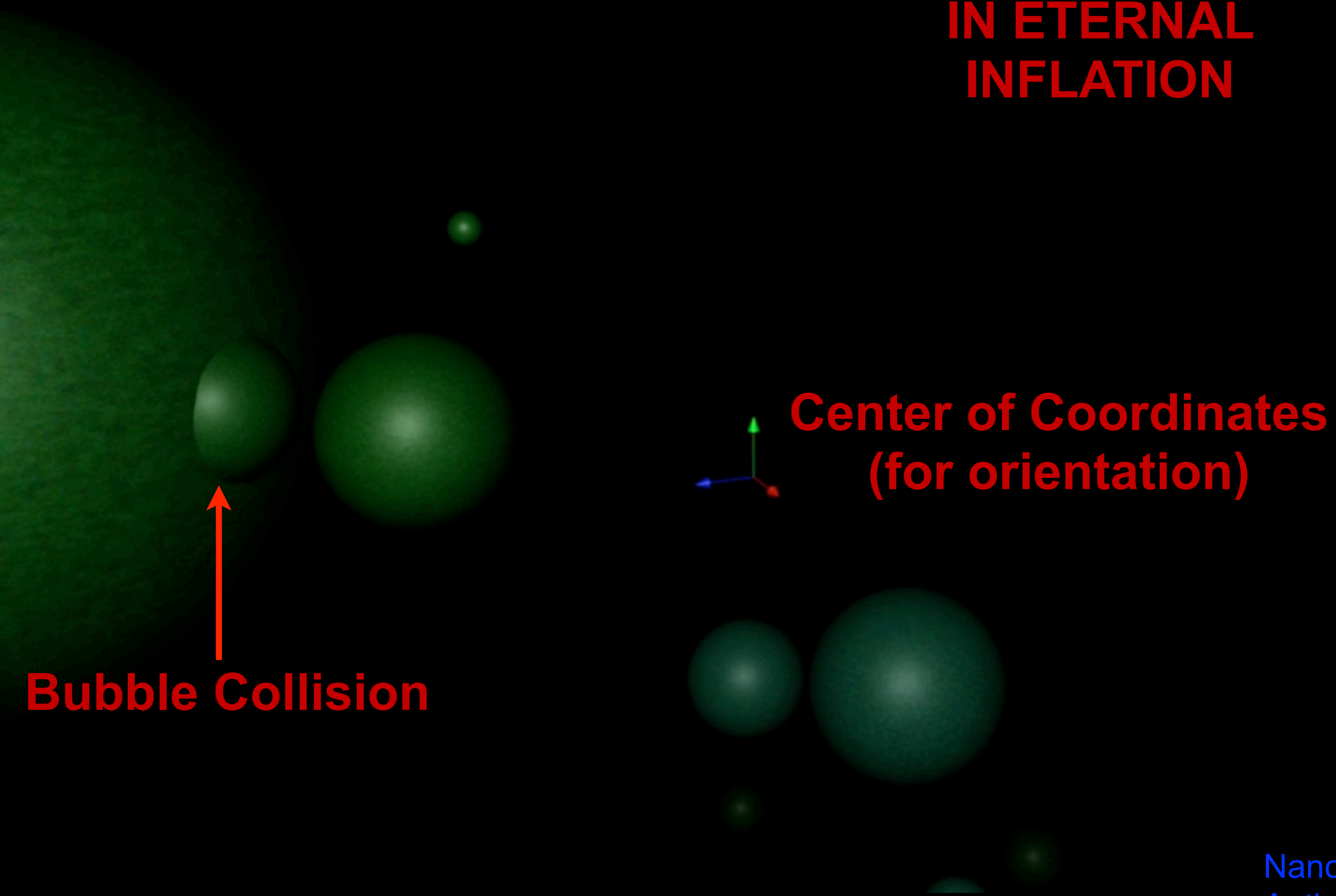
Consider a coin that has a run of tails. It becomes so small it can pass through the grating on the floor.





OUR COSMIC BUBBLE IN ETERNAL INFLATION

BUBBLE UNIVERSES IN ETERNAL INFLATION



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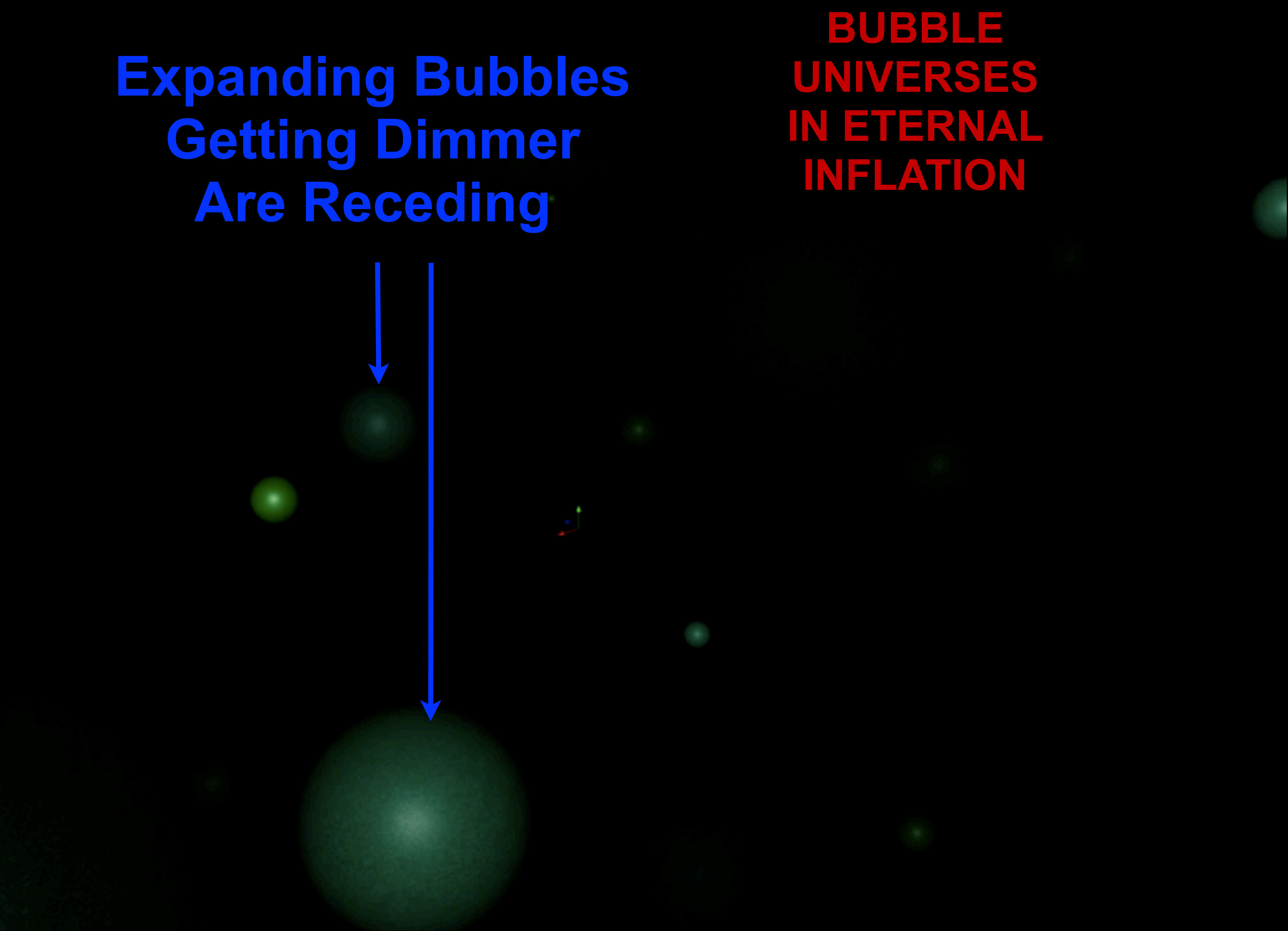
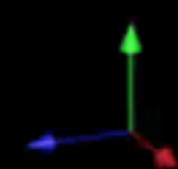

Expanding Bubbles
Getting Dimmer
Are Receding
BUBBLE
UNIVERSES
IN ETERNAL
INFLATION



Supersymmetric Inflation

When Pagels and I (1982) first suggested that the lightest supersymmetric partner particle (LSP), stable because of R-parity, might be the dark matter particle, that particle was the gravitino in the early version of supersymmetry then in fashion. Weinberg (1982) immediately pointed out that if the gravitino were not the LSP, it could be a source of real trouble because of its long lifetime $\sim M_{\rm Pl}^2/m_{3/2}^3 \sim (m_{3/2}/{\rm TeV})^{-3}10^3$ s, a consequence of its gravitational-strength coupling to other fields. Subsequently, it was realized that supersymmetric theories can naturally solve the gauge hierarchy problem, explaining why the electroweak scale $M_{\rm EW} \sim 10^2$ GeV is so much smaller than the GUT or Planck scales. In this version of supersymmetry, which has now become the standard one, the gravitino mass will typically be $m_{3/2} \sim$ TeV; and the late decay of even a relatively small number of such massive particles can wreck BBN and/or the thermal spectrum of the CBR. The only way to prevent this is to make sure that the reheating temperature after inflation is sufficiently low: $T_{\rm RH} \lesssim 2 \times 10^9$ GeV (for $m_{3/2} =$ TeV) (Ellis, Kim, & Nanopoulos 1984, Ellis et al. 1992).

Inflation Summary

The key features of all inflation scenarios are a period of superluminal expansion, followed by ("re-")heating which converts the energy stored in the inflaton field (for example) into the thermal energy of the hot big bang.

Inflation is *generic*: it fits into many versions of particle physics, and it can even be made rather natural in modern supersymmetric theories as we have seen. The simplest models have inflated away all relics of any pre-inflationary era and result in a flat universe after inflation, i.e., $\Omega = 1$ (or more generally $\Omega_0 + \Omega_\Lambda = 1$). Inflation also produces scalar (density) fluctuations that have a primordial spectrum

$$\left(\frac{\delta\rho}{\rho}\right)^2 \sim \left(\frac{V^{3/2}}{m_{Pl}^3 V'}\right)^2 \propto k^{n_p}, \tag{1.12}$$

where V is the inflaton potential and n_p is the primordial spectral index, which is expected to be near unity (near-Zel'dovich spectrum). Inflation also produces tensor (gravity wave) fluctuations, with spectrum

$$P_t(k) \sim \left(\frac{V}{m_{Pl}}\right)^2 \propto k^{n_t}, \tag{1.13}$$

where the tensor spectral index $n_t \approx (1 - n_p)$ in many models.

The quantity $(1 - n_p)$ is often called the "tilt" of the spectrum; the larger the tilt, the more fluctuations on small spatial scales (corresponding to large k) are suppressed compared to those on larger scales. The scalar and tensor waves are generated by independent quantum fluctuations during inflation, and so their contributions to the CMB temperature fluctuations add in quadrature. The ratio of these contributions to the quadrupole anisotropy amplitude Q is often called $T/S \equiv Q_t^2/Q_s^2$; thus the primordial scalar fluctuation power is decreased by the ratio $1/(1+T/S)$ for the same COBE normalization, compared to the situation with no gravity waves ($T = 0$). In power-law inflation, $T/S = 7(1 - n_p)$. This is an approximate equality in other popular inflation models such as chaotic inflation with $V(\phi) = m^2\phi^2$ or $\lambda\phi^4$. But note that the tensor wave amplitude is just the inflaton potential during inflation divided by the Planck mass, so the gravity wave contribution is negligible in theories like the supersymmetric model discussed above in which inflation occurs at an energy scale far below m_{Pl}. Because gravity waves just redshift after they come inside the horizon, the tensor contributions to CMB anisotropies corresponding to angular wavenumbers $\ell \gg 20$, which came inside the horizon long ago, are strongly suppressed compared to those of scalar fluctuations.

Joel Primack, in *Formation of Structure in the Universe*, ed. Dekel & Ostriker (Cambridge Univ Press, 1999)

Basic Predictions of Inflation

1. **Flat universe.** This is perhaps the most fundamental prediction of inflation. Through the Friedmann equation it implies that the total energy density is always equal to the critical energy density; it does not however predict the form (or forms) that the critical density takes on today or at any earlier or later epoch.

2. **Nearly scale-invariant spectrum of Gaussian density perturbations.** These density perturbations (scalar metric perturbations) arise from quantum-mechanical fuctuations in the field that drives inflation; they begin on very tiny scales (of the order of 10^{-23} cm, and are stretched to astrophysical size by the tremendous growth of the scale factor during inflation (factor of e^{60} or greater). Scale invariant refers to the fact that the fuctuations in the gravitational potential are independent of length scale; or equivalently that the horizon-crossing amplitudes of the density perturbations are independent of length scale. While the shape of the spectrum of density perturbations is common to all models, the overall amplitude is model dependent. Achieving density perturbations that are consistent with the observed anisotropy of the CBR and large enough to produce the structure seen in the Universe today requires a horizon crossing amplitude of around 2×10^{-5}.

3. **Nearly scale-invariant spectrum of gravitational waves**, from quantum-mechanical fluctuations in the metric itself . These can be detected as CMB “B-mode” polarization, or using special gravity wave detectors such as LIGO and LISA.

Observational Status of Inflation

I. The predictions of inflation are right:

(i) the universe is flat with a critical density
(ii) superhorizon fluctuations
(iii) density perturbation spectrum nearly scale invariant: $P(k) = Ak^n$, $n \approx 1$
(iv) Single slow-roll field models vindicated: Gaussian perturbations, not much running of spectral index

- If primordial fluctuations are Gaussian distributed, then they are completely characterized by their two-point function $\xi(r)$, or equivalently by the power spectrum. All odd-point functions are zero.
- If nonGaussian, there is additional info in the higher order correlation functions
- The lowest order statistic that can differentiate is the 3-point function, or bispectrum in Fourier space: $\langle \Phi(\boldsymbol{k}_1)\Phi(\boldsymbol{k}_2)\Phi(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_\Phi(k_1,k_2,k_3)$. Here $B_\Phi(k_1,k_2,k_3) = f_{\rm NL}F(k_1,k_2,k_3)$. The quantity $f_{\rm NL}$ is known as the nonlinearity parameter. Planck data: $f_{\rm NL} = 2.7 \pm 5.8$ -- small!

II. Data differentiate between models

-- each model makes specific predictions for density perturbations and gravity modes
-- WMAP and Planck rule out many models (see graph on next page, from Planck 2013 Results XXII, http://arxiv.org/abs/1303.5082).

Observational Status of Inflation

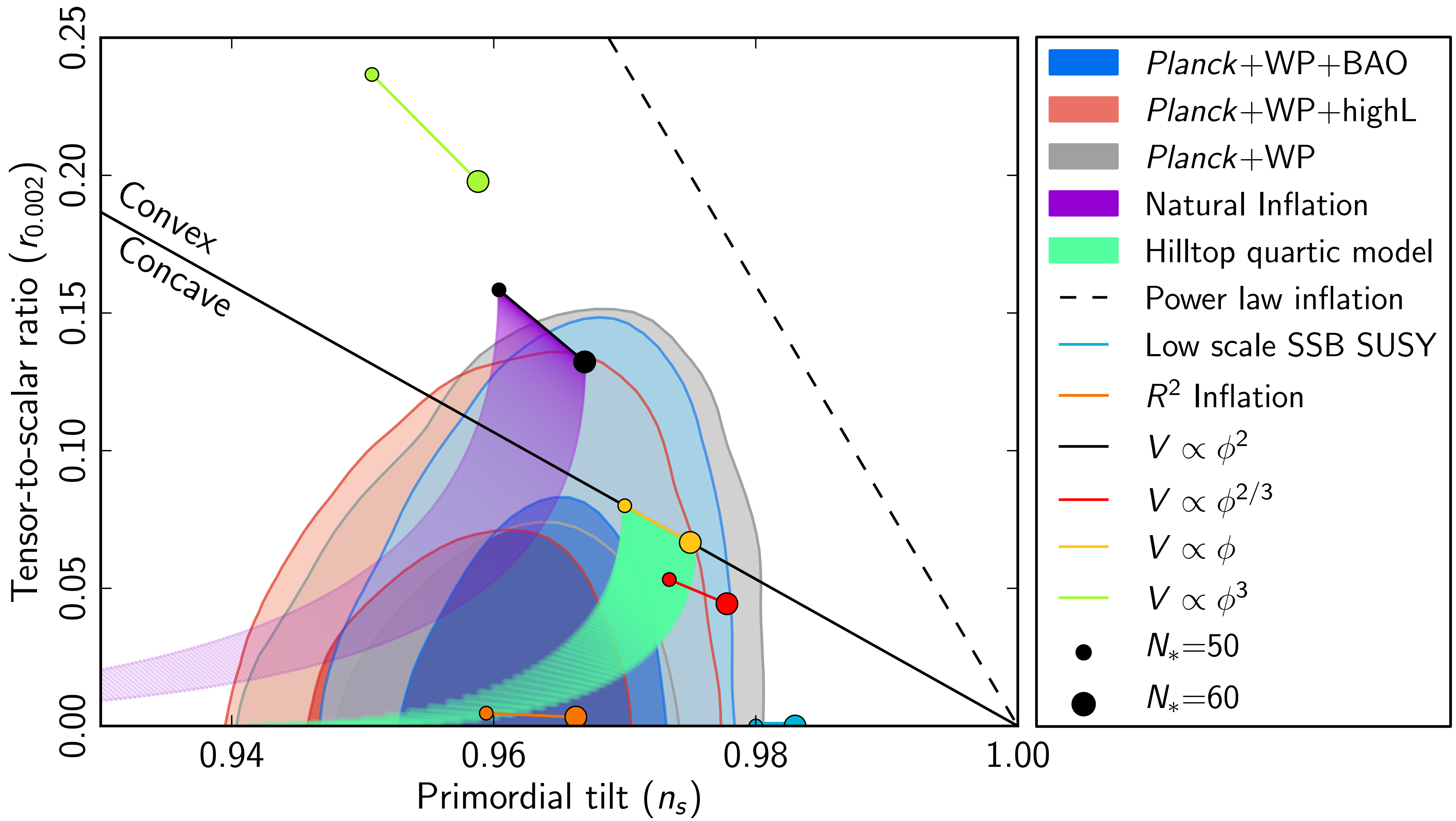


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

CONCLUSIONS: We find that standard slow-roll single field inflation is compatible with the Planck data. Planck in combination with WMAP 9-year large angular scale polarization (WP) yields Ω_K −0.006 ±0.018 at 95%CL by combining temperature and lensing information (Planck Collaboration XVI, 2013; Planck Collaboration XVII, 2013). The bispectral non-Gaussianity parameter f_{NL} measured by Planck is consistent with zero (Planck Collaboration XXIV, 2013). These results are compatible with zero spatial curvature and a small value of f_{NL}, as predicted in the simplest slow-roll inflationary models. Planck+WP data give n_s =0.9603±0.0073 (and n_s = 0.9629±0.0057 when combined with BAO). The 95% CL bound on the tensor-to-scalar ratio is $r_{0.002} < 0.12$; this implies an upper limit for the inflation energy scale of 1.9×10^{16} GeV.

Post-Inflation

Baryogenesis: generation of excess of baryon (and lepton) number compared to anti-baryon (and anti-lepton) number. In order to create the observed baryon number today

$$\frac{n_B}{n_\gamma} = (6.1\,^{+0.3}_{-0.2}) \times 10^{-10}$$

it is only necessary to create an excess of about 1 quark and lepton for every ~10^9 quarks+antiquarks and leptons +antileptons.

Other things that might happen Post-Inflation:

Breaking of Pecci-Quinn symmetry so that the observable universe is composed of many PQ domains.

Formation of cosmic topological defects if their amplitude is small enough not to violate cosmological bounds.