Physics 129 LECTURE 3 January 14, 2014

- Review: Different fundamental forces (Gravity, Weak and Strong Interactions, Electromagnetism) are important at different size scales. Other forces beyond the Standard Model may be discovered on small size scales.
- Introduction to Special Relativity

  Thought Experiments, Resolution of the Twin Paradox

  Rotations and Lorentz transformations

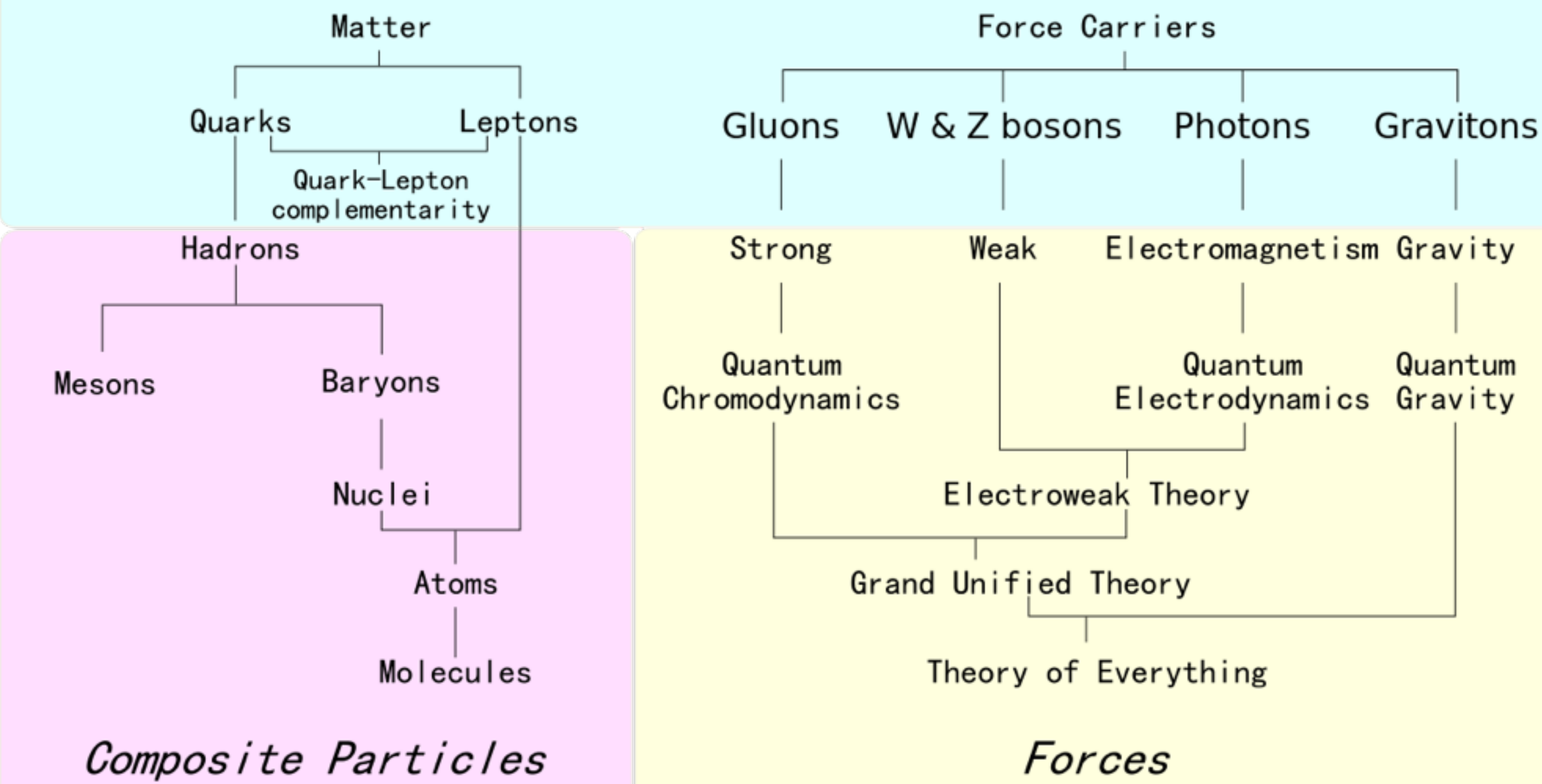
  The invariant interval $c^2t^2 - x^2$ and the Light Cone

  Special Relativity with 4-vectors

  Introduction to General Relativity

- In accelerating reference frames there are "pseudoforces" F_{pseudo} = ma, where a is the acceleration of the frame. General Relativity: gravity is a pseudoforce which is eliminated in inertial (freely falling) reference frames

Elementary Particles and Forces



http://en.wikipedia.org/wiki/Fundamental_interaction

Galilean–Newtonian Relativity

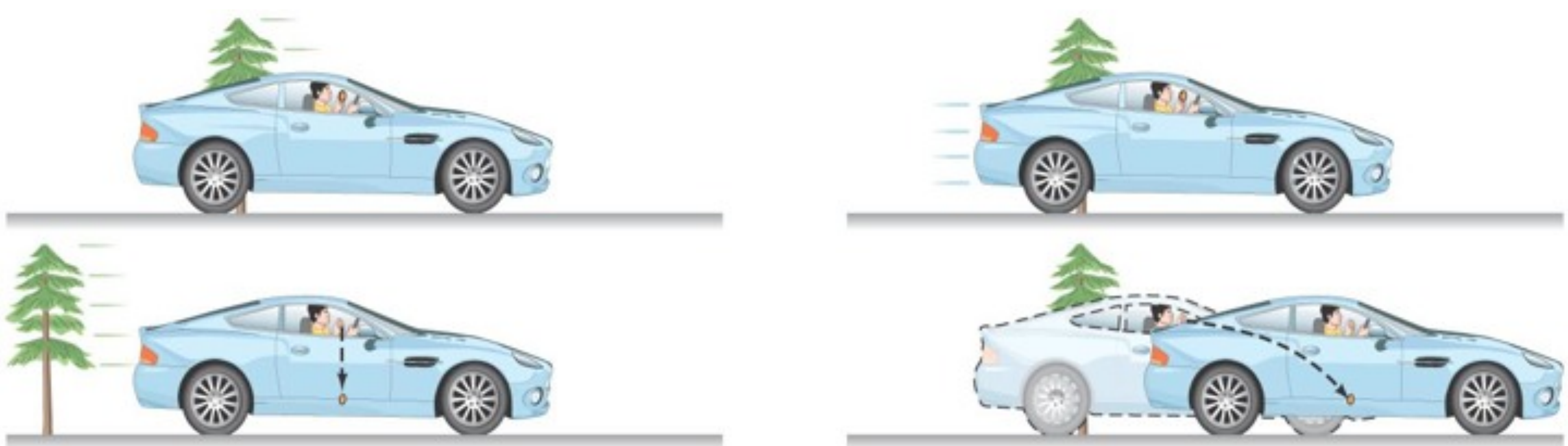
Definition of an inertial reference frame: One in which Newton's first law is valid.

A frame moving with a constant velocity with respect to an inertial reference frame is itself inertial.

Earth is rotating and has gravity, and therefore is not an inertial reference frame -- but we can treat it as one for many purposes.

Relativity principle:

The basic laws of physics are the same in all inertial reference frames.



(a)
Reference frame = car

(b)
Reference frame = Earth

Galilean–Newtonian Relativity

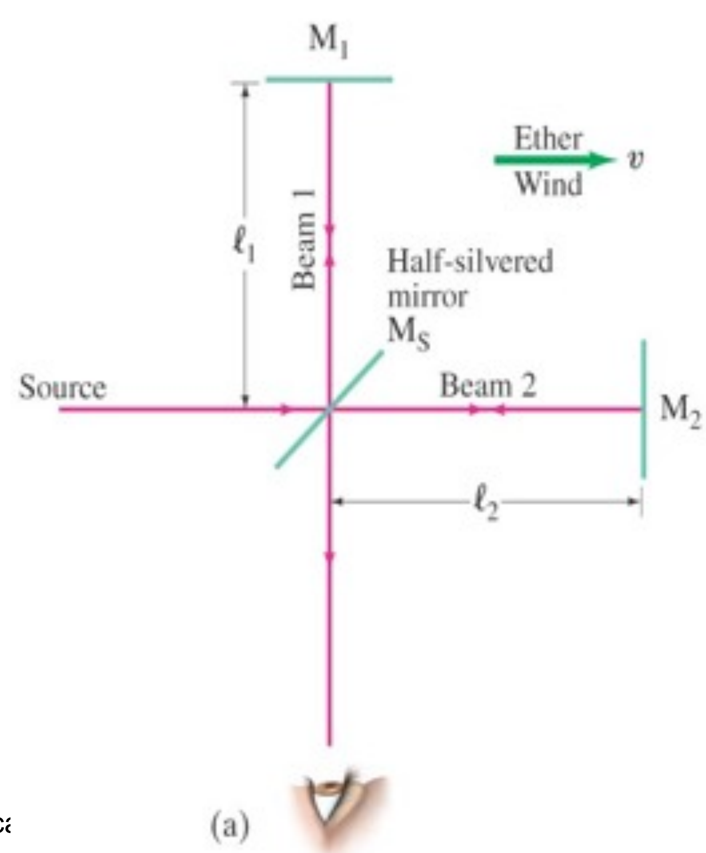
This principle works well for mechanical phenomena on or near Earth.

However, Maxwell's equations yield the velocity of light c; it is 3.0 x 10^8 m/s.

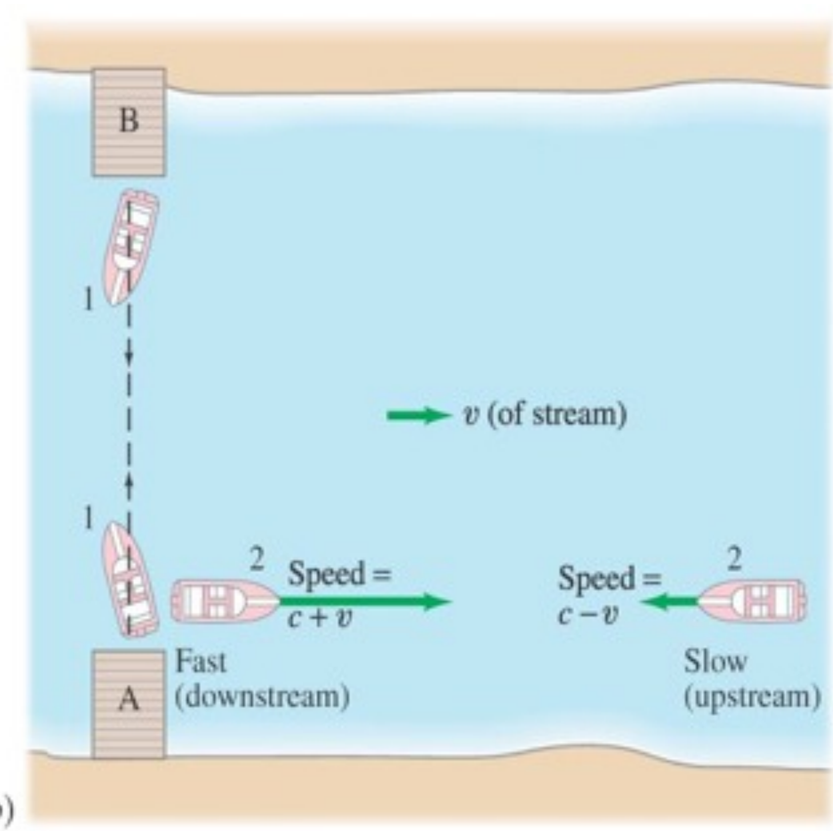
So, which is the ether reference frame in which light travels at that speed?

The 1887 Michelson-Morley experiment was designed to measure the speed of the Earth with respect to the ether.

The Earth's motion around the Sun should produce small changes in the speed of light, which would be detectable through interference when the split beam is recombined.

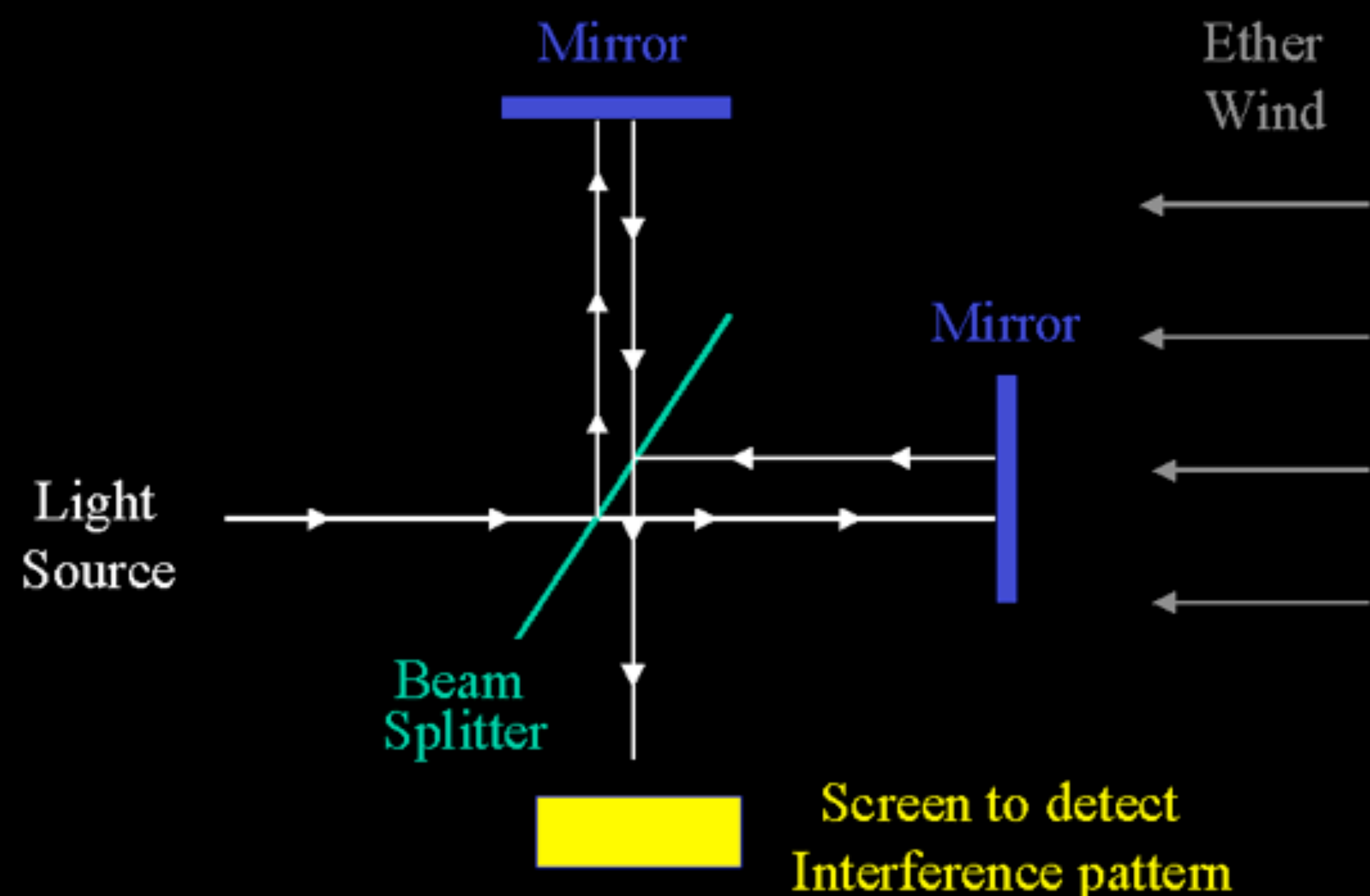


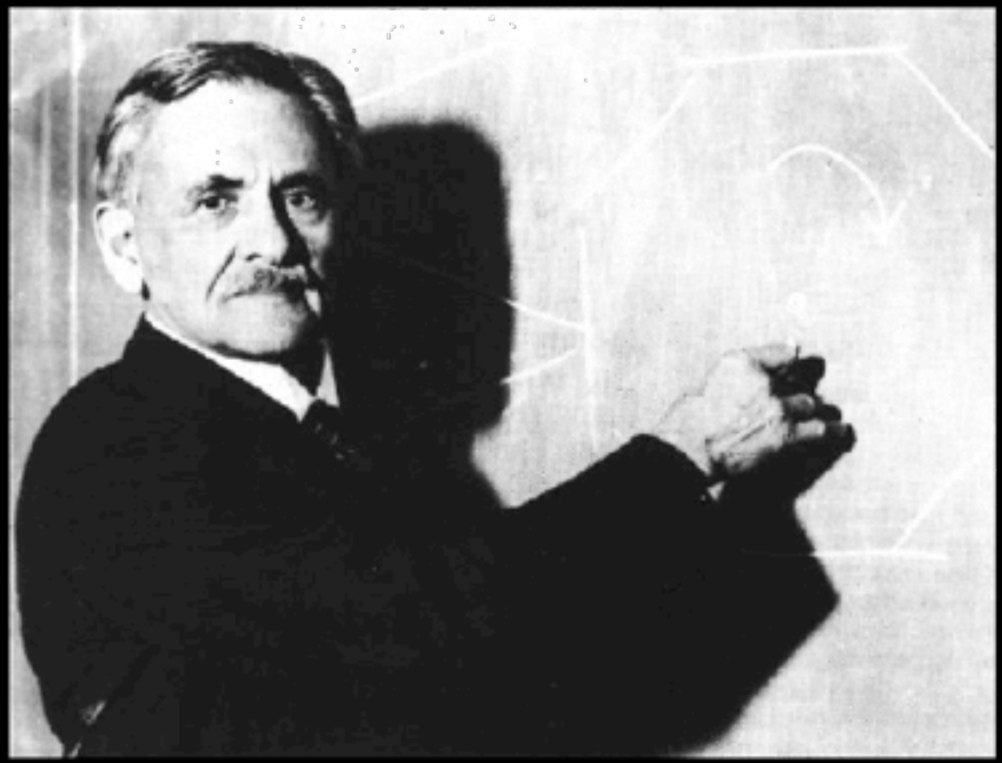
(a)



(b)

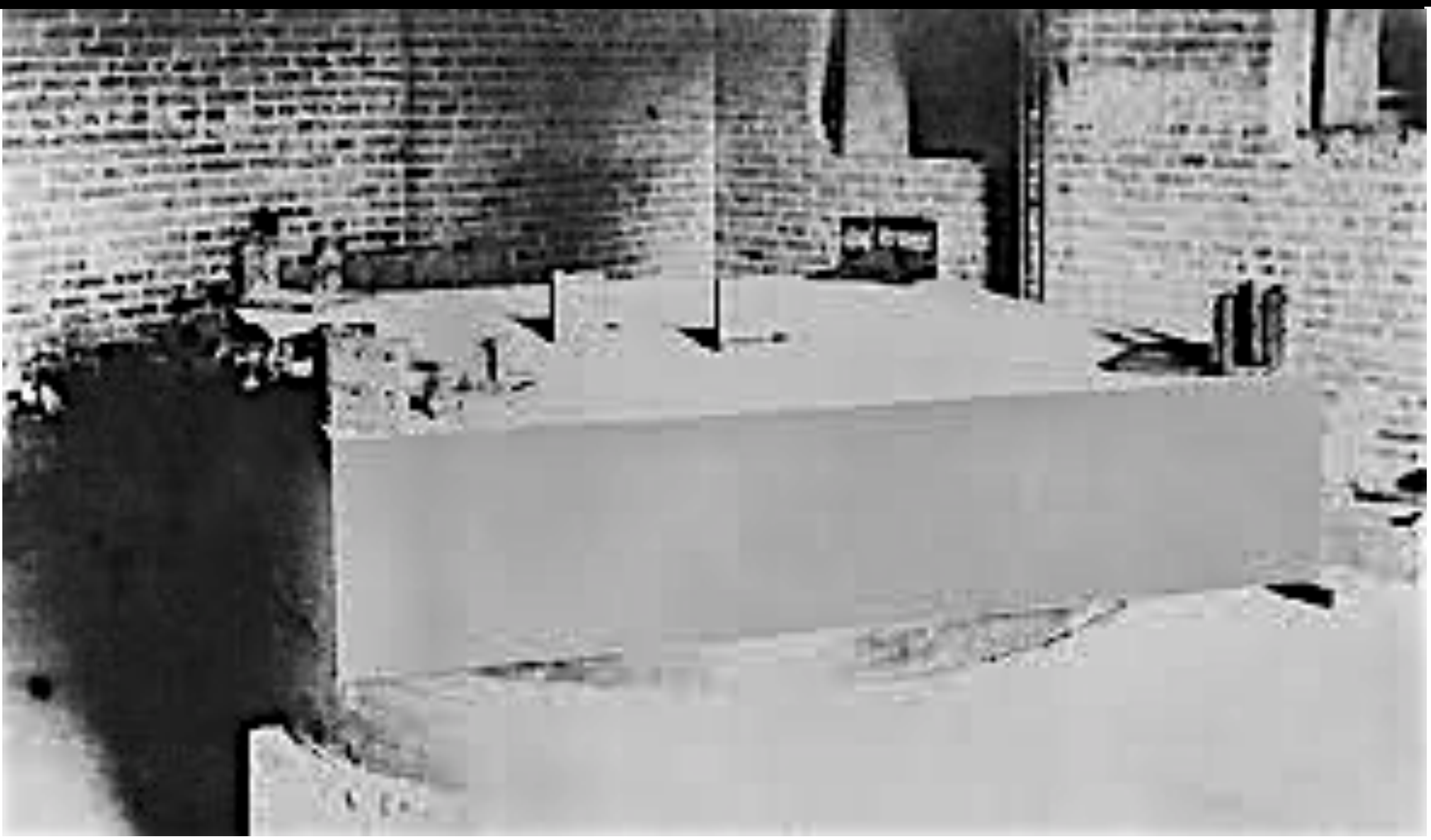
Michelson-Morley Experiment

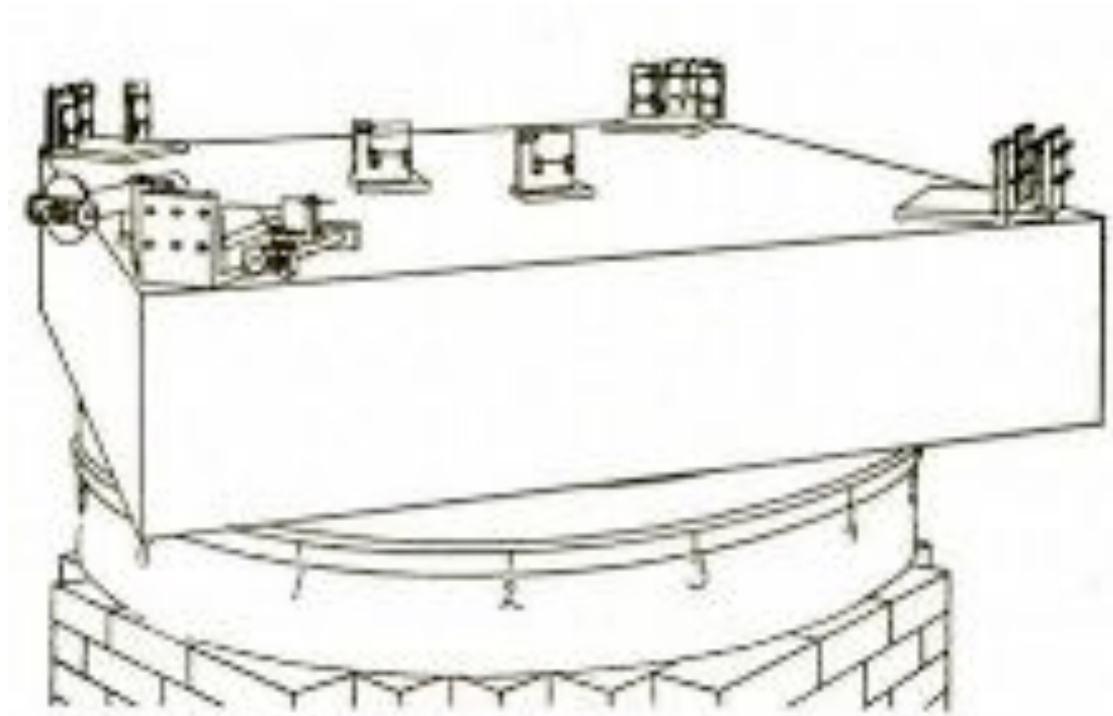




Albert Michelson, 1852-1931

(the first American to win a Nobel Prize)





Einstein's Special Theory of Relativity

Special Relativity is based on two postulates:

1. The Principle of Relativity: If a system of coordinates K is an inertial reference system [i.e., Newton's 1st law is valid], the same laws of physics hold good in relation to any other system of coordinates K' moving in uniform translation relatively to K.

2. Invariance of the speed of light: Light in vacuum propagates with the speed *c* in terms of any system of inertial coordinates, regardless of the state of motion of the light source.

The consequences are illustrated by the Einstein's Rocket thought experiments.

Einstein's Rocket - Thought Experiments and Games

use
Safari ⟶ http://physics.ucsc.edu/~snof/er.html

with latest
version of
java

Lorentz Transformations

The Lorentz transformation between inertial reference frame (x,t) and inertial frame (x',t') moving at speed v in the x-direction when the origins of the reference frames coincide at $t = t' = 0$:

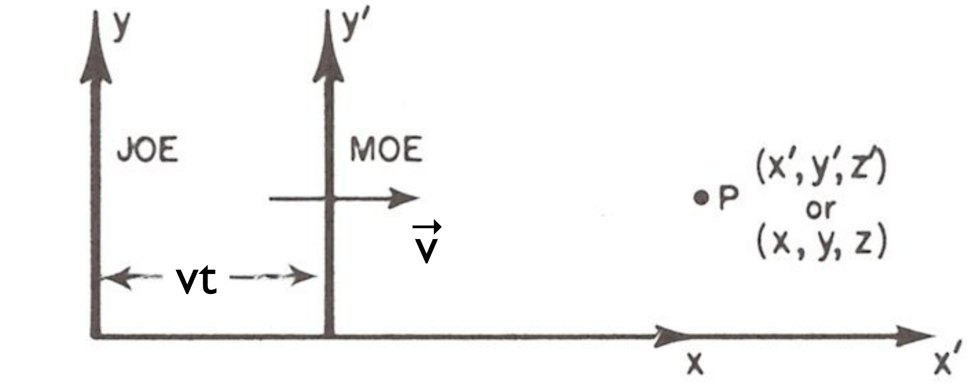


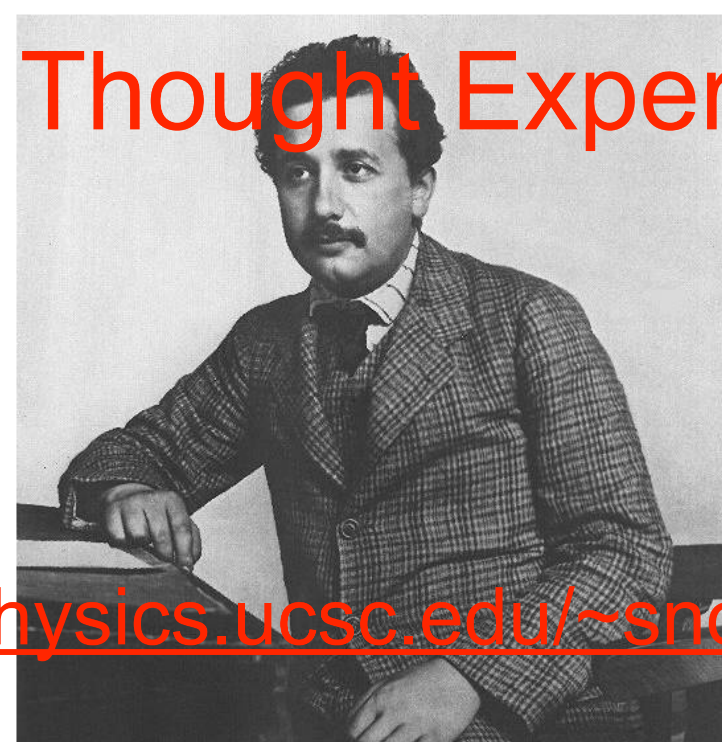
Fig. 15–1. Two coordinate systems in uniform relative motion along their x-axes.

$$x' = \gamma(x - vt), \qquad ct' = \gamma(ct - vx/c).$$

As usual $\gamma = (1-v^2/c^2)^{-1/2}$. Note that $\gamma = 1$ for $v = 0$, and $\gamma \longrightarrow \infty$ as $v \longrightarrow c$.

These transformations reflect the time dilation and length contraction illustrated in the Einstein's Rocket thought experiments.

Einstein's Rocket - Thought Experiments and Games



use Safari $\longrightarrow$ http://physics.ucsc.edu/~snof/er.html

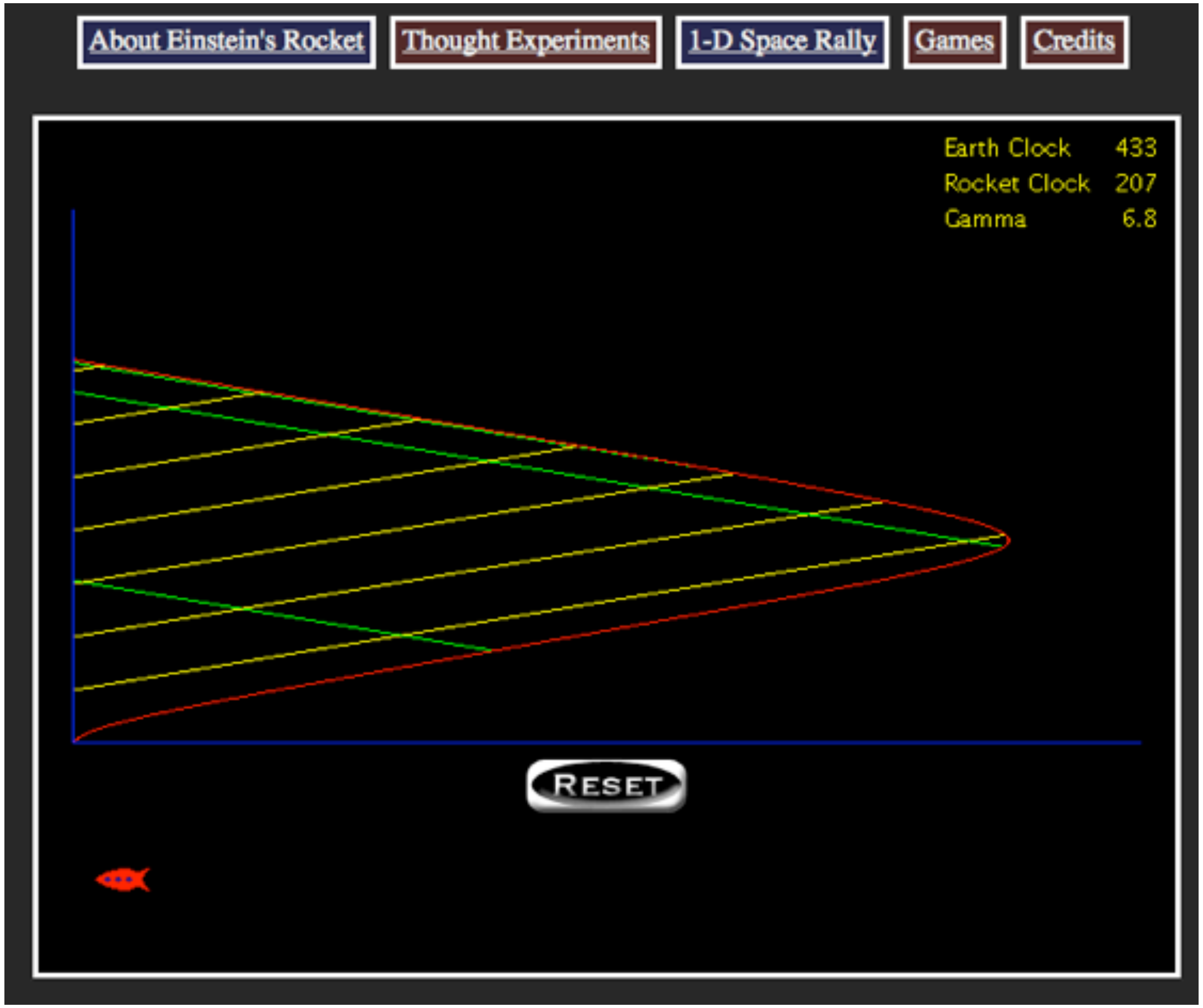
with latest version of java

The “twin paradox” of Special Relativity

If Albert stays home and his twin sister Berta travels at high speed to a nearby star and then returns home, Albert will be much older than Berta when he meets her at her return.

But how can this be true? Can’t Berta say that, from her point of view, it is Albert who traveled at high speed, so Albert should actually be younger?

To clarify why more time elapses on Albert’s clock than on Berta’s, we can use the Einstein’s Rocket “1-D Space Rally”.



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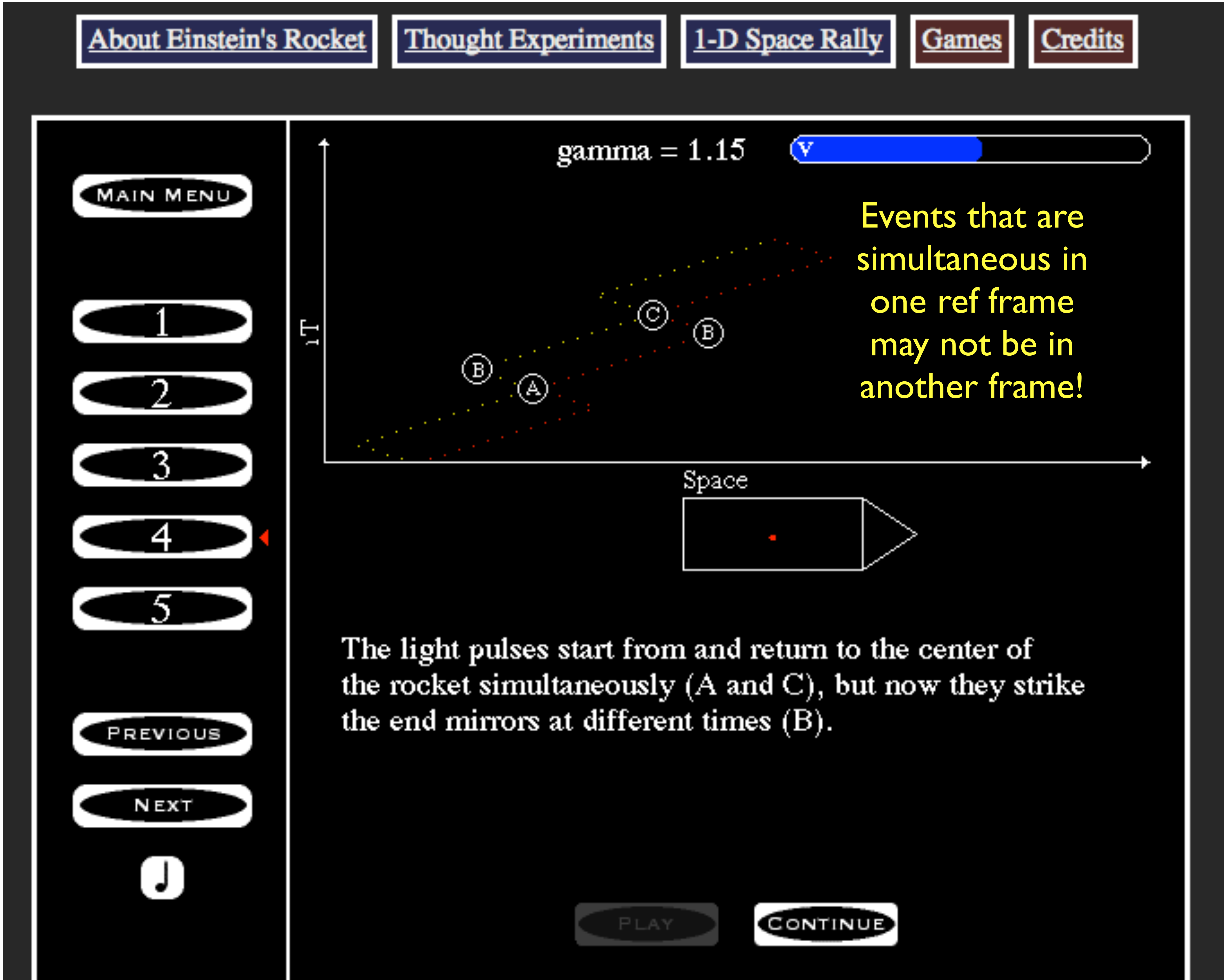


http://physics.ucsc.edu/~snof/er.html

The invariant interval c^2t^2 - x^2 and the Light Cone

Lorentz transformations preserve the space-time interval ($c^2t^2 - x^2 = c^2t'^2 - x'^2$), as they must since that guarantees that the speed of light is c in both frames.

This means that relatively moving observers will agree as to whether the space-time interval between two events is negative (space-like ①), positive (time-like ② or ③), or zero (light-like). If two events are time-like or light-like separated, the earlier one can affect the latter one, but if they space-like separated then they can have no effect on each other. The Light Cone separates events into these three different classes.

Before relativity, people thought "Now" had an invariant meaning. But we have already seen (in the Einstein's Rocket thought-experiment concerning the flash sent from the center to the ends of the rocket) that events that one observer says are simultaneous will not be so according to an observer in relative motion. That's also the explanation of the pole-and-barn paradox. The pole is entirely inside the barn in its rest frame, but in the rest frame of the pole the barn is much shorter and both doors are simultaneously open.

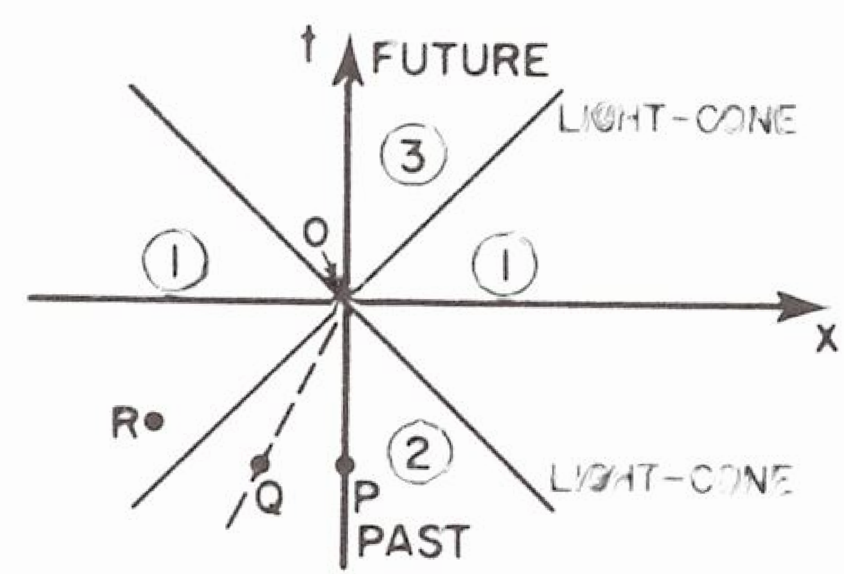
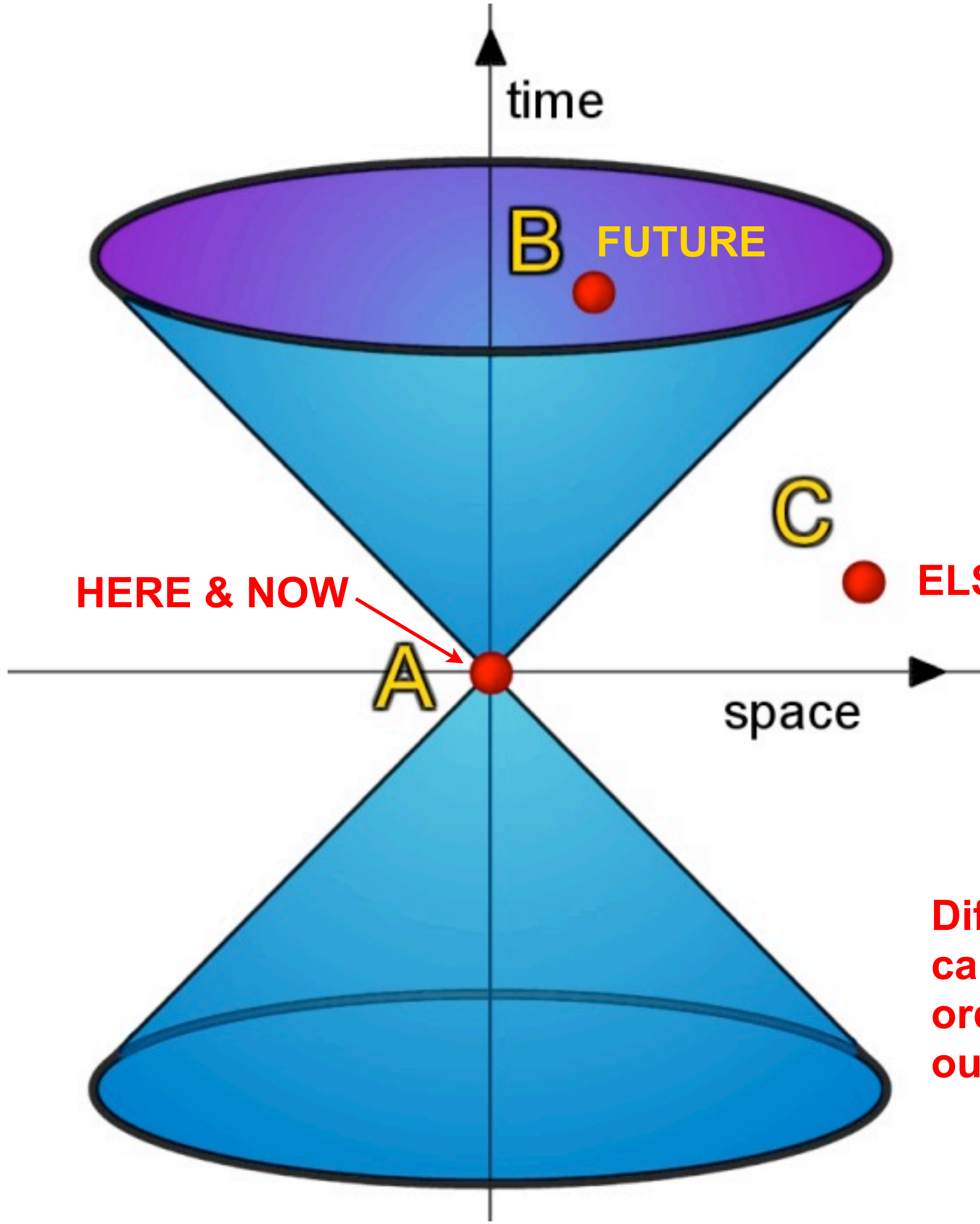


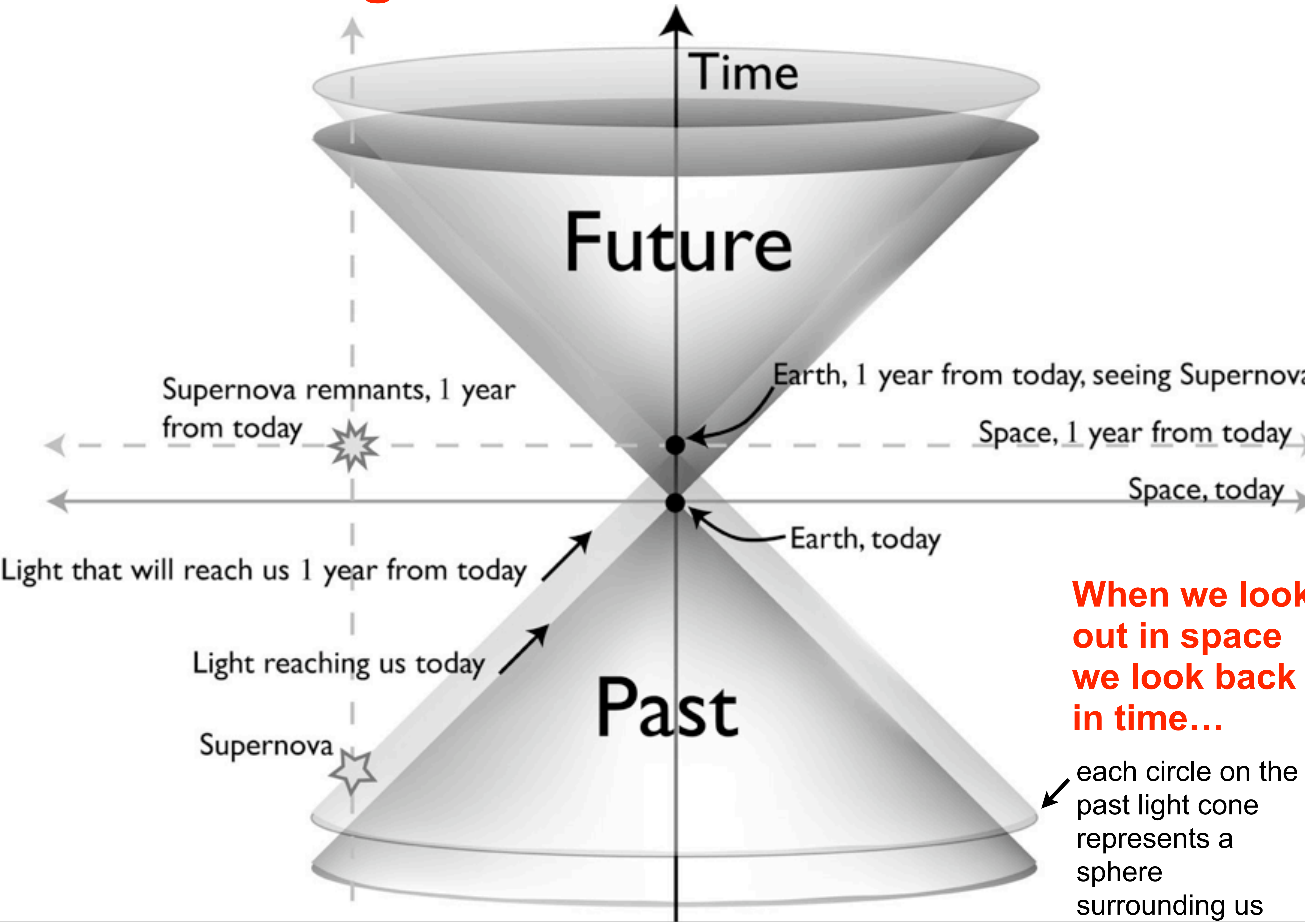
Fig. 17–3. The space-time region surrounding a point at the origin.

Lightcone of Past and Future



Different observers can see different time orders of events outside the lightcone!

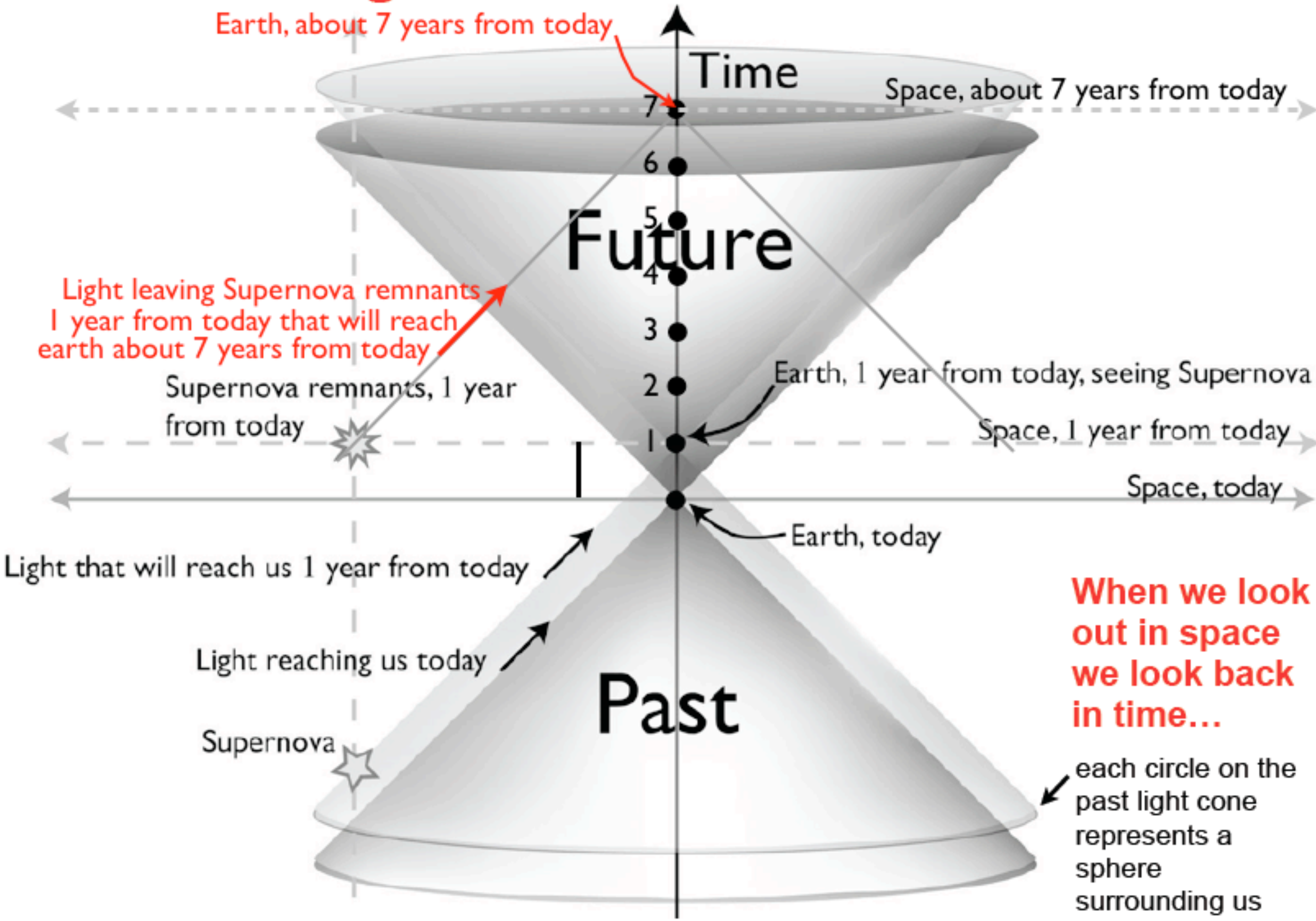
Lightcone of Past and Future



When we look out in space we look back in time…

each circle on the past light cone represents a sphere surrounding us

Lightcone of Past and Future



When we look out in space we look back in time...

Rotations in 2D

Recall that a rotation by angle θ in the x-y plane is given by

$$x = x'c_\theta - y's_\theta \qquad y = x's_\theta + y'c_\theta.$$

where $c_\theta = \cos\theta$, $s_\theta = \sin\theta$. The reverse rotation corresponds to $\theta \rightarrow -\theta$:

$$x' = x\,c_\theta + y\,s_\theta \qquad y' = -x\,s_\theta + y\,c_\theta.$$

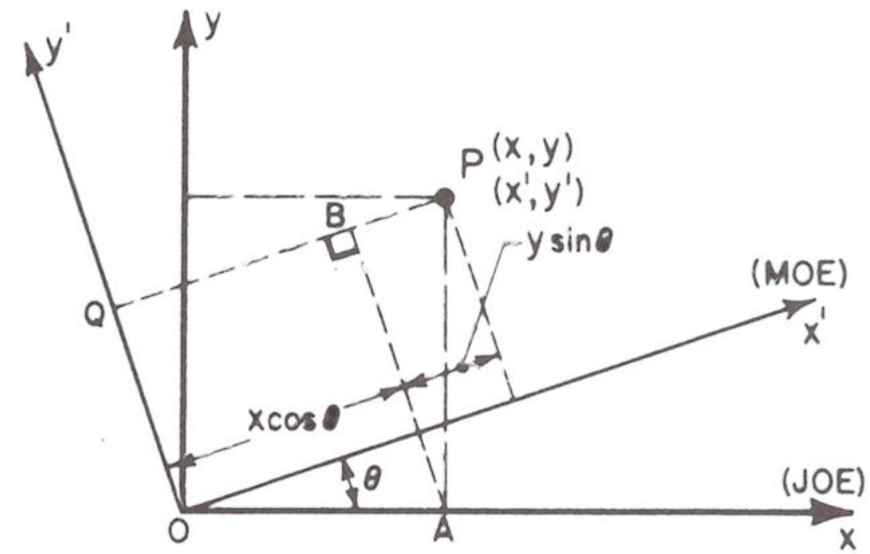


Fig. 11–2. Two coordinate systems having different angular orientations.

From the *Feynman Lectures on Physics*, vol. 1.

These transformations preserve the lengths of the vectors ($x^2 + y^2 = x'^2 + y'^2$) since $\sin^2\theta + \cos^2\theta = 1$.

Successive rotations by angles θ_1 and θ_2 correspond to rotation through angle $\theta_3 = \theta_1 + \theta_2$.

Lorentz transformations

The Lorentz transformation between inertial reference frame (x,t) and the inertial frame (x',t') moving at speed v in the x-direction when the origins of the reference frames coincide at $t = t' = 0$ is

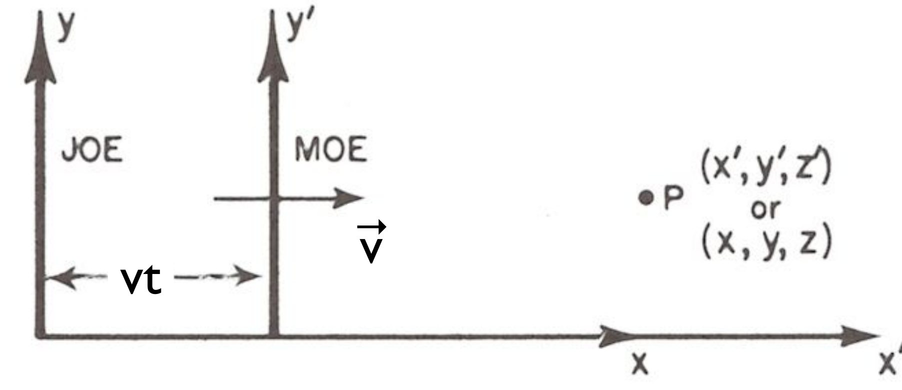


Fig. 15–1. Two coordinate systems in uniform relative motion along their x-axes.

$$x = x' C_\theta + ct' S_\theta \qquad ct = x' S_\theta + ct' C_\theta.$$

where $\gamma = (1-v^2/c^2)^{-1/2} = \cosh\theta = C_\theta$, $\gamma v/c = \sinh\theta = S_\theta$, and $\theta = \tanh^{-1} v/c$ since $\tanh\theta = \sinh\theta / \cosh\theta = v/c$. The reverse transformation is just $\theta \rightarrow -\theta$:

$$x' = x C_\theta - ct S_\theta \qquad ct' = -x S_\theta + ct C_\theta.$$

Reminder: $\cosh\theta = (e^\theta + e^{-\theta})/2$ and $\sinh\theta = (e^\theta - e^{-\theta})/2$ are the hyperbolic functions, and $\tanh\theta = \sinh\theta / \cosh\theta$. With the correspondence above between θ and v/c, it follows that $v/c = \tanh\theta$, so $\theta = \tanh^{-1} v/c$.

Successive Lorentz transformations: The reason the θ approach is useful is that the product of two Lorentz transformations that correspond to θ_1 and θ_2 is $\theta_3 = \theta_1 + \theta_2$, which greatly simplifies things. It turns out that Lorentz transformations form a group that is a generalization of the group of rotations.

Lorentz transformations preserve the space-time interval ($c^2t^2 - x^2 = c^2t'^2 - x'^2$) because $\cosh^2\theta - \sinh^2\theta = 1$.

Special Relativity with 4-Vectors

An quantity that transforms the same way as $(ct, \boldsymbol{x})$ is called a 4-vector. It turns out that the combination $V^\mu = (\gamma, \gamma \boldsymbol{v}/c) = \gamma(1, \boldsymbol{v}/c)$ where $\boldsymbol{v}$ is the velocity vector, is a 4-vector, called the velocity 4-vector. Here $V^0 = \gamma$, and $\boldsymbol{V} = \gamma \boldsymbol{v}/c$ (i.e., $V^i = \gamma v^i/c$, for $i = 1, 2, 3$). Its invariant length-squared is $V \cdot V = (V^0)^2 - \boldsymbol{V}^2 = \gamma^2 (1 - v^2/c^2) = 1$.

Multiply the rest energy of a particle mc^2 by its velocity 4-vector and you get its momentum 4-vector:

$$P = (E, \boldsymbol{p}c) = mc^2\, \gamma(1, \boldsymbol{v}/c)\ .$$

Its invariant length-squared is $P \cdot P = E^2 - \boldsymbol{p}^2 c^2 = m^2 c^4\, \gamma^2\, (1 - \boldsymbol{v}^2/c^2) = m^2 c^4$.

For a particle of mass m, this says that $E^2 = \boldsymbol{p}^2 c^2 + m^2 c^4$, $E = mc^2\, \gamma$, $\boldsymbol{v}/c = \boldsymbol{p}/E$. For the special case of a massless particle like the photon, this says that $E^2 = \boldsymbol{p}^2 c^2$ or $E = |\boldsymbol{p}|c$. The momentum carried by a photon of energy E is $p = E/c$.

Using "natural units" where $c = 1$, we understand that powers of c are included as needed to get the right units. Then the energy-momentum-mass relation becomes $E^2 = p^2 + m^2$. We often measure mass in energy units, for example we say that the mass m_e of the electron is 0.511 MeV, even though what we really mean is that $m_e c^2 = 0.511$ MeV. And of course we measure distances in time units: light-years.

Special Relativity with 4-Vectors

We have discussed several 4-vectors

position $X^\mu = (ct, \boldsymbol{x})$, or $dX^\mu = (c\,dt, d\boldsymbol{x})$

velocity $V^\mu = (\gamma, \gamma \boldsymbol{v}/c) = \gamma(1, \boldsymbol{v}/c)$

momentum $P^\mu = (E, \boldsymbol{p}c) = mc^2\,\gamma(1, \boldsymbol{v}/c)$

where $\boldsymbol{v}$ is the velocity 3-vector.

The corresponding invariants are

$X \cdot X = c^2t^2 - \boldsymbol{x}^2$, $dX \cdot dX = c^2(dt)^2 - d\boldsymbol{x}^2 = ds^2$

$V \cdot V = (V^0)^2 - \boldsymbol{v}^2 = \gamma^2\,(1 - v^2/c^2) = 1$

$P \cdot P = E^2 - \boldsymbol{p}^2c^2 = m^2c^4$.

We introduce the metric tensor $g_{\mu\nu}$ (where $\mu\nu = 0, 1, 2, 3$): $g_{00}=1$, $g_{11}= g_{22}= g_{33}= -1$, and the off-diagonal terms all vanish. Also $g^{00}=1$, $g^{11}= g^{22}= g^{33}= -1$. Thus as a matrix

$$g^{\mu\nu} = g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

We use $g_{\mu\nu}$ to lower indices: $X_\nu = X^\mu g_{\mu\nu}$ where repeated upper-lower indices are summed over: $X_0 = X^0 g_{00} = X^0 = ct$, $X_i = X^j g_{ij} = -X^i$, so $X_\mu = (ct, -\boldsymbol{x})$. We similarly use $g^{\mu\nu}$ to raise indices.

We can write the Lorentz transformation as $X'^\mu = L^\mu{}_\nu X^\nu$. The same transformation applies to any 4-vector.

Lorentz tensors such as the electromagnetic field tensor $F^{\mu\nu}$ or the stress-energy tensor $T^{\mu\nu}$ are generalizations of the 4-vector concept: each index transforms the same way under the the Lorentz transformation.

Lorentz transformations and the Doppler effect

The Lorentz transformation between inertial reference frame (x,t) and inertial frame (x',t') moving at speed v in the x-direction is (setting $c = 1$ and writing $v/c = \beta$)

$$t' = \gamma(t - \beta x), \quad x' = \gamma(x - \beta t), \quad y' = y, \quad z' = z$$

Since all 4-vectors transform the same way, the Lorentz transformation between the same frames for the energy-momentum 4-vector $P^\mu = (E,\boldsymbol{p})$ is

$$E' = \gamma(E - \beta p_x), \quad p_x' = \gamma(p_x - \beta E), \quad p_y' = p_y, \quad p_z' = p_z$$

Since $|\boldsymbol{p}| = p = E$ for photons, this says that for a photon moving in the x-direction $E' = \gamma E\,(1 - \beta) = E\,[(1 - \beta)/(1 + \beta)]^{1/2}$. Since $E = hf$ where h = Planck's constant and f is the frequency, the frequencies are related by $f' = f\,[(1 - \beta)/(1 + \beta)]^{1/2}$, which is 1st order in β. The frequency decreases, so the wavelength $\lambda = c/f$ increases, or "red-shifts". (We will derive a different formula in the expanding universe.)

For a photon moving in the transverse (y or z) direction, the only effect is time dilation: $v = \gamma v' = (1 + \frac{1}{2}\beta^2 + ...)v'$, a redshift that's second order in β.

Special Relativity Kinematics

Consider a scattering experiment a + b $\longrightarrow$ c + d. By 4-momentum conservation, the total 4-momentum $P = P_a + P_b = P_c + P_d = P'$. The momentum transfer 4-vector is defined as $q = P_c - P_a$. It is usually easiest to work in the center of momentum (CM) frame, in which $\boldsymbol{p}_a = -\boldsymbol{p}_b$ and $\boldsymbol{p}_c = -\boldsymbol{p}_d$ so $P = (E_a + E_b, 0) = (E_{CM}, 0) = P'$. We define s = $(E_{CM})^2$. Note that since s = $P \cdot P$, it is a relativistic invariant, with the same value in every inertial reference frame.

In the extreme relativistic limit where $E_i >> p_i$, we can neglect the masses of the particles. In the CM frame, $\boldsymbol{p}_a = -\boldsymbol{p}_b$ so $E_a = E_b = \frac{1}{2}E_{CM}$, and similarly $E_c = E_d = \frac{1}{2}E_{CM}$. Also in the CM frame, we can write $\boldsymbol{p}_c = \boldsymbol{p}_a \cos\theta$, where θ is the scattering angle. Then $q = P_c - P_a = (0, \boldsymbol{p}_c - \boldsymbol{p}_a)$, so the square of the 4-momentum transfer is $t = q^2 = (0, \boldsymbol{p}_c - \boldsymbol{p}_a)^2 = -(\boldsymbol{p}_c - \boldsymbol{p}_a)^2$.

Fixed Target Experiments: In the Lab frame $\boldsymbol{p}_b = 0$ so $E_b = m_b$ and $E_{CM}^2 =$ s $= (E_a+m_b)^2 - \boldsymbol{p}_a^2 = E_a^2 - \boldsymbol{p}_a^2 + 2E_a m_b + m_b^2 = m_b^2 + 2E_a m_b + m_b^2 \approx 2E_a m_b$ in the high energy regime where $E_a >> m_a, m_b$. The CM energy $E_{CM} = \sqrt{s} = (2E_a m_b)^{1/2}$.

Colliding Beam Experiments: With equal mass particles (e^+e^-, $p\bar{p}$, pp) the Lab frame is the CM frame, so the CM energy is the energy of each initial particle. Before shutting down for an upgrade, the LHC operated at $E_a = E_b = 4$ TeV, so $E_{CM} = 8$ TeV. To have an equal E_{CM} in a fixed target experiment with protons, $2E_a m_p = 64\ \text{TeV}^2$, or $E_a \approx 32{,}000$ TeV, only achievable with cosmic rays!

Schrodinger Equation: $i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$ where $\hat{H} = p^2/2m$ or $E = p^2/2m$

for free particle — Newtonian

Dirac Equation: $i\hbar\frac{\partial}{\partial t}\Psi = (\beta mc^2 + \boldsymbol{\alpha}\cdot\boldsymbol{p}c)\Psi$, with $E^2 = \boldsymbol{p}^2c^2 + m^2c^4$

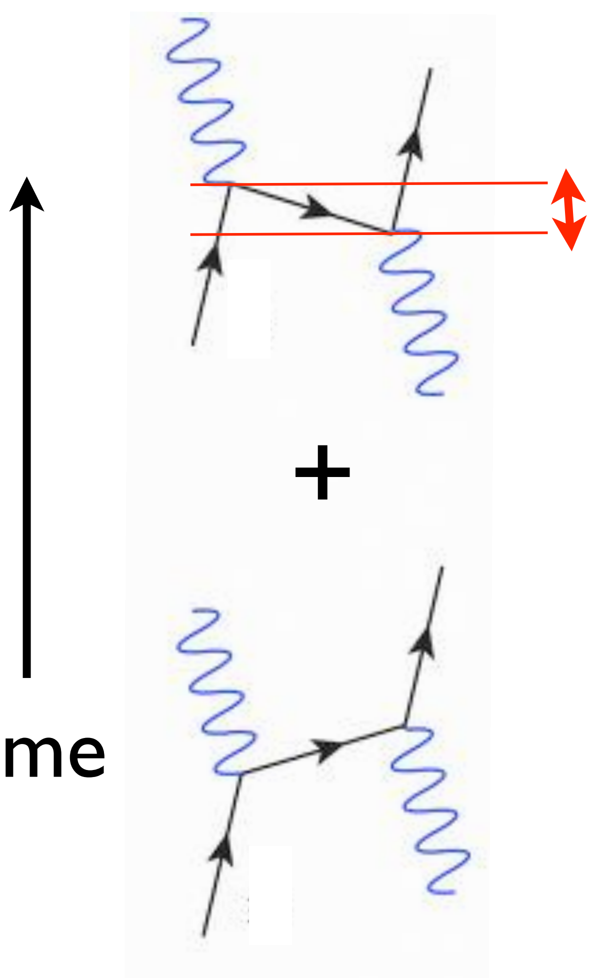
matrices (β, $\boldsymbol{\alpha}$)

Relativistic

Special Relativity + Quantum Mechanics

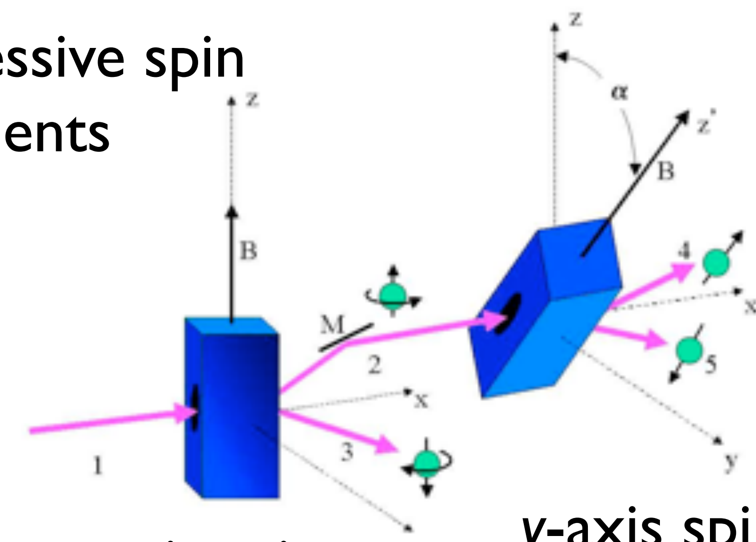
⇒ Antiparticles (& CPT) & Spin-Statistics

Feynman diagrams for electron-photon scattering (Compton scattering)



Electron goes backward in time = anti-electron (positron)

Two successive spin measurements



z-axis spin: "white-black"

2 possibilities (spin-1/2)

y-axis spin: "sheep-goats" (perpendicular: α=90°)

Two successive w-b or s-g measurements give the same answer, but if all the w's from the w-b measurement are subjected to a s-g measurement, half the s's will be w and half will be b, and the same for the g's. That is, measuring s-g interferes with measuring w-b!

Special Relativity is based on two postulates

- All the laws of physics are the same in all inertial reference frames.

- The speed of light is the same for all inertial observers, regardless of their velocity or that of the source of the light.

Einstein realized that Newton's theory of gravity, with instantaneous action at a distance, could not be compatible with special relativity -- which undermined the concept of simultaneous events at a distance. It took 10 years for Einstein to get the right idea for the right theory, but then in only two months in late 1915 he worked out the theory and its main initial predictions: the precession of the orbit of Mercury, bending of light by the sun, and the slowing of clocks by gravity.

General Relativity is also based on two postulates

- Equivalence Principle: All the effects of gravity on small scales are the same as those of acceleration. (Thus gravity is eliminated in local inertial = free fall frames.)

- Einstein's Field Equations: $G_{\mu\nu} = -(8\pi G/c^4)\, T_{\mu\nu}$ where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu}$ describes the curvature of space-time at each point and $T_{\mu\nu}$ describes the mass-energy, momentum, and stress density at the same point.

General Relativity and Cosmology

CURVED SPACE TELLS MATTER HOW TO MOVE

$$\frac{du^\mu}{ds} + \Gamma^\mu{}_{\alpha\beta}\, u^\alpha u^\beta = 0$$

MATTER TELLS SPACE HOW TO CURVE

Einstein Field Equations

$$G^{\mu\nu} \equiv R^{\mu\nu} - \tfrac{1}{2} R g^{\mu\nu} = - 8\pi G T^{\mu\nu} - \Lambda g^{\mu\nu}$$

Einstein's Cosmological Principle: on large scales, space is uniform and isotropic.

COBE-Copernicus Theorem: If all observers observe a nearly-isotropic Cosmic Background Radiation (CBR), then the universe is locally nearly homogeneous and isotropic – i.e., is approximately described by the Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)\,[dr^2 (1 - kr^2)^{-1} + r^2\, d\Omega^2]$$

with curvature constant $k = -1$, 0, or +1. Substituting this metric into the Einstein equations above, we get the Friedmann equations.

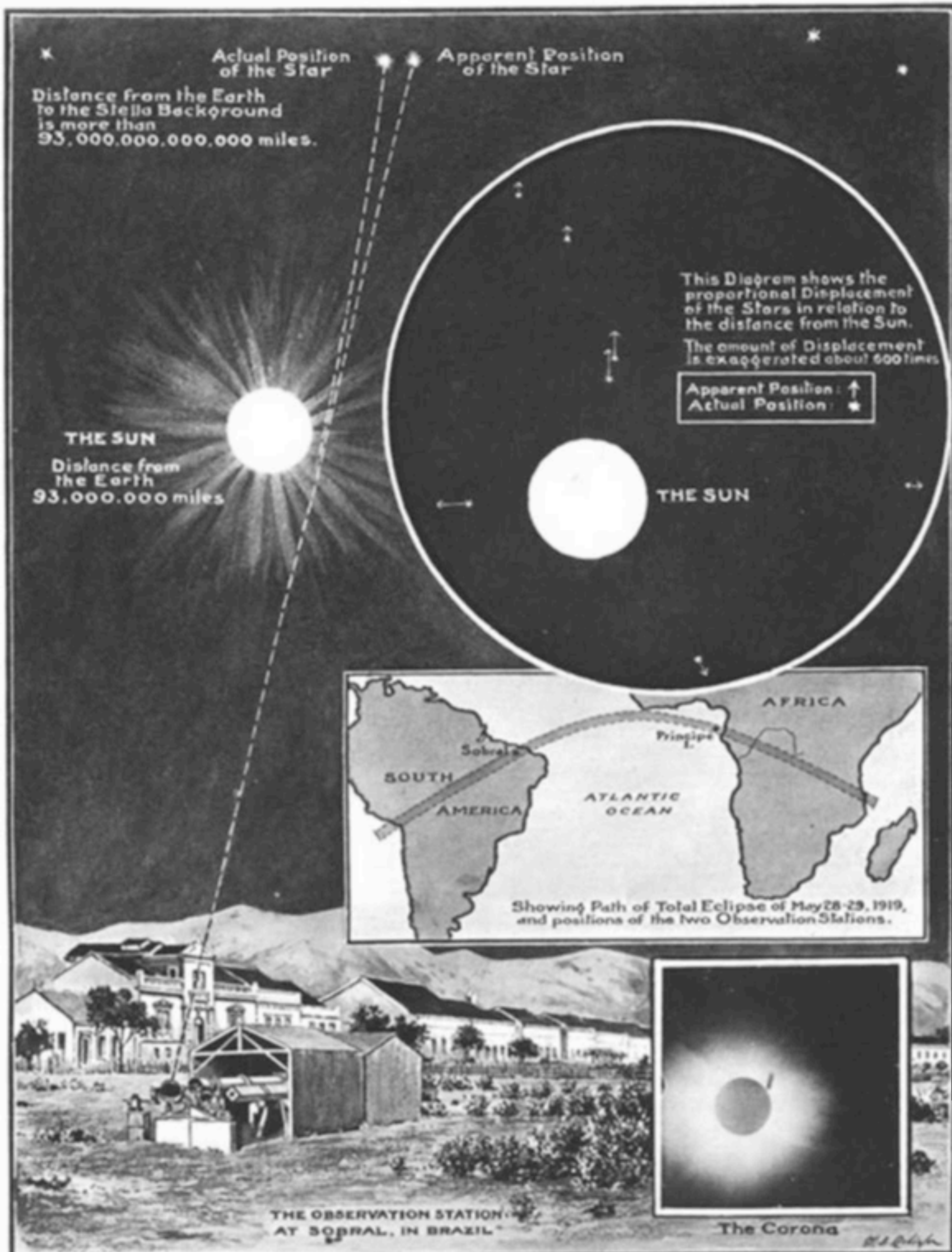
Equivalence Principle:

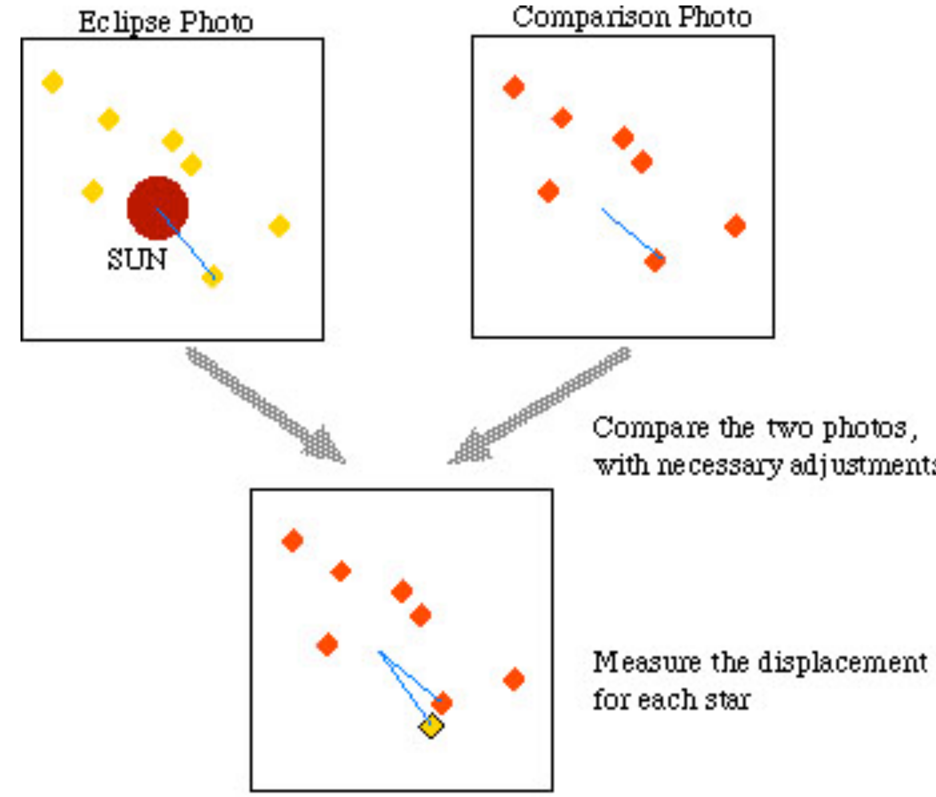
All the effects of gravity on small scales are the same as those of acceleration.

This predicts the path of a beam of light in a gravitational field -- it will be a parabola.

(a)

(b)





Arthur Stanley Eddington

LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.