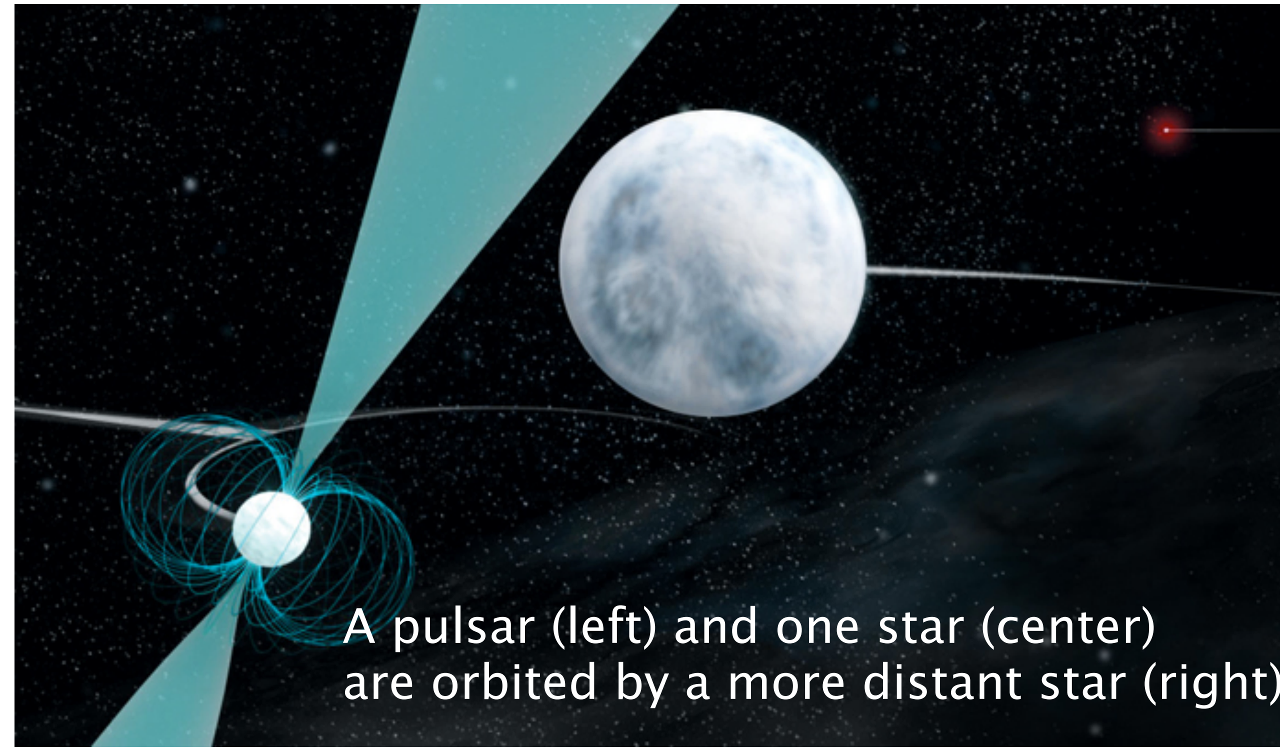
Physics 129 LECTURE 7 January 28, 2014

- Newly Discovered Pulsar System Allows New Test of the Einstein Equivalence Principle
- Running Coupling Constants
- Grand Unification and Proton Decay (Perkins Ch 4)
- CP Violation
  - In Neutral K Mesons
  - In the CKM Matrix
  - In the Strong Interactions, Avoided with Axions
  - In Physics Beyond the Standard Model
- Neutrino Masses and Oscillations
- Supersymmetry

Science 10 January 2014: **343,** pp. 126-127

Rare Celestial Trio to Put Einstein's Theory to the Test Adrian Cho


A pulsar (left) and one star (center) are orbited by a more distant star (right).

In a cosmic coup, astronomers have found a celestial beacon known as a pulsar in orbit with two other stars. The first-of-its-kind trio could soon be used to put Einstein's theory of gravity to an unprecedented test. "It's a wonderful laboratory that nature has given us," says Paulo Freire, a radio astronomer at the Max Planck Institute for Radio Astronomy in Bonn, Germany, who was not involved in the work. "It's almost made to order."

A pulsar consists of a neutron star, the leftover core of a massive star that has exploded in a supernova. The core's intense gravity squeezes atomic nuclei into a single sphere of neutrons. The spinning neutron star also emits a beam of radio waves that sweeps the sky as steadily as the ticking of an atomic clock. Tiny variations in the flashing can reveal whether the pulsar is in orbit with another object: As the pulsar cycles toward and away from Earth, the pulse frequency oscillates. Roughly 80% of the more than 300 fast-spinning "millisecond" pulsars have a partner.

But a single partner couldn't explain the peculiar warbles in the frequency of pulsar PSR J0337+1715, which Scott Ransom, an astronomer at the National Radio Astronomy Observatory in Charlottesville, Virginia, and colleagues discovered in 2007 with the Robert C. Byrd Green Bank Telescope in West Virginia. Training other radio telescopes on the object, Ransom and colleagues monitored it almost constantly for a year and a half. Eventually, Anne Archibald, a graduate student at McGill University in Montreal, Canada, figured out exactly what's going on.

The pulsar, which has 1.4 times the sun's mass and spins 366 times a second, is in a tight orbit lasting 1.6 days with a white dwarf star only 20% as massive as the sun. A second white dwarf that weighs 41% as much as the sun orbits the inner pair every 327 days, Ransom and colleagues reported this week in Nature. "We think that there are not more than 100 of these [trios] in our galaxy," Ransom says. "They really are one-in-a-billion objects."

Rare Celestial Trio to Put Einstein's Theory to the Test (continued)

The distinctive new system opens the way for testing a concept behind Einstein's theory of gravity, or general relativity. Called the equivalence principle, it relates two different conceptions of mass: inertial mass, which quantifies how an object resists accelerating when it's pushed or pulled, and gravitational mass, which determines how much a gravitational field pulls on it.

The simplest version of the principle says inertial mass and gravitational mass are equal. It explains why ordinary objects like baseballs and bricks fall to Earth at the same rate regardless of their mass. The strong equivalence principle takes things an important step further. According to Einstein's famous equation, $E = mc^2$, energy equals mass. So energy in an object's own gravitational field can contribute to its mass. The strong equivalence principle states that even if one includes mass generated through such "self-gravitation," gravitational and inertial mass are still equal.

The strong equivalence principle holds in Einstein's general theory of relativity but not in most alternative theories, says Thibault Damour, a theoretical physicist at the Institute for Advanced Scientific Studies in Bures-sur-Yvette, France. So poking a pin in the principle would prove that general relativity is not the final word on gravity.

Researchers have tried to test the strong equivalence principle by scrutinizing how the moon and Earth orbit in the gravitational field of the sun and how pulsar-white dwarf pairs cavort in the gravitational field of the galaxy. But Earth's self-gravitation is tiny, and the galaxy's gravity is weak. So such tests have yielded a precision of only parts per thousand, Damour says.

The new pulsar system opens the way to a much more stringent test by combining the powerful self-gravitation of the pulsar with the strong gravitational field of the outer white dwarf. By tracking whether either the inner white dwarf or the pulsar falls faster toward the outer white dwarf, Ransom and colleagues should be able to test strong equivalence about 100 times as precisely as before, Damour says. If strong equivalence falters, Freire says, the result would mark "a complete revolution" in physics.

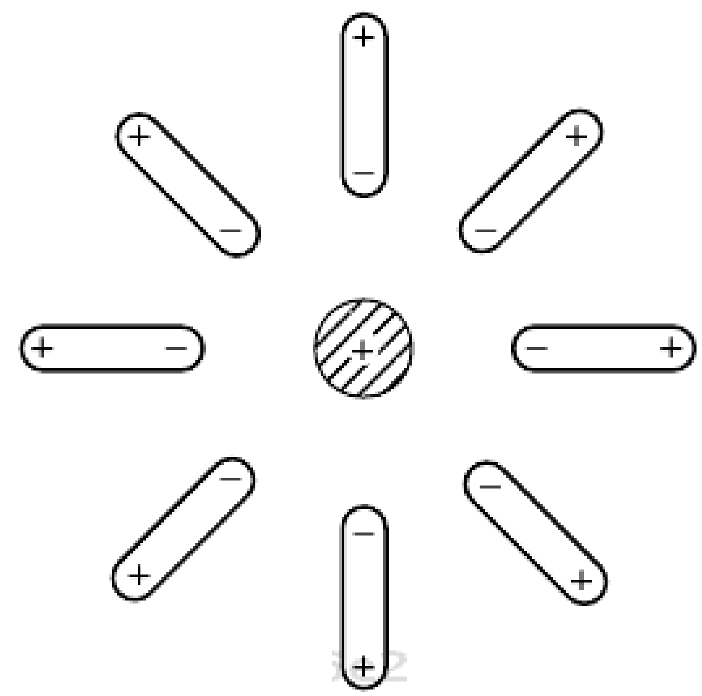
Ransom says his team should be able to test the principle within a year.

Running Coupling Constants

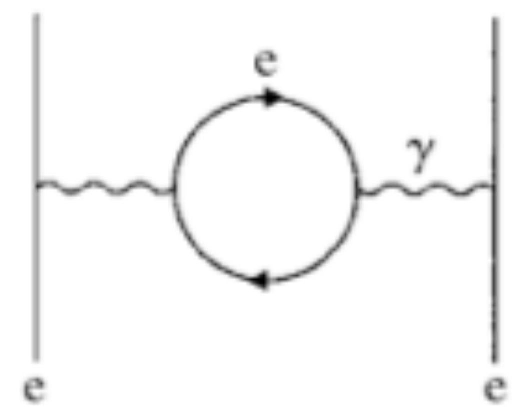
In the Standard Model of particle physics, the gauge theory SU(2)xU(1) describes the Electroweak interaction, and “color” SU(3) describes the Strong interaction. It turns out to be possible to calculate the values of the effective couplings of these gauge fields as a function of the momentum transfer q^2 through the interaction. To first order in terms that are logarithmic in q^2 (called “leading log approximation”), the electromagnetic coupling, which is $\alpha \approx 1/137$ at low q^2, becomes

$$\alpha\left(q^2\right) = \frac{\alpha\left(\mu^2\right)}{\left[1 - (1/\pi)\,\alpha\left(\mu^2\right)\ln\left(q^2/\mu^2\right)\right]}$$

This relates the coupling at one momentum transfer μ^2 to that at another q^2. Note that as q^2 increases, the effective electro-magetic coupling constant $\alpha(q^2)$ increases. This is because higher q^2 corresponds to shorter distances and greater penetration of the cloud of e^+e^- pairs that shields the bare charge at larger distances.



Cartoon

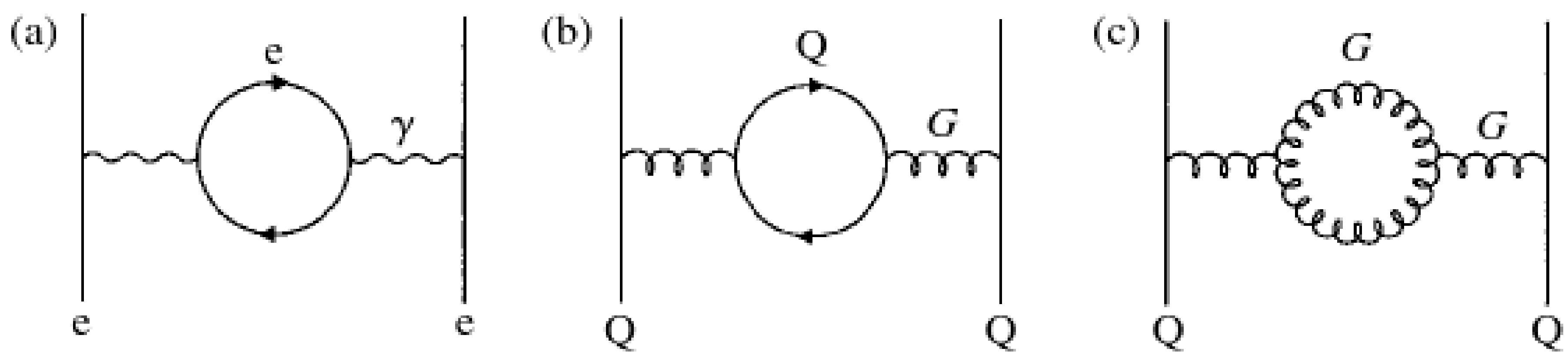


Feynman Diagram

Perkins’ Example 3.3 uses the above formula to calculate the value of $1/\alpha(q^2)$ at two values of q^2, the electroweak scale $q \sim 100$ GeV, where $1/\alpha \sim 129$, and a grand unified theory (GUT) scale $q \sim 3\times10^{14}$ GeV, where $1/\alpha \sim 111$. (Homework 3 problem 1 is about running couplings.)

Running Coupling Constants

In the Standard Model of particle physics, for the gauge theories SU(2) and "color" SU(3), the effective couplings of these nonabelian gauge fields decrease as a function of the momentum transfer q^2, unlike for the abelian U(1) electromagnetic case. The reason is that the photon interacts only with electrically charged particles but not with itself since it is uncharged, while in the nonabelian theories the gauge particles interact with themselves. The diagrams below are examples of how gluons interact with quarks (b) but also with themselves (c).



The leading log approximation is shown at right for the "running" of the Strong coupling $\alpha_S(q^2)$, which decreases as q^2 increases. This behaviour is known as "asymptotic freedom". The effective Weak coupling also decreases as q^2 increases.

$$\alpha_s\left(q^2\right) = \frac{\alpha_s\left(\mu^2\right)}{\left[1 + B\alpha_s\left(\mu^2\right)\ln\left(q^2/\mu^2\right)\right]} = \frac{1}{\left[B\ln\left(q^2/\Lambda^2\right)\right]}$$

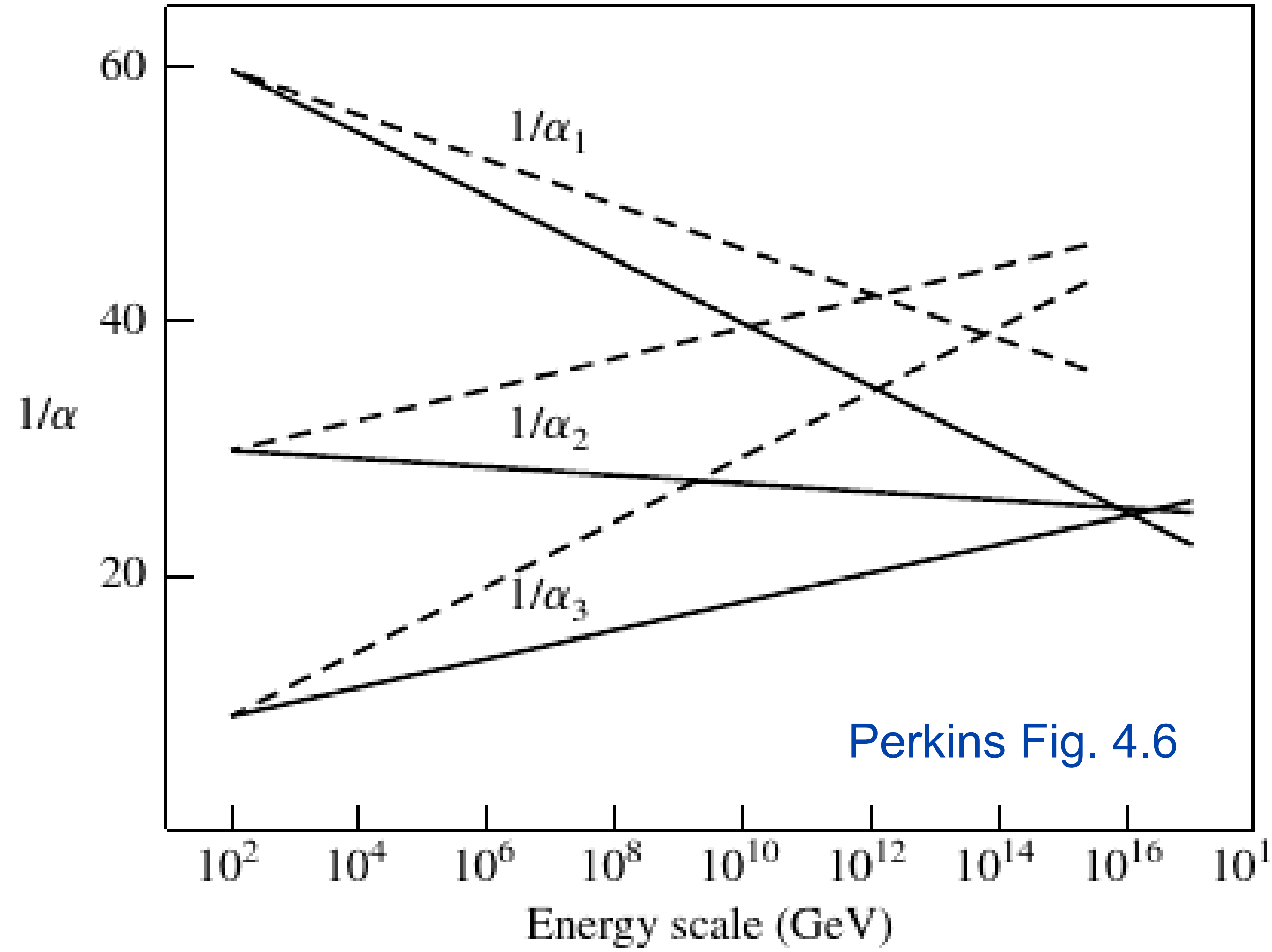
where $B = 7/4\pi$ and

$$\Lambda^2 = \mu^2 \exp\left[-1/B\alpha_s\left(\mu^2\right)\right]$$

Experiments have abundantly confirmed these predictions of the Standard Model, as discussed in Perkins Section 3.12.

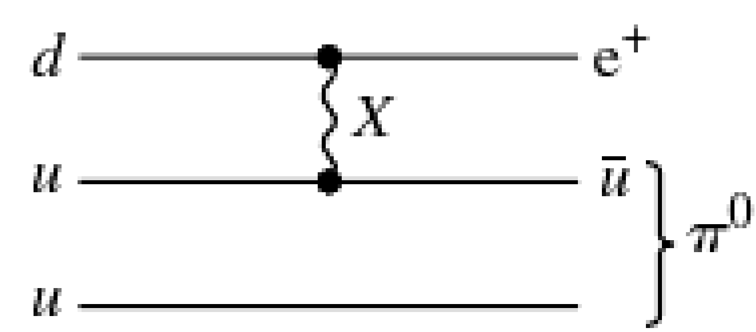
Grand Unified Theories (GUTs)

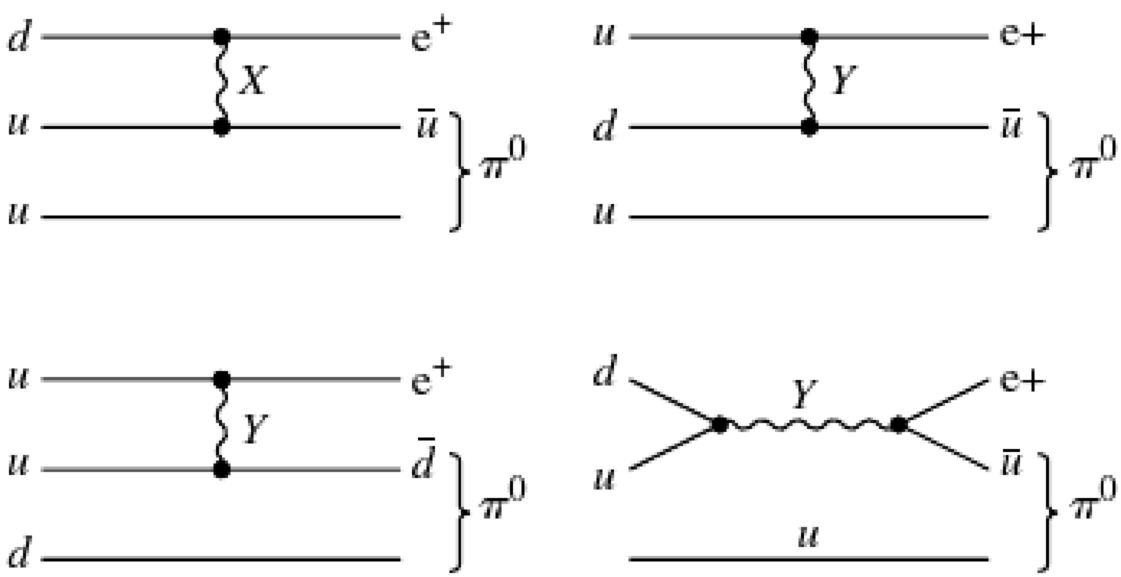
If the Electromagnetic, Weak and Strong interactions are "grand unified", then they will all meet at some value of q^2 when we run the coupling constants. The dashed lines at right show what happens in the Standard Model: they do not all meet. The solid lines show what happens in a supersymmetric version of the same model: they all meet at about $q = 10^{16}$ GeV. In this figure, α_1 is the U(1) Electroweak coupling, α_2 the Weak coupling, and α_3 the Strong coupling.



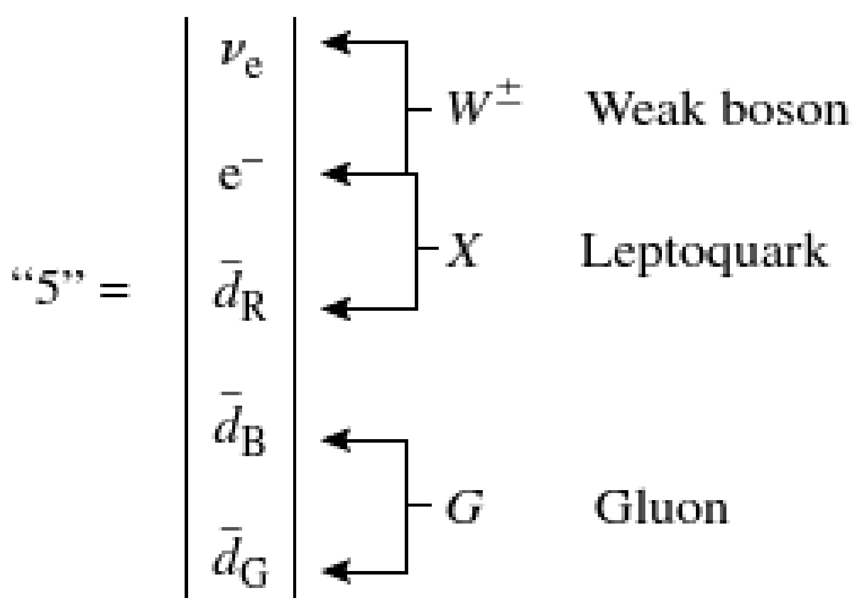
Perkins Fig. 4.6

In GUTs there are, besides the Standard Model gauge bosons (photon, $W^{\pm}$, Z^0, and gluons), there are additional leptoquark gauge bosons X and Y that transform leptons into quarks and vice versa. These bosons would have masses of order the unification energy scale, where the couplings meet. The first energy scale where the non-supersymmetric Standard Model couplings meet is $q \sim 10^{14.5}$ GeV. If quarks can turn into leptons, the proton could decay. The proton lifetime would be $\tau_p \sim M_X^4/(\alpha_S m_p^5)$, which for $M_X \sim 10^{14.5}$ GeV is $\tau_p \sim 10^{30}$ yr. However the SuperKamiokande experimental lower limit is $\tau_p > 8\text{x}10^{33}$ yr.

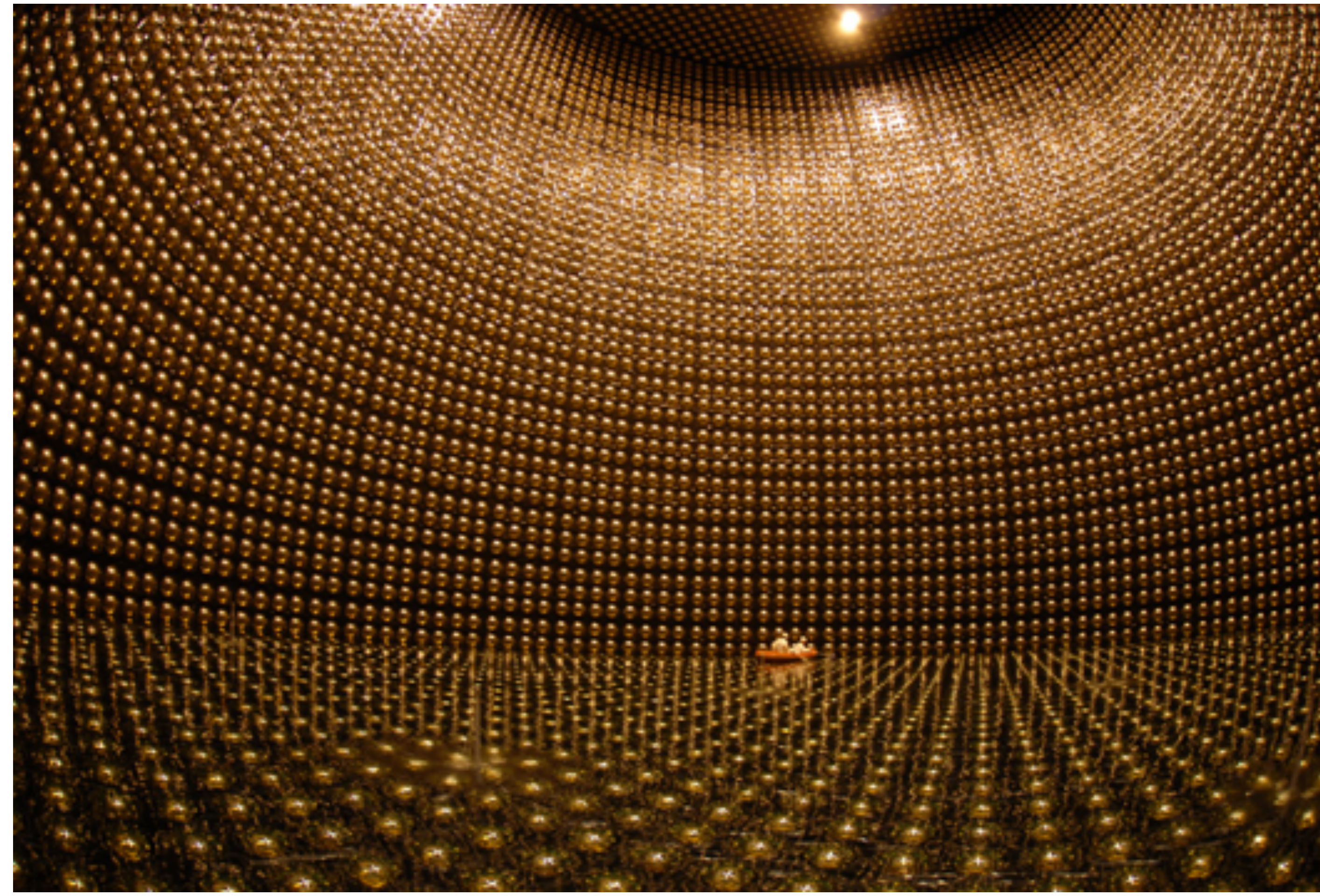




Feynman Diagrams for GUT Proton Decay

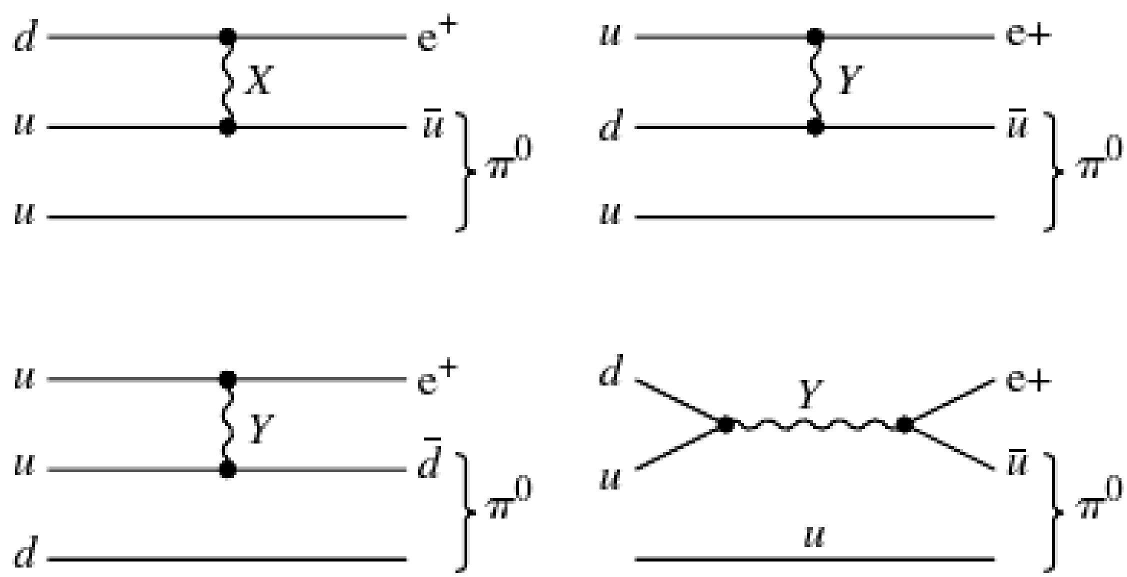


GUT "5" Multiplet of Leptons & Quarks

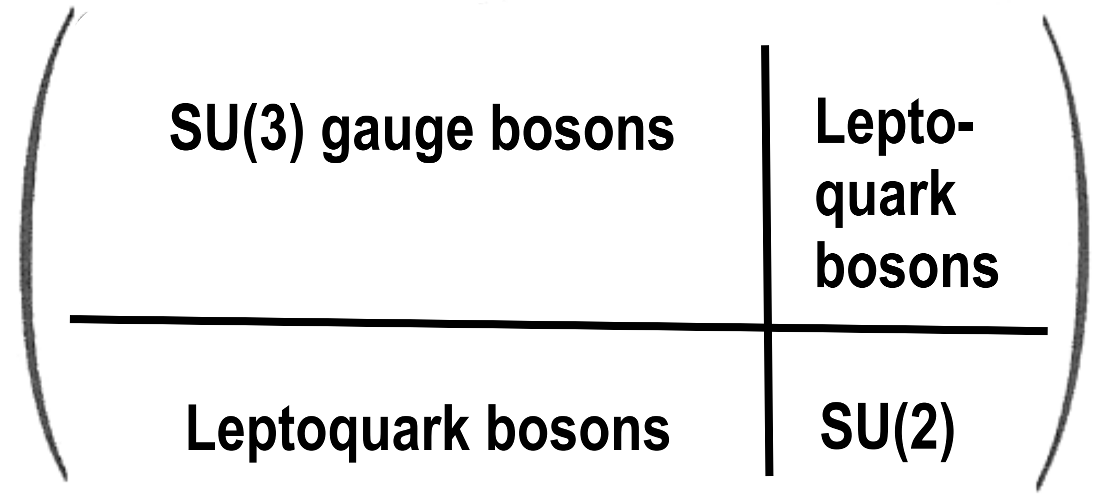


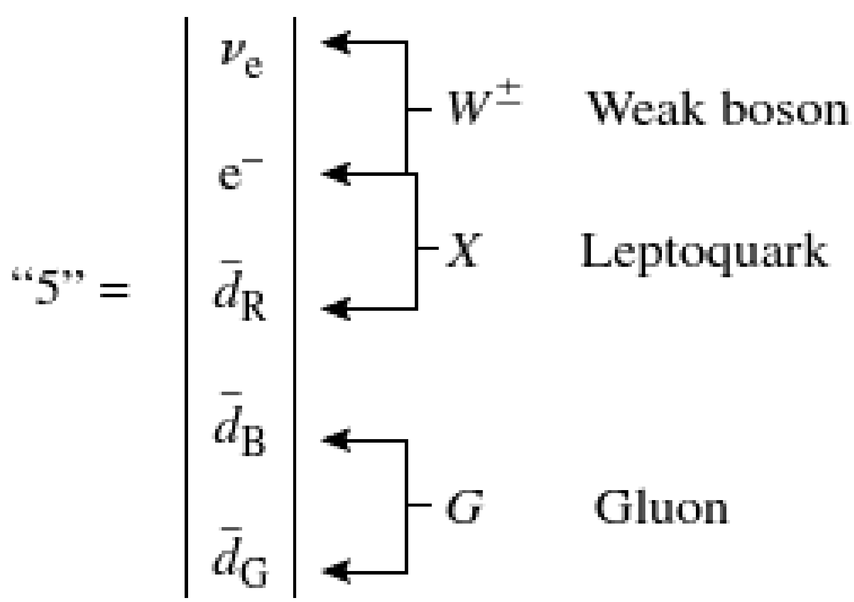
SuperKamiokande Neutrino and Proton Decay Detector
(see also Perkins Fig. 4.7 for more information)

Note that the possibility of "baryogenesis" -- making more quarks than antiquarks in the early universe -- is required in modern cosmology. If this is possible, proton decay should also be possible, which is a motivation to consider GUTs. If we do the same calculation of the proton lifetime but with $M_X \sim 10^{16}$ GeV, the proton lifetime is $\tau_p \sim 10^{36}$ yr, perfectly compatible with the current limits, and challenging to test!



Feynman Diagrams for GUT Proton Decay





SU(5) GUT "5" Multiplet
of Leptons & Quarks

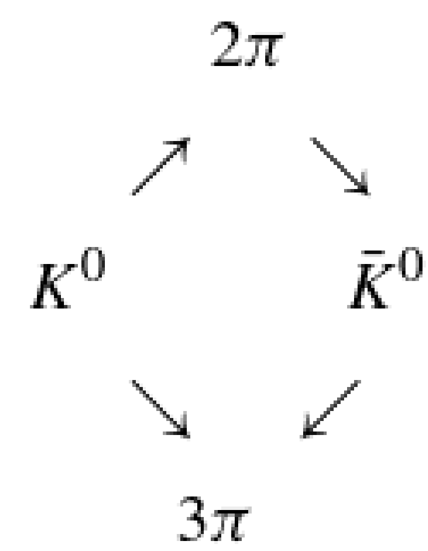
$$\begin{pmatrix} d^{red} \\ d^{blue} \\ d^{green} \\ e^{+} \\ \overline{\nu}_e \end{pmatrix}_R \qquad \begin{pmatrix} 0 & \overline{u}^{\,green,} & \overline{u}^{\,blue,} & u^{\,red} & d^{\,red} \\ & 0 & \overline{u}^{\,red,} & u^{\,blue} & d^{\,blue} \\ & & 0 & u^{\,green} & d^{\,green} \\ & & & 0 & e^{+} \\ & & & & 0 \end{pmatrix}_L$$

SU(5) "5" and "10" Multiplets of Leptons & Quarks

CP Violation in the K^0-$\bar{K}^0$ System

The kaons are the lightest mesons formed from the combination of a strange quark or antiquark with a non-strange antiquark or quark. They are produced in strong interactions of hadrons and occur in four states, all of spin-parity $J^P = 0^-$ and with masses of 0.494 GeV/c^2 for $K^+(= u\bar{s})$ and $K^-(= \bar{u}s)$, and 0.498 GeV/c^2 for $K^0(= d\bar{s})$ and $\bar{K}^0(= \bar{d}s)$. The states with a strange quark have $S = -1$, while those with a strange antiquark have $S = +1$. All the kaon states are unstable. The charged kaons, being particle and antiparticle, have the same mean lifetime of 12.4 ns. For the neutral kaons, however, two different lifetimes are observed. The state called K_S has $\tau = 0.089$ ns and that called K_L has $\tau = 51.7$ ns (the subscripts standing for 'short' and 'long'). The existence of two lifetimes arises because the decaying states the experimentalist detects are superpositions of K^0 and $\bar{K}^0$ amplitudes. This mixing occurs through virtual 2π and 3π intermediate states and involves a *second-order weak interaction of*

$\Delta S = 2$:



First, we can form CP eigenstates from the neutral kaon states as follows:

$$K_S = \sqrt{\frac{1}{2}}\left(K^0 + \bar{K}^0\right) \qquad CP = +1$$

$$K_L = \sqrt{\frac{1}{2}}\left(K^0 - \bar{K}^0\right) \qquad CP = -1$$

where, since the kaons have spin zero, the operation CP on the wavefunction has the same effect as that of charge conjugation, C. On taking into account the negative intrinsic parity of the pion mentioned in Section 3.4, the decay modes will be $K_S \to 2\pi$ where the final state consists of two pions in an S-state with CP $= +1$ and $K_L \to 3\pi$ with CP $= -1$. Thus, while the neutral kaons are *produced* as eigenstates of strangeness, K_0 and $\bar{K}^0$, they *decay* as superpositions of these states which are actually eigenstates of CP.

CP Violation in the K^0-$\overline{K}^0$ System

In 1964, it was found by Christenson *et al.* that the above states were in fact *not* pure CP eigenstates. If we denote a pure CP $= +1$ state by K_1, and a pure CP $= -1$ state by K_2, the K_L and K_S amplitudes are written as

$$K_S = N\,(K_1 - \varepsilon K_2)$$

$$K_L = N\,(K_1 + \varepsilon K_2)$$

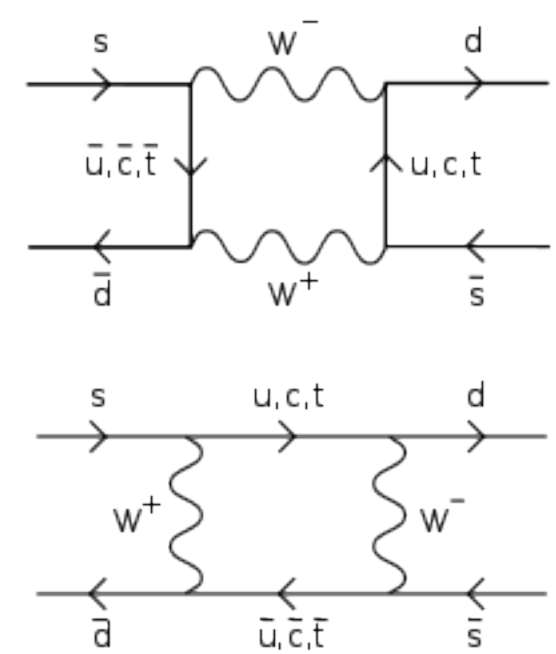
where the normalizing factor $N = (1 + |\varepsilon|^2)^{-1/2}$ and $\varepsilon \approx 2.3 \times 10^{-3}$ is a small parameter quantifying the level of CP violation. The experiment commenced with a beam of K^0 generated in a strong interaction. After coasting for several K_S mean lives, the experimenters were left with a pure K_L beam. It was observed that a small proportion of the K_L decays were to a two-pion state, with $CP = +1$

CP violation is also demonstrated in the leptonic decay modes of K_L. If we denote the rate for $K_L \to e^+ + \nu_e + \pi^-$ by R^+, and for $K_L \to e^- + \nu_e + \pi^+$ by R^-, then it is observed that

$$\Delta = \frac{(R^+ - R^-)}{(R^+ + R^-)} = (3.3 \pm 0.1) \times 10^{-3}$$

CP violation because the K_S and K_L are not pure CP eigenstates is known as indirect CP violation. Direct CP violation also occurs in the actual decay process, a smaller effect that can be described by the CKM matrix. For the discovery of CP violation in K^0 decay, Jim Cronin and Val Fitch won the 1980 Nobel Prize. Direct CP violation was observed starting in 1999, especially in the B^0-$\overline{B}^0$ system. The slight difference in K_S - K_L masses was the basis for the 1972 prediction of the mass of the c quark.

Feynman diagrams for K_S - K_L mass difference:



CP Violation in the CKM Matrix

The charged W boson causes transitions not from the *u* quark to the *d* quark, but actually to the *d'* quark, a linear combination of *d* and *s* quarks. This mixing, described by the "Cabibbo angle" $\theta_C = 12.7^o$, is why the weak interactions allow $\Delta S = \pm 1$ transitions. Thus the first and second generation quark doublets can be written as

$$\begin{pmatrix} u \\ d' \end{pmatrix} \text{ and } \begin{pmatrix} c \\ s' \end{pmatrix} \quad \text{where} \quad \begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

After the third generation of quarks, *t* and *b*, were discovered, this matrix was generalized by Kobayashi and Maskawa in 1972 to include the *b'* quark:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad \text{where} \quad V_{\text{CKM}} = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix} \approx \begin{vmatrix} 0.975 & 0.221 & 0.004 \\ 0.221 & 0.975 & 0.039 \\ 0.008 & 0.038 & 0.999 \end{vmatrix}$$

What does this have to do with CP violation? An N by N unitary matrix has N^2 real parameters, of which N(N−1)/2 pairs are generalized mixing angles. This is 1 for N=1 (the Cabibbo angle) and 3 mixing angles for N=3. Kobayashi and Maskawa pointed out in 1973 that the 3x3 matrix also in general has an additional complex phase, which introduces CP violation; they shared half of the 2008 Nobel Prize for this work.

We will see that CP violation is crucial to explaining why the Universe is made of matter rather than a mixture of matter and anti-matter. However, the level of CP violation in V_{CKM} is not nearly large enough to explain this.

CP Violation in the Strong Interaction?

There is no experimental evidence for CP violation in the Strong Interaction, but there is also no fundamental reason why the color SU(3) theory, called Quantum Chromodynamics (QCD) in analogy with Quantum Electrodynamics (QED), should not violate CP. In quantum field theory, all interaction term that are permitted by symmetries must be included in the Lagrangian. Gell-Mann called this the **anti-totalitarian principle: everything that is not forbidden is compulsory.**

If the Strong Interactions violate CP, it will give rise to electric dipole moments for elementary particles. For the neutron the expected value of the edm is $d_n \sim 10^{-18}$ e · cm. The upper limit on the edm of the neutron is about 10^{-12} of the expected value if CP is violated in the strong interactions. There is an a parameter in QCD Lagrangian that could just happen to be this small, a "fine-tuning" problem. It is considered to be much more plausible that there is some physical reason why CP violation is extremely small or nonexistent. The best reason discovered thus far is that there is a thus-far undiscovered particle, called the **axion**, with just the right interactions. For certain values of the axion mass, the axion could also be the dark matter in the Universe. (The axion is discussed in Section 7.9 of Perkins, and we will discuss it again in the context of dark matter.)

CP Violation in Physics Beyond the Standard Model?

The axion is an example of "physics beyond the Standard Model." But other extensions to the Standard Model such as supersymmetry lead to small CP violations that would produce an electron edm $d_e \sim 10^{-27}$ to 10^{-30} e·cm. The ACME collaboration just reported a new upper limit: $|d_e| < 8.7 \times 10^{-29}$ **e·cm** with 90% confidence, an order of magnitude improvement in sensitivity relative to the previous best limit. This new result constrains CP-violating physics at the TeV energy scale. (Baron et al., Science, 17 January 2014, 343, pp. 269-272.)

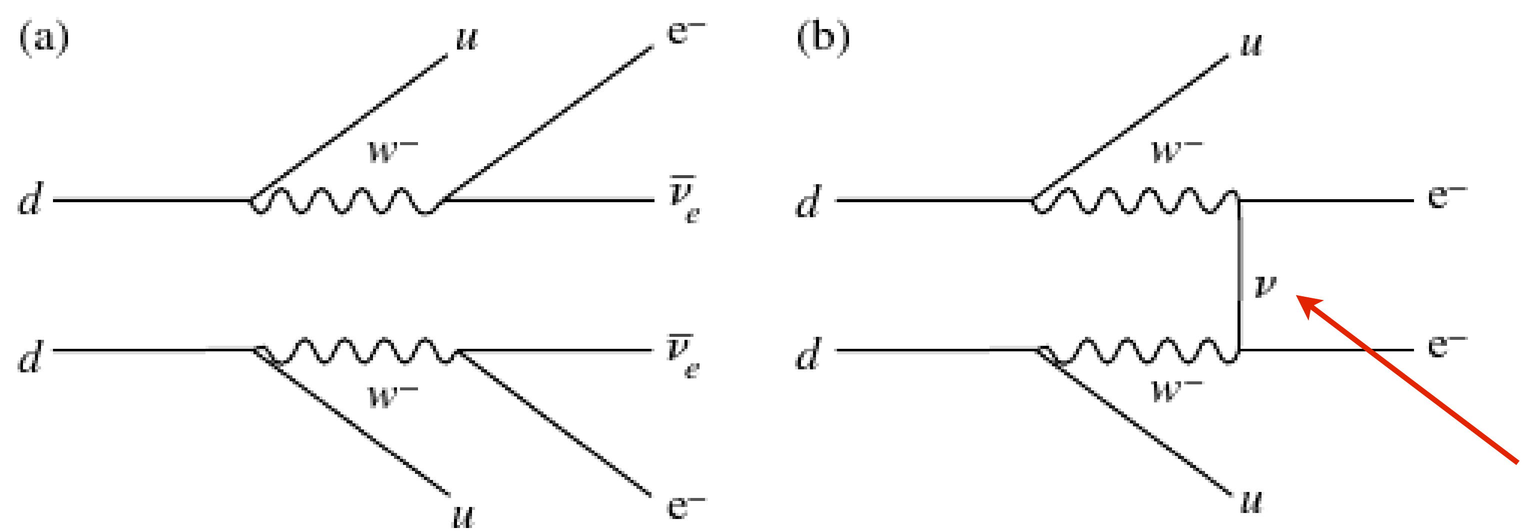
The excitement of the discovery of the Higgs boson at the Large Hadron Collider (LHC) and the award of the Nobel prize for its prediction are testaments to the power of high energy experimenal physics. At present the results at the LHC are consistent with the Standard Model. The lack of detection of other new particles also limits possible theoretical models, and as a result, the value of the electron electric dipole moment. It is unclear at the moment whether investigation at high precision such as the ACME experiment or investigation at high energy will be first to reveal additional physics beyond the Standard Model.

Neutrinos Beyond the Standard Model

In the Standard Model, neutrinos are massless. However, neutrino flavor oscillations, to be discussed next, show that all three neutrinos have very small masses, and cosmological evidence shows that the sum of the three neutrino masses is less than 0.23 eV.

Neutrinos could be Majorana particles, i.e. their own antiparticles, as originally suggested by Fermi's student Ettore Majorana in 1937. In this case the right-handed antineutrino would be the antiparticle of the left-handed neutrino. This would permit neutrinoless double beta decay, illustrated in the Feynman diagram at the right below

ordinary double beta decay



neutrinoless double beta decay

Majorana neutrino

In ordinary double beta decay, a nucleus (Z, A) ⟶ (Z+2, A) + $2e^-$ + $2\bar{\nu}_e$, but in neutrinoless beta decay there are no emitted neutrinos and the beta rays (electrons) therefore have more energy and are emitted almost perfectly collimated. Such double beta decays are 2nd order weak interactions, so the decay rates are very low and the corresponding lifetimes are ~10^{-20} yr. Neutrinoless double beta decay is proportional to the neutrino mass squared, and the current upper limit of ~ 10^{-25} yr corresponds to an upper limit $m(\nu_e) \lesssim 1$ eV. Improving this limit, and possibly detecting neutrinoless double beta decay, is a major experimental challenge.

Neutrino Masses and Flavor Oscillations

The fact that electron neutrino beams interact with matter to produce electrons, muon neutrinos produce muons, and tau neutrinos produce tau leptons suggested that all three lepton flavor numbers are conserved in the weak interactions. But neutrino oscillations violate such flavor conservation just as the mixing of the *d, s,* and *b* quarks in the weak doublets into *d', s'*, and *b'* allows weak violation of quark flavor conservation.

In order for the sun to fuse four protons into a helium nucleus, two weak transformations of protons into neutrons are required: $p \longrightarrow n + e^+ + \nu_e$, which requires the emission of two electron-type neutrinos. Neutrino flavor oscillations explain the experimental measurement that the sun emits only about a third of the number of electron-type neutrinos. Such oscillations have now been measured involving all three types of neutrinos. Further experimental evidence has allowed mesurement of the differences of the squared neutrino masses, but not yet their actual masses.

Neutrinos are produced as flavor eigenstates ν_e, ν_μ, or ν_τ, but these are mixtures of the mass eigenstates ν_1, ν_2, ν_3. Since the masses differ, the superposition that corresponds to any particular flavor eigenstate will oscillate into a mixture of the other flavor eigenstates.

It is simplest to discuss just two types of neutrinos, for example ν_μ and ν_τ, which for simplicity we can regard as mixtures of ν_2 and ν_3. This is a pretty good description of atmospheric neutrinos. Pions are abundantly produced by cosmic rays hitting air molecules the upper atmosphere, and the charged pions mainly decay to muons: e.g, $\pi^+ \longrightarrow \mu^+ + \nu_\mu$. Then the muons decay to muon and electron neutrinos: $\mu^+ \longrightarrow e^+ + \nu_e + \overline{\nu}_\mu$. The result is that we expect two ν_μ for every ν_e (and similarly for antineutrinos). This is true for downward going ν_μ, but the neutrinos coming from larger zenith angles or coming up through the earth have a lower ν_μ/ν_e ratio because of these atmospheric neutrino oscillations.

Neutrino Masses and Flavor Oscillations

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The correspoding neutrino mixing is described by

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}$$

and the neutrino mass eigenstates will propagate as

$$\nu_2(t) = \nu_2(0)\exp(-iE_2 t)$$

$$\nu_3(t) = \nu_3(0)\exp(-iE_3 t)$$

It is always a good approximation the write $E_i = p_i\,(1+m_i^2/p^2)^{1/2} = p + m_i^2/2\text{p}$ since the neutrino masses m_i are are so much smaller than the neutrino energies.

Neutrino Masses and Flavor Oscillations

Recall that

$$\nu_2(t) = \nu_2(0)\exp(-iE_2 t)$$

$$\nu_3(t) = \nu_3(0)\exp(-iE_3 t)$$

If we start off with muon neutrinos, i.e. $\nu_\mu(0) = 1$, then

$$\nu_2(0) = \nu_\mu(0)\ \cos\theta$$

$$\nu_3(0) = \nu_\mu(0)\ \sin\theta$$

and

$$\nu_\mu(t) = \nu_2(t)\ \cos\theta + \nu_3(t)\ \sin\theta$$

The time dependence of the muon neutrino amplitude becomes

$$A_\mu(t) = \frac{\nu_\mu(t)}{\nu_\mu(0)} = \cos^2\theta\ \exp(-iE_2 t) + \sin^2\theta\ \exp(-iE_3 t)$$

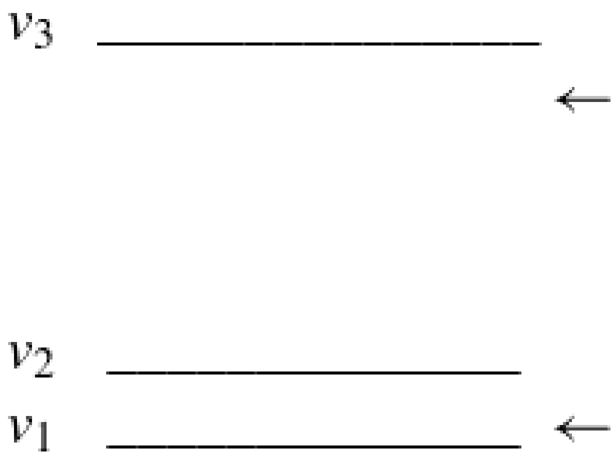
and the corresponding intensity is

$$\frac{I_\mu(t)}{I_\mu(0)} = AA^* = 1 - \sin^2 2\theta\ \sin^2\left[\frac{(E_3 - E_2)\,t}{2}\right] = 1 - \sin^2 2\theta \cdot \sin^2\left(1.27\frac{\Delta m^2 L}{E}\right)$$

where we define $\Delta m_{23}{}^2 \equiv m_3{}^2 - m_2{}^2$ (and assume for definiteness that $m_3 > m_2$) and where L is in km, E is in GeV and Δm^2 is in $(\mathrm{eV})^2$ (see homework 3 problem 6).

Neutrino Masses and Flavor Oscillations

The mixing angles θ_{ij} and squared mass differences Δm_{ij}^2 are experimentally found to be



$$\sin^2(2\theta_{23}) > 0.95$$
$$\Delta m^2_{32} = (2.32^{+0.12}_{-0.08}) \times 10^{-3}\ \mathrm{eV}^2$$

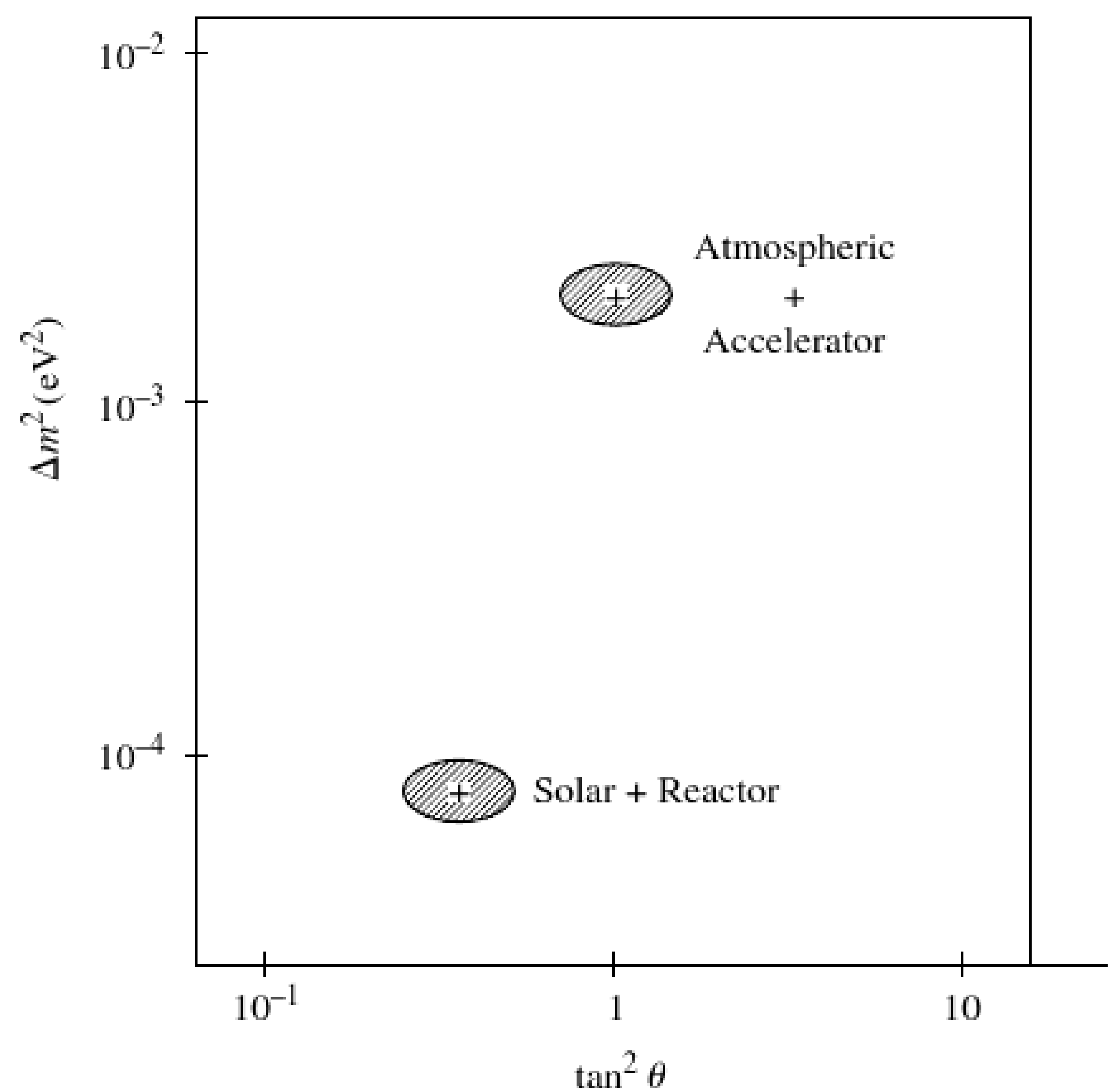
$$\sin^2(2\theta_{12}) = 0.857 \pm 0.024$$
$$\Delta m^2_{21} = (7.50 \pm 0.20) \times 10^{-5}\ \mathrm{eV}^2$$

These are from the latest Particle Data Group summary table, which also says that $\sin^2(2\theta_{13}) = 0.095 \pm 0.010$

The plot at the right shows Δm_{23}^2 and Δm_{12}^2 vs. the corresponding $\tan^2\theta_{ij}$. If the neutrino masses are hierarchical rather than nearly degenerate, then

$m_3 \sim (2.3\times10^{-3}\ \mathrm{eV}^2)^{1/2} = 0.05\ \mathrm{eV}$
$m_2 \sim (7.5\times10^{-5}\ \mathrm{eV}^2)^{1/2} = 0.007\ \mathrm{eV}$

As already meantioned, cosmological data shows that $m_1 + m_2 + m_2 < 0.23$ eV.



See-saw Mechanism for Neutrino Masses

It is puzzling that the neutrino masses, ~0.1 eV or less, are so much smaller than the other fermion masses. A plausible explanation is the "see-saw" mechanism, in which neutrino masses are a mixture of Majorana masses m_L and m_R, which are separate for left- and right-handed neutrinos, and a Dirac mass, which mixes L and R. The corresponding mass matrix is

$$\begin{vmatrix} m_L & m_D \\ m_D & m_R \end{vmatrix}$$

Diagonalizing this matrix gives

$$m_{1,2} = \frac{1}{2}\left[(m_R + m_L) \pm \sqrt{(m_R - m_L)^2 + 4m_D^2}\right]$$

If m_L is so small that we can neglect it, and $M \equiv m_R >> m_D$, then

$$m_1 \approx \frac{(m_D)^2}{M}, \quad m_2 \approx M$$

If we take $m_D \sim 10$ GeV, then $M \sim 10^{12}$ GeV gives $m_1 \sim 0.1$ eV, as required. This is another indication, besides Grand Unification, that there might be interesting new physics at high mass scales. Decay of these hypothetical very massive right-handed neutrinos is also a plausible mechanism to help explain the cosmic asymmetry between matter and antimatter.

Supersymmetric WIMPs

When the British physicist Paul Dirac first combined Special Relativity with quantum mechanics, he found that this predicted that for every ordinary particle like the electron, there must be another particle with the opposite electric charge – the anti-electron (positron). Similarly, corresponding to the proton there must be an anti-proton. Supersymmetry appears to be required to combine General Relativity (our modern theory of space, time, and gravity) with the other forces of nature (the electromagnetic, weak, and strong interactions). The consequence is another doubling of the number of particles, since supersymmetry predicts that for every particle that we now know, including the antiparticles, there must be another, thus far undiscovered particle with the same electric charge but with *spin* differing by half a unit.

Spin	**Matter** (fermions)	**Forces** (bosons)
2		graviton
1		photon, $W^{\pm}, Z^0$ gluons
1/2	quarks u,d,... leptons $e, \nu_e, \ldots$	
0		Higgs bosons axion

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after doubling

Spin	**Matter** (fermions)	**Forces** (bosons)	**Hypothetical Superpartners**	**Spin**
2		graviton	gravitino	3/2
1		photon, $W^{\pm}$, Z^0 gluons	photino, winos, zino, gluinos	1/2
1/2	quarks u,d,... leptons e, ν_e, ...		squarks $\tilde{u}$, $\tilde{d}$, ... sleptons $\tilde{e}$, $\tilde{\nu}_e$, ...	0
0		Higgs bosons axion	Higgsinos axinos	1/2

Note: Supersymmetric cold dark matter candidate particles are underlined.

Supersymmetric WIMPs

Spin is a fundamental property of elementary particles. Matter particles like electrons and quarks (protons and neutrons are each made up of three quarks) have spin ½, while force particles like photons, W,Z, and gluons have spin 1. The supersymmetric partners of electrons and quarks are called selectrons and squarks, and they have spin 0. The supersymmetric partners of the force particles are called the photino, Winos, Zino, and gluinos, and they have spin ½, so they might be matter particles. The lightest of these particles might be the photino. Whichever is lightest should be stable, so it is a natural candidate to be the dark matter WIMP, as first suggested by Pagels & Primack 1982. A supersymmetric WIMP also naturally has about the observed dark matter density. Its mass is not predicted by supersymmetry, but it will be produced soon at the LHC if it exists and its mass is not above ~1 TeV!

Supersymmetry thus helps unify gravity with the other forces, and it provides a natural candidate for the dark matter particle. The boson-fermion cancellation built into supersymmetry also helps to control the vacuum energy (related to the cosmological constant) and to explain the "gauge hierarchy problem" (why the Electroweak scale is so much less than the GUT or Planck scales).