Physics 129 LECTURE 9 February 4, 2014

- Introduction to Cosmology
  - The Expanding Universe
  - The Friedmann Equation
  - The Age of the Universe
  - Cosmological Parameters
  - History of Cosmic Expansion
  - The Benchmark Model
  - The Backward Lightcone
  - Cosmic Particle and Event Horizons
  - Distances in the Expanding Universe
  - Ned Wright's Cosmology Calculator

The in-class open-book Midterm Exam will be Thursday February 13.

Reserve Books for Physics 129

Prof/ta	Primack, J.
Course	Physics 129 PHYS 129 Nuclear and Particle Astrophysics
Cour note	Winter 2014

Materials for this course

Title	Author	Call #	
Astrophysics in a nutshell / Dan Maoz	Maoz, Dan	Reserves S&E Desk QB461 .M32 2007 NOT CHECKD OUT	24 Hours
Cosmology : the science of the universe / Edward Harrison	Harrison, Edward Robert	Reserves S&E Desk QB981 .H32 2000 NOT CHECKD OUT	24 Hours
The early universe / Edward W. Kolb, Michael S. Turner.	Kolb, Edward W.	Reserves S&E Desk QB981.K687 1990 NOT CHECKD OUT	24 Hours
Introduction to cosmology / Barbara Ryden	Ryden, Barbara Sue	Reserves S&E Desk QB981 .R93 2003 NOT CHECKD OUT	24 Hours
Modern cosmology / Scott Dodelson	Dodelson, Scott	Reserves S&E Desk QB981 .D634 2003 NOT CHECKD OUT	24 Hours
The physical universe : an introduction to astronomy / Frank H. Shu.	Shu, Frank H.	Reserves S&E Desk QB43.2.S54 1982 c.2 NOT CHECKD OUT	24 Hours
The physical universe : an introduction to astronomy / Frank H. Shu.	Shu, Frank H.	Reserves S&E Desk QB43.2.S54 1982 c.3 NOT CHECKD OUT	24 Hours
The view from the center of the universe : discovering our extraordinary place in the cosmos / by Joel R. Primack and Nancy Ellen Abrams	Primack, J. R. (Joel R.)	Reserves S&E Desk QB981 .P85 2006 NOT CHECKD OUT	24 Hours

The Expanding Universe

Edwin Hubble discovered the expansion of the universe by discovering a linear relation between the expansion velocity v of a galaxy and its distance D:

$$v = H_0 D$$

where the constant of proportionality H_0, called the Hubble constant or Hubble parameter, has the value (according to Perkins) $H_0 = 72 \pm 3 \text{ km s}^{-1} \text{ Mpc}^{-1}$

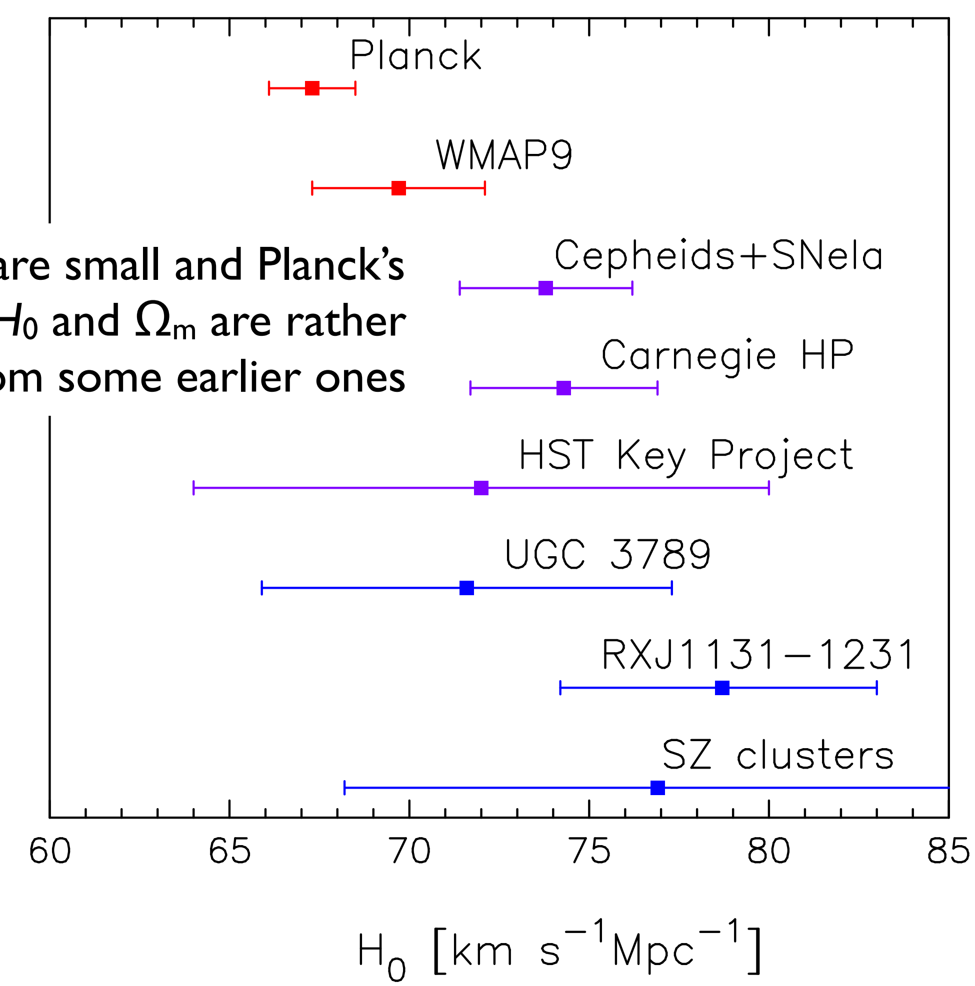
Actually, the latest value for H_0, from the Planck satellite data plus much other astronomical data, is 67.80 ± 0.77 . And the actual recession velocity of a galaxy is the sum of its expansion velocity and its "peculiar velocity" v_p, generated mostly by local gravitational effects:

$$v = H_0 D + v_p$$

A galaxy's redshift is given by

$$z = (\lambda_o - \lambda_e)/\lambda_e$$

where λ_e and λ_o are the emitted and observed wavelengths. Measuring galaxy redshifts is easy, but measuring their distances is hard. Milton Humason and others had measured a number of galaxy redshifts, but Hubble figured out how to measure distances to galaxies using Cepheid variable stars. He got the relative distances more or less right, although his distance scale was later recalibrated as Cepheid variables were better understood.



Planck errors are small and Planck's values for H_0 and Ω_m are rather different from some earlier ones

General Relativity

CURVED SPACE TELLS MATTER HOW TO MOVE

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = 0$$

MATTER TELLS SPACE HOW TO CURVE

Einstein Field Equations

$$G^{\mu\nu} \equiv R^{\mu\nu} - \tfrac{1}{2}Rg^{\mu\nu} = -8\pi G T^{\mu\nu} - \Lambda g^{\mu\nu}$$

Here u^α is the velocity 4-vector of a particle. The Riemann curvature tensor $R^\lambda{}_{\mu\sigma\nu}$, Ricci curvature tensor $R_{\mu\nu} \equiv R_{\lambda\mu\sigma\nu} g^{\lambda\sigma}$, curvature scalar $R \equiv R_{\mu\nu} g^{\mu\nu}$, and affine connection $\Gamma^\mu_{\alpha\beta}$ can be calculated from the metric tensor $g_{\lambda\sigma}$. If the metric is just that of flat space, then $\Gamma^\mu_{\alpha\beta} = 0$ and the first equation above just says that the particle is unaccelerated -- i.e., it satisfies the law of inertia (Newton's 1st law).

General Relativity and Cosmology

CURVED SPACE TELLS MATTER HOW TO MOVE

$$\frac{du^\mu}{ds} + \Gamma^\mu{}_{\alpha\beta}\, u^\alpha u^\beta = 0$$

MATTER TELLS SPACE HOW TO CURVE

Einstein Field Equations

$$G^{\mu\nu} \equiv R^{\mu\nu} - \tfrac{1}{2} R g^{\mu\nu} = -\, 8\pi G T^{\mu\nu} - \Lambda g^{\mu\nu}$$

Einstein's Cosmological Principle: on large scales, space is uniform and isotropic.

COBE-Copernicus Theorem: If all observers observe a nearly-isotropic Cosmic Background Radiation (CBR), then the universe is locally nearly homogeneous and isotropic – i.e., is approximately described by the Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)\,[dr^2\,(1 - kr^2)^{-1} + r^2\, d\Omega^2]$$

with curvature constant $k = -1$, 0, or +1. Substituting this metric into the Einstein equations above, we get the Friedmann equations. Here r is the comoving coordinate, and the expansion factor $a(t) = 1/(1+z)$, where z is the redshift. At the present epoch $t = t_0$, $a_0 = a(t_0) = 1$ and $z(t_0) = 0$. The distance $D(t) = a(t)\, r$. [Perkins $R(t) = a(t)$.]

Friedmann-Robertson-Walker Metric

(homogeneous, isotropic universe)

FRW E(00) $$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$ ⟵ **Friedmann equation**

FRW E(ii) $$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p - \frac{k}{a^2} + \Lambda$$

$$H_0 \equiv 100h\,\mathrm{km\,s^{-1}Mpc^{-2}} \equiv 70h_{70}\,\mathrm{km\,s^{-1}Mpc^{-2}}$$

$$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda \text{ with } H \equiv \frac{\dot{a}}{a},\ a_0 \equiv 1,\ \Omega_0 \equiv \frac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2},$$

at t_0, with $a(t_0)=1$

$$\rho_{c,0} \equiv \frac{3H_0^2}{8\pi G} = 1.36 \times 10^{11} h_{70}^2 M_\odot \mathrm{Mpc}^{-3}$$

$$E(ii) - E(00) \Rightarrow \frac{2\ddot{a}}{a} = -\frac{8\pi}{3}G\rho - 8\pi G p + \frac{2}{3}\Lambda$$

Divide by $2E(00) \Rightarrow$ $$q_0 \equiv -\left(\frac{\ddot{a}}{a}\frac{a^2}{\dot{a}^2}\right)_0 = \frac{\Omega_0}{2} - \Omega_\Lambda$$ ⟵ **deceleration parameter**

(note that $p_0 = 0$)

$$E(00) \Rightarrow t_0 = \int_0^1 \frac{da}{a}\left[\frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}\right]^{-\frac{1}{2}} = H_0^{-1}\int_0^1 \frac{da}{a}\left[\frac{\Omega_0}{a^3} - \frac{k}{H_0^2 a^2} + \Omega_\Lambda\right]^{-\frac{1}{2}}$$

$$t_0 = H_0^{-1} f(\Omega_0, \Omega_\Lambda) \qquad H_0^{-1} = 9.78h^{-1}\mathrm{Gyr}$$

$$f(1,0) = \tfrac{2}{3}$$
$$f(0,0) = 1$$
$$f(0,1) = \infty$$

age of the universe

$$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda \text{ with } H \equiv \frac{\dot{a}}{a},\ a_0 \equiv 1,\ \Omega_0 \equiv \frac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2},$$

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$$t_0 = H_0^{-1} f(\Omega_0, \Omega_\Lambda) \qquad H_0^{-1} = 9.78h^{-1}\mathrm{Gyr} = 13.97 h_{70}^{-1}\,\mathrm{Gyr}$$

age of the universe

$$f(1,0) = \tfrac{2}{3}, \quad f(0,0) = 1, \quad f(0,1) = \infty$$

$$[E(00)a^3]' \text{ vs. } E(ii) \Rightarrow \frac{\partial}{\partial a}(\rho a^3) = -3pa^2 \text{ (“continuity”)}$$

Given eq. of state $p = p(\rho)$, integrate to determine $\rho(a)$,
integrate $E(00)$ to determine $a(t)$

Examples:

$$p = 0 \Rightarrow \rho = \rho_0 a^{-3} \text{ (assumed above in } q_0,\ t_0 \text{ eqs.)}$$

$$p = \frac{\rho}{3},\ k = 0 \Rightarrow \rho \propto a^{-4}$$

$$p = w\rho,\ k = 0 \Rightarrow \rho \propto a^{-3(1+w)}$$

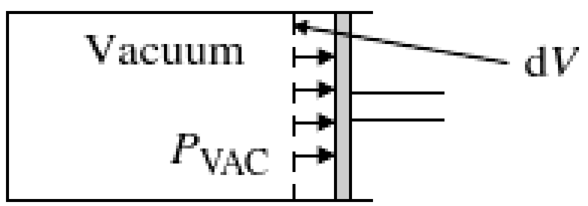
Cosmological Parameters (observations and simulations)

Parameter	*WMAP9**	*Bolshoi*	*Planck+WP+highL+BAO***	*Bolshoi-Planck****	*Millennium*
Ω_Λ	0.7135 ± 0.0096	0.73	0.692 ± 0.010	0.6929	0.75
Ω_m	0.2865 ± 0.0088	0.27	$(1-\Omega_\Lambda)$	0.3071	0.25
σ_8	0.820 ± 0.014	0.82	0.826 ± 0.012	0.8225	0.90
H_0	69.32 ± 0.80	70.0	67.80 ± 0.77	67.77	73.0
n_s	0.9608 ± 0.0080	0.95	0.9608 ± 0.0054	0.96	1.00
t_0 (Gyr)	13.772 ± 0.059	13.86	13.798 ± 0.037	13.814	13.573

*WMAP9 is WMAP+eCMB+BAO+H_0 from Table 17 of Bennett et al. arXiv:1212.5225v2 (30 Jan 2013)

**The 4th column is the 68% limits for *Planck+WP+highL+BAO* from Table 5 of of the Planck Collaboration: Cosmological parameters paper, Planck 2013 results. XVI. Cosmological parameters

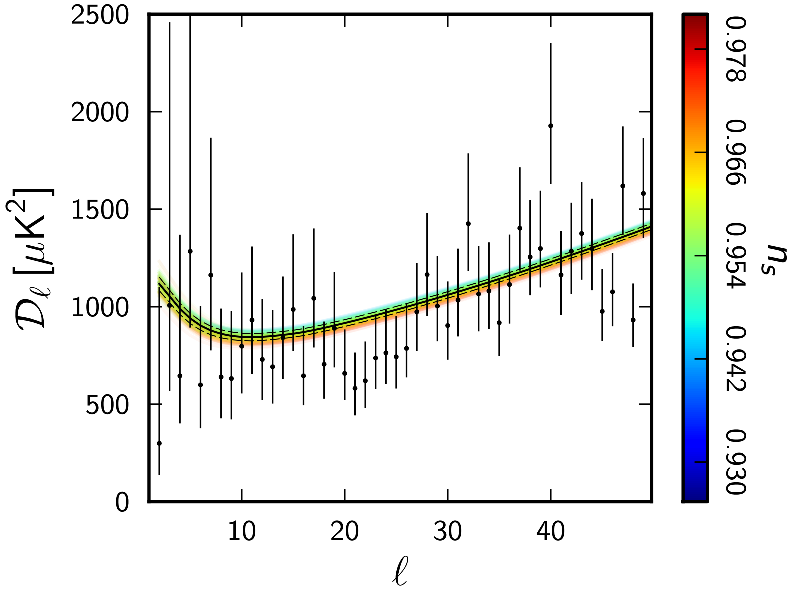
***Bolshoi-Planck parameters were used for the Bolshoi-Planck and MultiDark-Planck simulations



Negative vacuum pressure
= gravitational repulsion

Why a cosmological constant corresponds to negative pressure:

When gas pushes the piston out it does work $p dV$ and the internal energy of the gas is *reduced*. But when the vacuum expands, the energy *increases* by $\rho_V\, dV$. Hence $p = -\rho_V$, so $w = -1$.

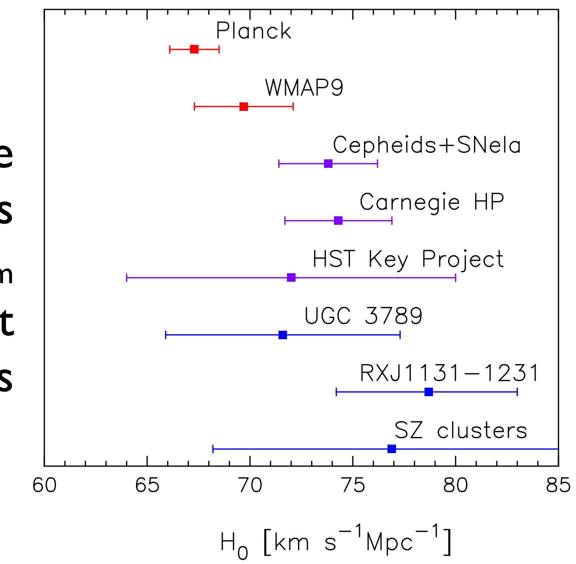


Planck 2013 results. XVI. Cosmological parameters

The main Planck anomaly is the low amplitudes at $\ell \approx$ 21-27

Planck errors are small and Planck's values for H_0 and Ω_m are rather different from WMAP's

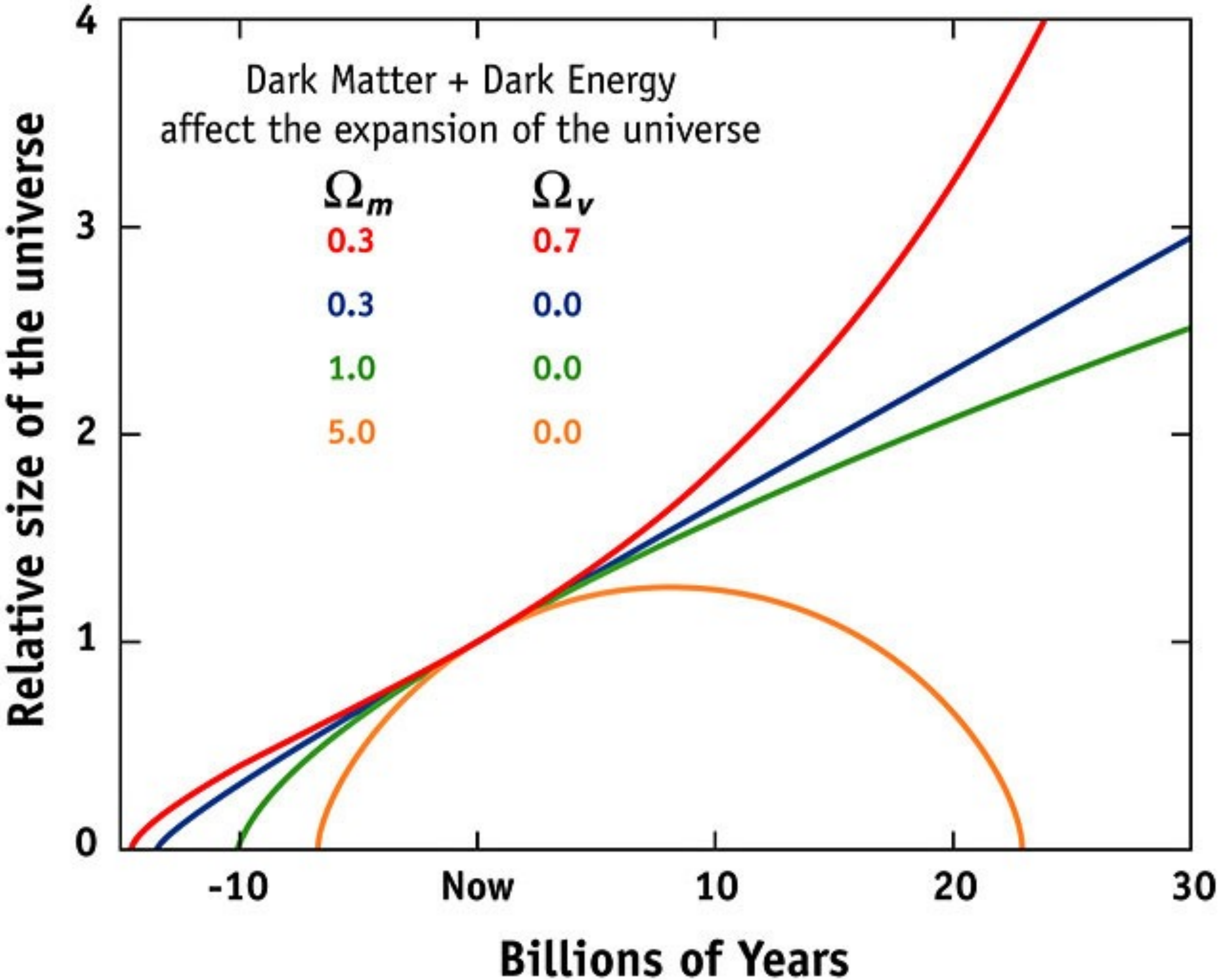
Planck Collaboration: Cosmological parameters



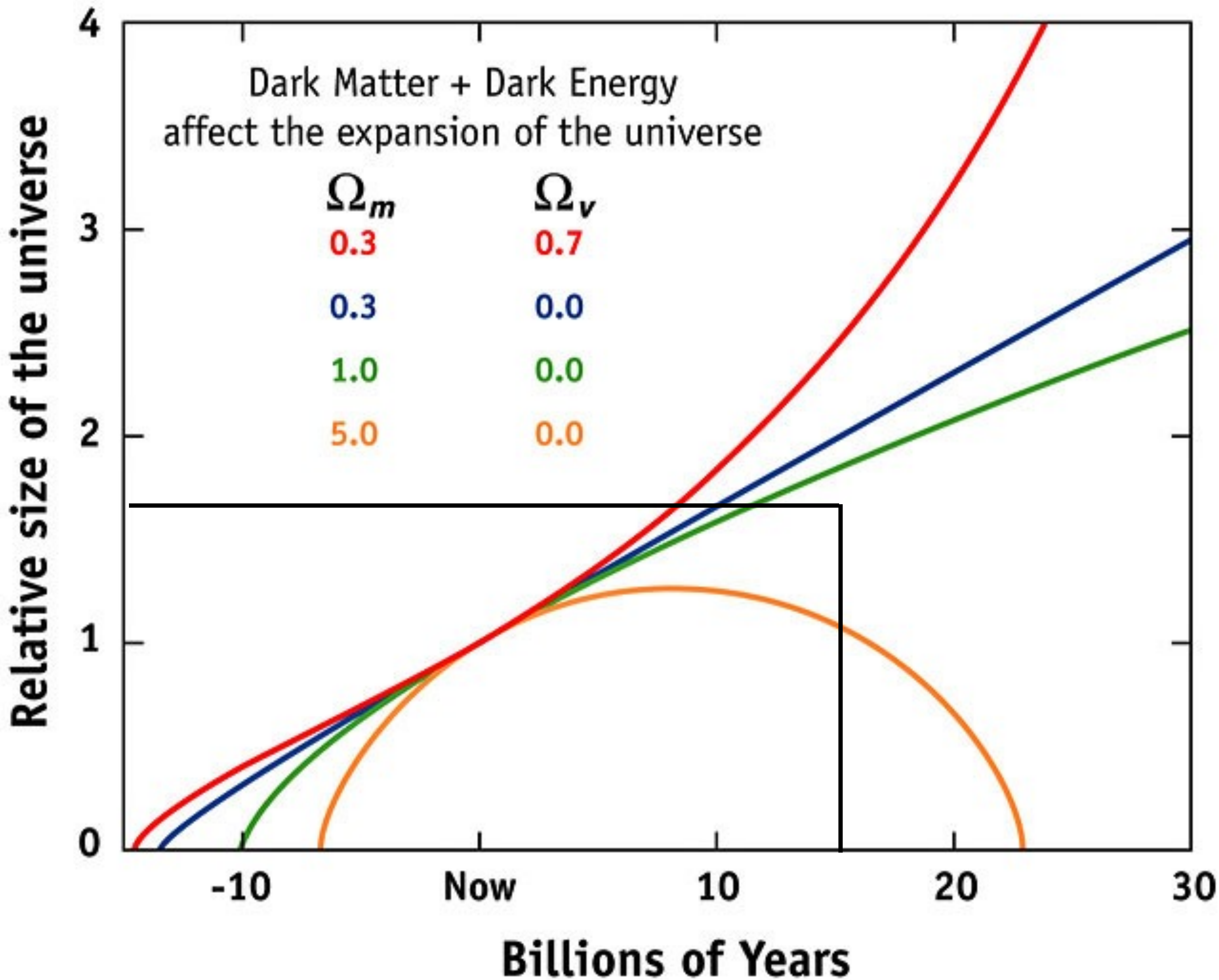
	Planck+WP		*Planck*+WP+highL		*Planck*+lensing+WP+highL		*Planck*+WP+highL+BAO	
Parameter	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.022032	0.02205 ± 0.00028	0.022069	0.02207 ± 0.00027	0.022199	0.02218 ± 0.00026	0.022161	0.02214 ± 0.00024
$\Omega_c h^2$	0.12038	0.1199 ± 0.0027	0.12025	0.1198 ± 0.0026	0.11847	0.1186 ± 0.0022	0.11889	0.1187 ± 0.0017
$100\theta_{MC}$	1.04119	1.04131 ± 0.00063	1.04130	1.04132 ± 0.00063	1.04146	1.04144 ± 0.00061	1.04148	1.04147 ± 0.00056
τ	0.0925	$0.089^{+0.012}_{-0.014}$	0.0927	$0.091^{+0.013}_{-0.014}$	0.0943	$0.090^{+0.013}_{-0.014}$	0.0952	0.092 ± 0.013
n_s	0.9619	0.9603 ± 0.0073	0.9582	0.9585 ± 0.0070	0.9624	0.9614 ± 0.0063	0.9611	0.9608 ± 0.0054
$\ln(10^{10} A_s)$	3.0980	$3.089^{+0.024}_{-0.027}$	3.0959	3.090 ± 0.025	3.0947	3.087 ± 0.024	3.0973	3.091 ± 0.025
Ω_Λ	0.6817	$0.685^{+0.018}_{-0.016}$	0.6830	$0.685^{+0.017}_{-0.016}$	0.6939	0.693 ± 0.013	0.6914	0.692 ± 0.010
σ_8	0.8347	0.829 ± 0.012	0.8322	0.828 ± 0.012	0.8271	0.8233 ± 0.0097	0.8288	0.826 ± 0.012
z_{re}	11.37	11.1 ± 1.1	11.38	11.1 ± 1.1	11.42	11.1 ± 1.1	11.52	11.3 ± 1.1
H_0	67.04	67.3 ± 1.2	67.15	67.3 ± 1.2	67.94	67.9 ± 1.0	67.77	67.80 ± 0.77
Age/Gyr	13.8242	13.817 ± 0.048	13.8170	13.813 ± 0.047	13.7914	13.794 ± 0.044	13.7965	13.798 ± 0.037
$100\theta_*$	1.04136	1.04147 ± 0.00062	1.04146	1.04148 ± 0.00062	1.04161	1.04159 ± 0.00060	1.04163	1.04162 ± 0.00056
r_{drag}	147.36	147.49 ± 0.59	147.35	147.47 ± 0.59	147.68	147.67 ± 0.50	147.611	147.68 ± 0.45

Table 5. Best-fit values and 68% confidence limits for the base ΛCDM model.

History of Cosmic Expansion for General Ω_M & Ω_Λ



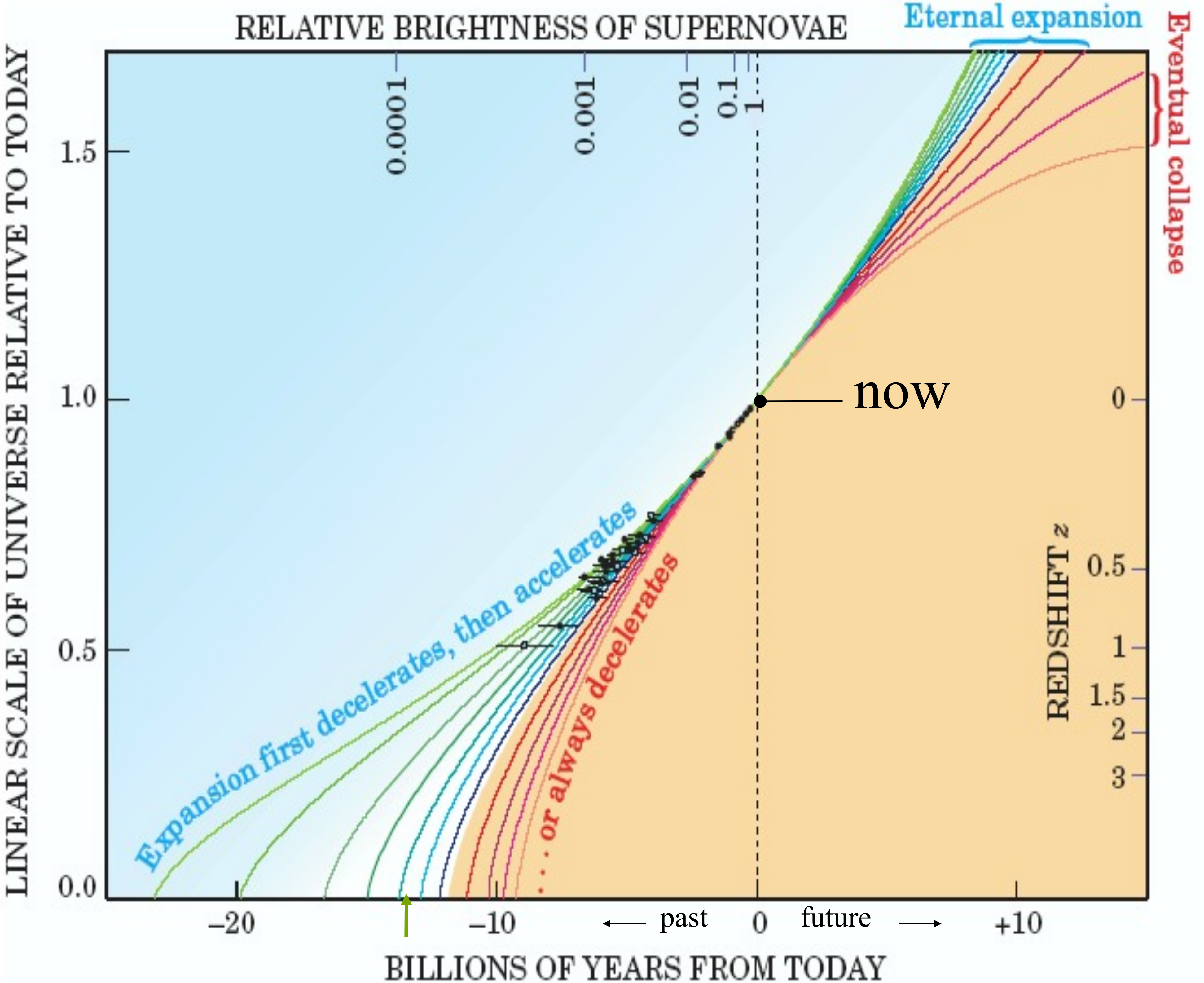
History of Cosmic Expansion for General Ω_M & Ω_Λ



History of Cosmic Expansion for $\Omega_\Lambda = 1 - \Omega_M$

With $\Omega_\Lambda = 0$ the age of the decelerating universe would be only 9 Gyr, but $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$ gives an age of 14 Gyr, consistent with stellar and radioactive decay ages

Figure 4. The history of cosmic expansion, as measured by the high-redshift supernovae (the black data points), assuming flat cosmic geometry. The scale factor R of the universe is taken to be 1 at present, so it equals $1/(1 + z)$. The curves in the blue shaded region represent cosmological models in which the accelerating effect of vacuum energy eventually overcomes the decelerating effect of the mass density. These curves assume vacuum energy densities ranging from $0.95\,\rho_c$ (top curve) down to $0.4\,\rho_c$. In the yellow shaded region, the curves represent models in which the cosmic expansion is always decelerating due to high mass density. They assume mass densities ranging (left to right) from $0.8\,\rho_c$ up to $1.4\,\rho_c$. In fact, for the last two curves, the expansion eventually halts and reverses into a cosmic collapse.



Saul Perlmutter, ***Physics Today***, Apr 2003

LCDM Benchmark Cosmological Model: Ingredients & Epochs

	List of Ingredients
photons:	$\Omega_{\gamma,0} = 5.0 \times 10^{-5}$
neutrinos:	$\Omega_{\nu,0} = 3.4 \times 10^{-5}$
total radiation:	$\Omega_{r,0} = 8.4 \times 10^{-5}$
baryonic matter:	$\Omega_{\mathrm{bary},0} = 0.04$
nonbaryonic dark matter:	$\Omega_{\mathrm{dm},0} = 0.26$
total matter:	$\Omega_{m,0} = 0.30$
cosmological constant:	$\Omega_{\Lambda,0} \approx 0.70$

	Important Epochs	
radiation-matter equality:	$a_{rm} = 2.8 \times 10^{-4}$	$t_{rm} = 4.7 \times 10^{4}$ yr
matter-lambda equality:	$a_{m\Lambda} = 0.75$	$t_{m\Lambda} = 9.8$ Gyr
Now:	$a_0 = 1$	$t_0 = 13.5$ Gyr

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

Benchmark Model: Scale Factor vs. Time

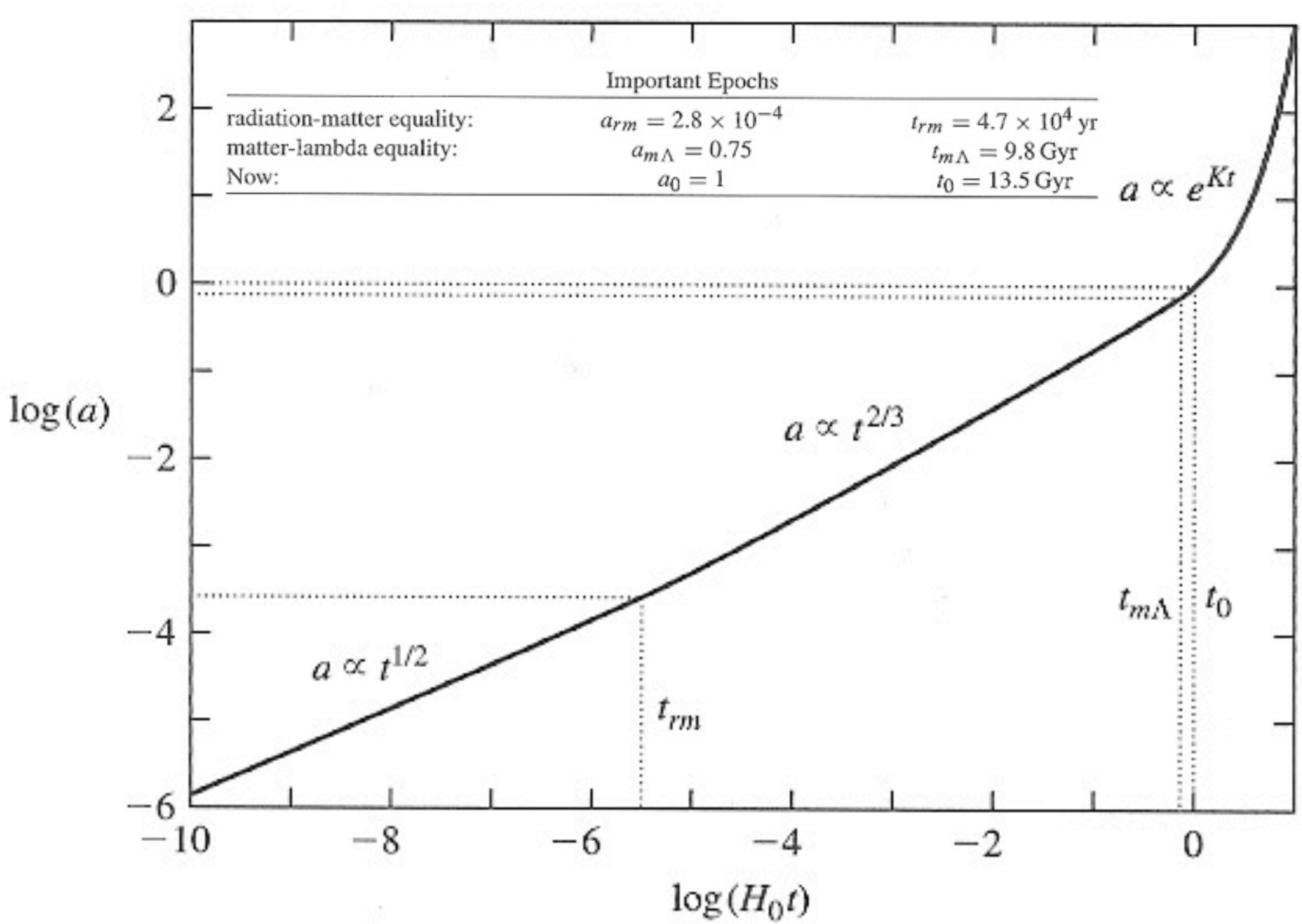
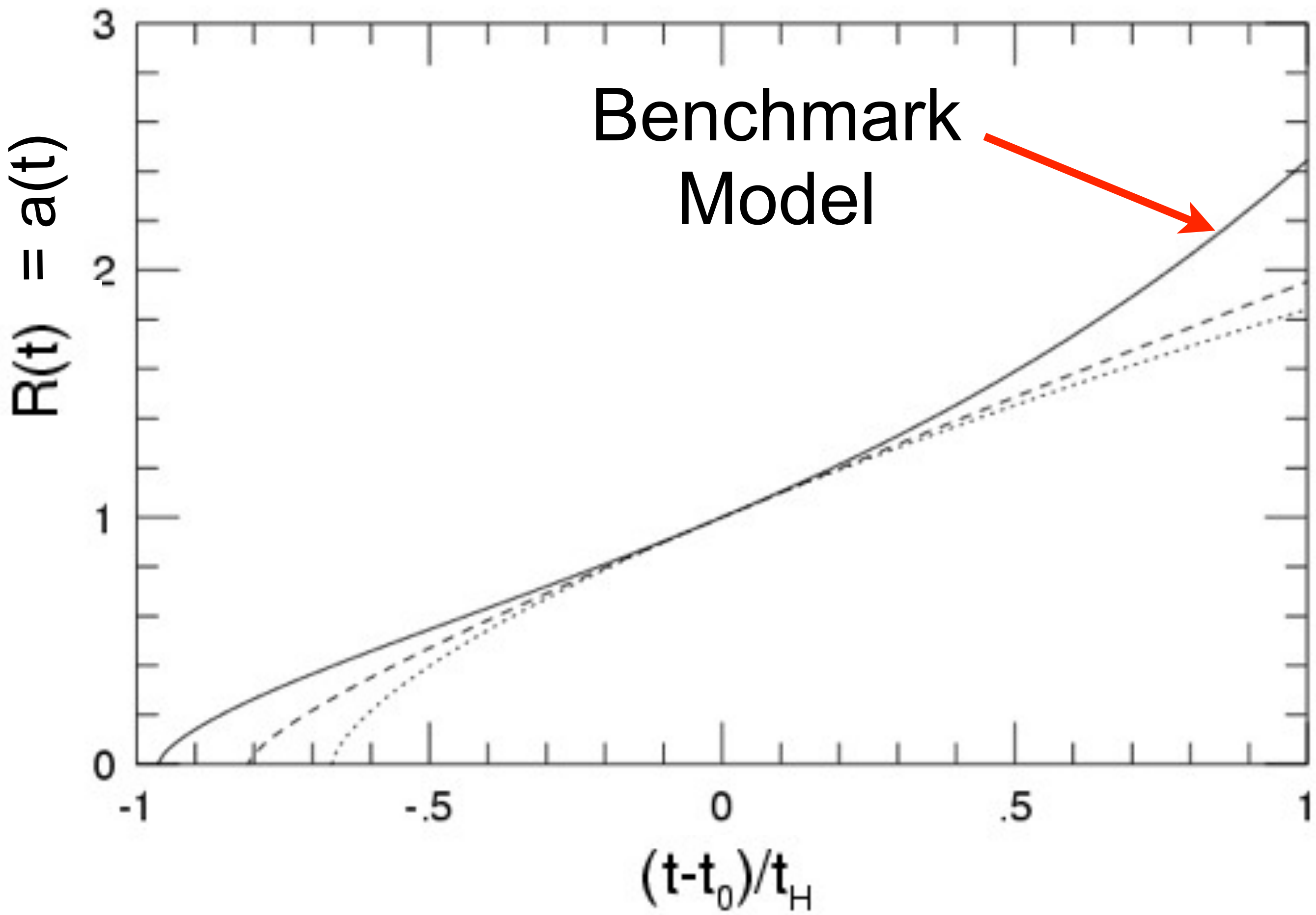
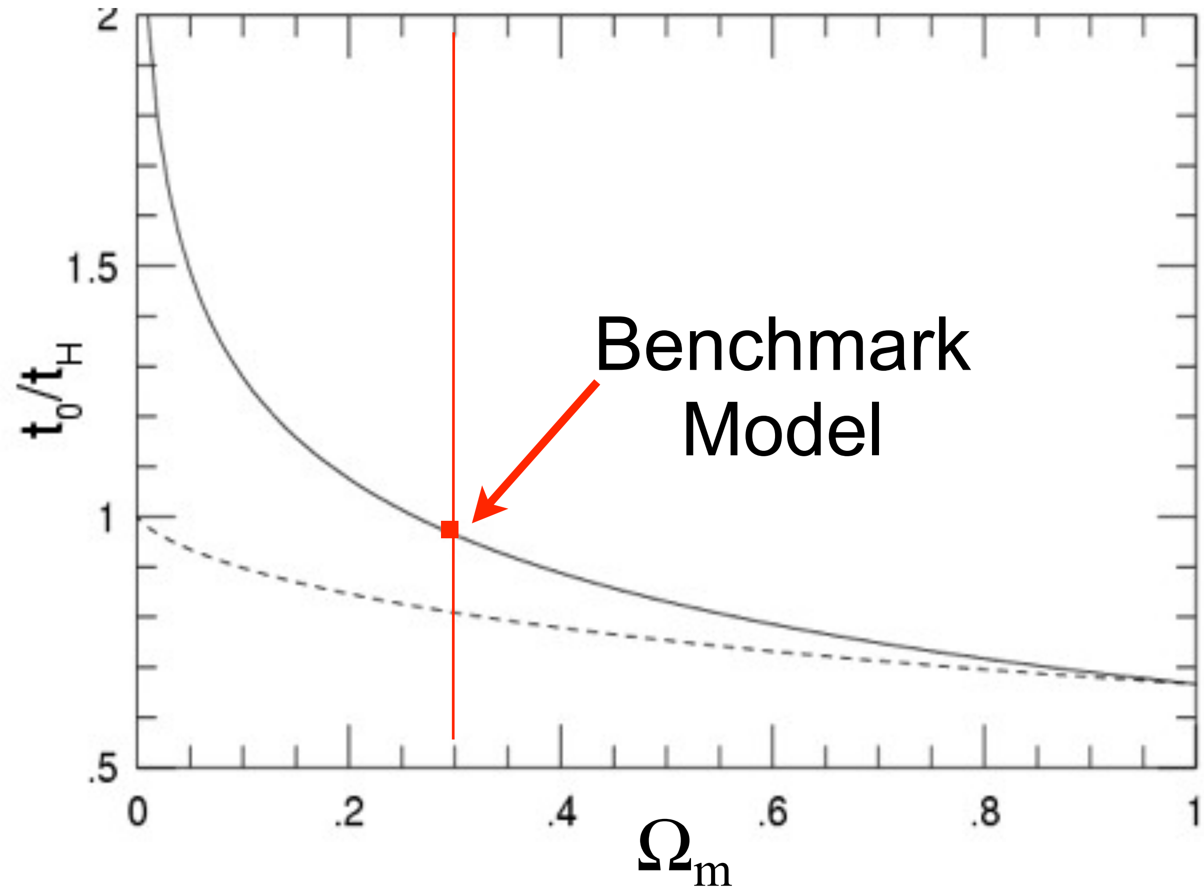


FIGURE 6.5 The scale factor a as a function of time t (measured in units of the Hubble time), computed for the Benchmark Model. The dotted lines indicate the time of radiation-matter equality, $a_{rm} = 2.8 \times 10^{-4}$, the time of matter-lambda equality, $a_{m\Lambda} = 0.75$, and the present moment, $a_0 = 1$.

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

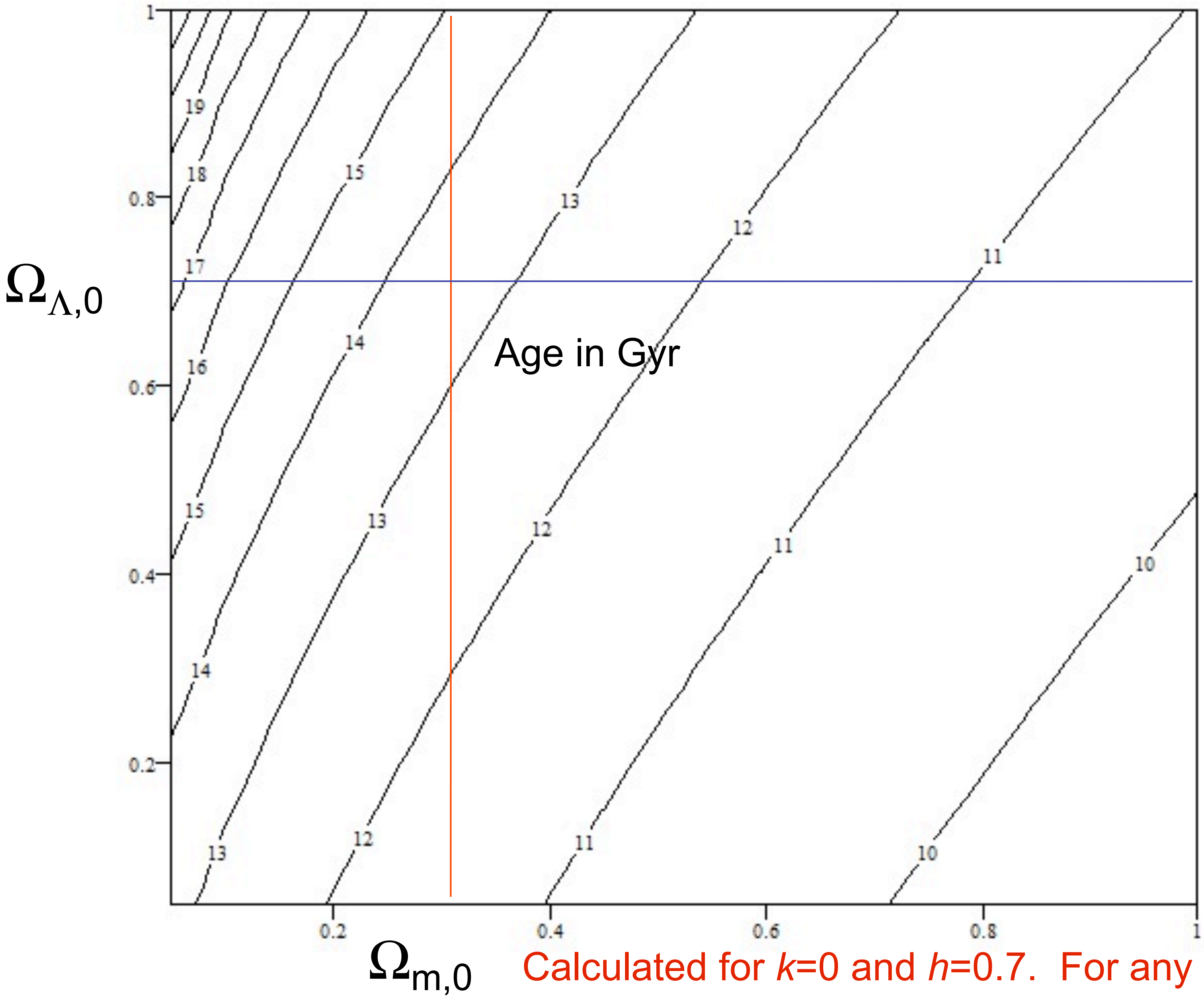
Age of the Universe t_0 in FRW Cosmologies





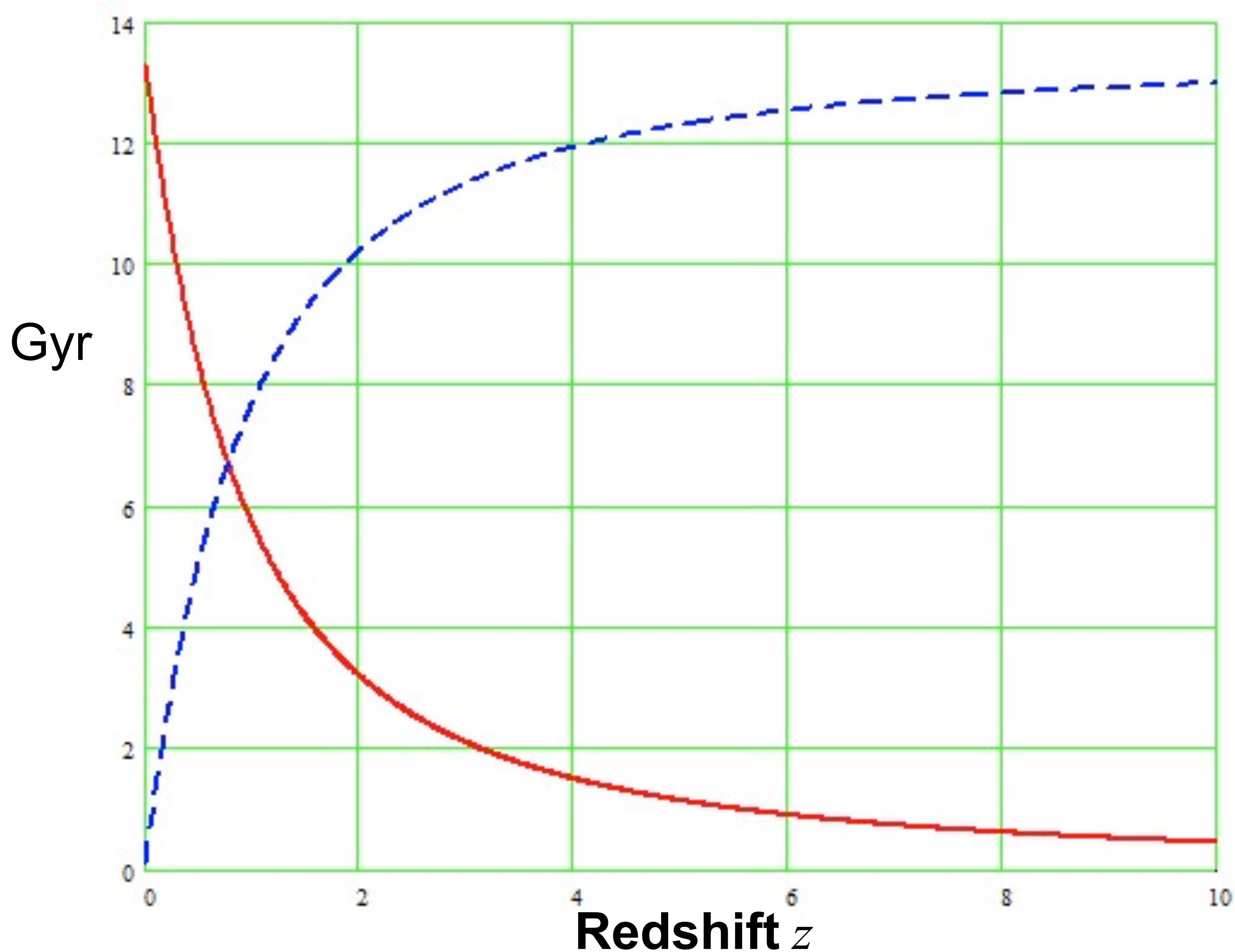
(a) Evolution of the scale factor $a(t)$ plotted vs. the time after the present $(t - t_0)$ in units of Hubble time $t_H \equiv H_0^{-1} = 9.78h^{-1}$ Gyr for three different cosmologies: Einstein-de Sitter ($\Omega_0 = 1, \Omega_\Lambda = 0$ dotted curve), negative curvature ($\Omega_0 = 0.3, \Omega_\Lambda = 0$: dashed curve), and low-Ω_0 flat ($\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$: solid curve). (b) Age of the universe today t_0 in units of Hubble time t_H as a function of Ω_0 for $\Lambda = 0$ (dashed curve) and flat $\Omega_0 + \Omega_\Lambda = 1$ (solid curve) cosmologies.

Age t_0 of the Double Dark Universe



Calculated for k=0 and h=0.7. For any other value of the Hubble parameter h, multiply the age by (h/0.7).

Age of the Universe and Lookback Time



These are for the Benchmark Model $\Omega_{m,0}$=0.3, $\Omega_{\Lambda,0}$=0.7, h=0.7.

Distances in the Expanding Universe:
Ned Wright's Javascript Calculator

Enter values, hit a button

70 H_o

0.30 $Omega_M$

0.83 z

Open | Flat

0.70 $Omega_{vac}$

General

Open sets $Omega_{vac}$ = 0 giving an open Universe [if you entered $Omega_M$ < 1]

Flat sets $Omega_{vac}$ = 1 - $Omega_M$ giving a flat Universe.

General uses the $Omega_{vac}$ that you entered.

For H_o = 70, $Omega_M$ = 0.300, $Omega_{vac}$ = 0.700, z = 0.830

- It is now 13.462 Gyr since the Big Bang.
- The age at redshift z was 6.489 Gyr.
- The light travel time was 6.974 Gyr.
- The comoving radial distance, which goes into Hubble's law, is 2868.9 Mpc or 9.357 Gly.
- The comoving volume within redshift z is 98.906 Gpc^3.
- The angular size distance D_A is 1567.7 Mpc or 5.1131 Gly.
- This gives a scale of 7.600 kpc/".
- The luminosity distance D_L is 5250.0 Mpc or 17.123 Gly.

$H_0D_L(z=0.83)$
$=17.123/13.97$
$=1.23$

1 Gly = 1,000,000,000 light years or $9.461*10^{26}$ cm.
1 Gyr = 1,000,000,000 years.
1 Mpc = 1,000,000 parsecs = $3.08568*10^{24}$ cm, or 3,261,566 light years.

Tutorial: Part 1 | Part 2 | Part 3 | Part 4
FAQ | Age | Distances | Bibliography | Relativity

Ned Wright's home page

Web app http://www.astro.ucla.edu/~wright/CosmoCalc.html

iPhone app http://itunes.apple.com/us/app/cosmocalc/id334569654?mt=8