HW #4 PHYS 129

Problem 1

(a) follow the book.

(b) (5.77 b) $1+z_{eq} = \dfrac{\Omega_m(0)}{\Omega_r(0)} \Rightarrow z_{eq} = \dfrac{\Omega_m(0)}{\Omega_r(0)} - 1 \approx 6403$

(c) $\dfrac{\Omega_m(z_\Lambda)}{\Omega_\Lambda(z_\Lambda)} = 1 \Rightarrow \dfrac{\Omega_m(0)(1+z_\Lambda)^3}{\Omega_\Lambda(0)} = 1$

$$\Rightarrow z_\Lambda = \left(\frac{\Omega_\Lambda(0)}{\Omega_m(0)}\right)^{1/3} - 1 \approx 0.31$$

(d) Now from part (a)

$$t_0 - t = \frac{1}{H_0}\int_0^\infty \frac{dz}{(1+z)\left[\Omega_m(0)(1+z)^3 + \Omega_R(0)(1+z)^4 + \Omega_\Lambda(0)\right]^{1/2}}$$

This is hard to integrate but from part (b) & (c) we have learned,

Ω_r dominates from $z=\infty$ to $z = 6403$

Ω_m " " $z = 6403$ to 0.31

& Ω_Λ " " $z = 0.31$ to 0

So we can integrate the above equation piecewise.

then.

$$H_0 t_0 = \frac{1}{\sqrt{\Omega_m(0)}} \int_{0.31}^{6403} (1+z)^{-5/2}\, dz + \frac{1}{\sqrt{\Omega_r(0)}} \int_{6403}^{\infty} (1+z)^{-3}\, dz +$$

$$+ \frac{1}{\sqrt{\Omega_\Lambda(0)}} \int_{0}^{0.31} dz$$

then the matter dominated era lasted for

$$H_0 \Delta t_m = -\frac{2}{3} \frac{1}{\sqrt{\Omega_m(0)}} \frac{1}{(1+z)^{3/2}} \Bigg|_{0.31}^{6403} = 0.799$$

$$\Rightarrow \boxed{\Delta t_m = \frac{0.799}{H_0}}$$

$$H_0 \Delta t_r = -\frac{1}{2\sqrt{\Omega_r(0)}} \frac{1}{(1+z)^2} \Bigg|_{6403}^{\infty} \approx 0.022\ 10^{-6}$$

$$H_0 \Delta t_\Lambda = +0.37$$

then $\boxed{t_0 = \Delta t_r + \Delta t_m + \Delta t_\Lambda = 1.16/H_0}$

Now $H_0 = 67.8 km/s/Mpc$

$1 Mpc = 3.09 \times 10^{19} km$

$\Rightarrow \quad H_0 = 21.9 \times 10^{-19} s^{-1}$

Then Hubble time $= \frac{1}{H_0} = 4.57 \times 10^{17} s$

$\approx 14.5 \times 10^{9}$ yrs.

thus $t_0 \approx 16$ Gyr.

We overestimate it.

Now

$\boxed{\text{Problem 3}}$

See the attached page.

For H_o = 67.8, $Omega_M$ = 0.310, $Omega_{vac}$ = 0.690, z = 0.000

- It is now 13.771 Gyr since the Big Bang.
- The age at redshift z was 13.771 Gyr.
- The light travel time was 0.000 Gyr.
- The comoving radial distance, which goes into Hubble's law, is 0.0 Mpc or 0.000 Gly.
- The comoving volume within redshift z is 0.000 Gpc^3.
- The angular size distance D_A is 0.000 Mpc or 0.000000 Gly.
- This gives a scale of 0.000 kpc/".
- The luminosity distance D_L is 0.0 Mpc or 0.000 Gly.

1 Gly = 1,000,000,000 light years or $9.461*10^{26}$ cm.
1 Gyr = 1,000,000,000 years.
1 Mpc = 1,000,000 parsecs = $3.08568*10^{24}$ cm, or 3,261,566 light years.

Tutorial: Part 1 | Part 2 | Part 3 | Part 4
FAQ | Age | Distances | Bibliography | Relativity

See the advanced and light travel time versions of the calculator.

James Schombert has written a Python version of this calculator.

Ned Wright's home page

Perkins 5.5

Total power radiated from the Sun:

$$P = 0.135 \text{ J/s-cm}^2 \times \left(4 \times \pi \times 1.5 \times 10^{13}\right)^2$$

$$= 3.82 \times 10^{26} \text{ Watt.}$$

energy flux on the surface of the Sun.

$$\phi = \frac{P}{4\pi R_s^2} = 6.19 \times 10^7 \frac{\text{J}}{\text{m}^2\,\text{s}}$$

Now Stefan Law $\Rightarrow$ $\phi = \sigma T^4$

$$\Rightarrow T \approx 5748 \text{ K}$$

Problem 4

We know that $ds^2 = -c\,dt^2 + R(t)^2\,dr^2$

Now for light $ds^2 = 0$.

then,

$$dr = c\,\frac{dt}{R(t)}$$

then,
$$\Delta r = \int dr = c\int_{t_0}^{t_1} \frac{dt}{R(t)}$$

Now for matter domination. $R(t) \propto t^{2/3}$

then
$$\Delta r = c\, t_0^{2/3} \int_{t_0}^{t_1} \frac{dt}{t^{2/3}}$$

$$= 3c\, t_0^{2/3}\left(\frac{1}{t_0^{-1/3}} - \frac{1}{t_1^{-1/3}}\right)$$

$$\boxed{\Delta r = 3c\, t_0\left[1 - \left(\frac{t_1}{t_0}\right)^{1/3}\right]}$$

Now $(1+z) = \dfrac{R(0)}{R(t)} = \left(\dfrac{t_0}{t_1}\right)^{2/3} \Rightarrow$

$$\boxed{\Delta = \frac{\Delta r}{c} = 3t_0\left[1 - \frac{1}{(1+z)^{1/2}}\right] \approx 1.26 \times t_0}$$