Homework Set 5

DUE: Tuesday February 25th

1. (10 points) Check Perkins equations 6.5, 6.8, 6.9, 6.14, and 6.15 by verifying that they are consistent with each other, and plugging in the numbers.

2. (10 points) Perkins problem 6.3.

3. (10 points) Perkins problem 6.4.

4. (10 points) Perkins problem 7.3.

HW # 5 (Solution)

(1) (6.5) ⇒ set $Q = 1.293\ \mathrm{MeV}/c^2$

& $KT = 0.80$ MeV

thus. $$\frac{N_n(0)}{N_p(0)} \approx 0.20.$$

(6.8) ⇒ Use (6.6) Here set ~~KT = 0.05~~ $t \sim 300s$.

then, $$r = \frac{N_n}{N_p} \approx 0.135$$

(6.9) ⇒ ~~Since~~ from ~~equation~~ reactions in

143 we see

$$Y = \frac{4N_{He}}{(4N_{He} + N_H)} = \frac{2r}{(1+r)}$$

plug in the numbers.

Perkins (6.3)

Freeze-out happens when

$$\frac{W}{H} \approx 1$$

Now (5.54) $\Rightarrow$ $H(t) = 1.66\, g^{*\,1/2} \dfrac{(kT)^2}{M_{PL}\hbar c^2}$

& $W = N\sigma c$

from (5.50) we see $g_\gamma = 2$.

$$N(E)\,dE = \frac{E^2\,dE/c^3}{\pi^2\hbar^3\{\exp(E/kT)-1\}}$$

then, for $E = Q \gg kT$ we have.

$$N(E)\,dE = \left(\frac{kT}{\hbar c}\right)^3 \frac{1}{\pi^2}\left(\frac{E}{kT}\right)^2 e^{-E/kT}\, d\left(\frac{E}{kT}\right)$$

then,

$$N(Q) \approx \left(\frac{kT}{\hbar c}\right)^3 \frac{1}{\pi^2} e^{-Q/kT}\left(\frac{Q}{kT}\right)^2$$

then, $W \simeq H$

$$\Rightarrow \left(\frac{kT}{\hbar c}\right)^3 \frac{\sigma c}{\pi^2} e^{-Q/kT}\left(\frac{Q}{kT}\right)^2 = 1.66\, g^{*\,1/2} \frac{(kT)^2}{M_{PL}\hbar c^2}$$

$$\Rightarrow \quad e^{-Q/KT} = (KT)\left[1.66 g^{*1/2} \frac{(\hbar c)^2}{\sigma} \frac{\pi^2}{Q^2} \frac{1}{M_{PL}c^2}\right]$$

Now we know $\hbar c$, σ, $M_{PL}c^2$, Q,

So we can solve this transcendental equation to numerically.

$$\boxed{KT \approx 0.06 \text{ MeV}}$$

<u>Perkins 6.4</u>

1 MeV = 1.6×10^{-13} J.

then, 9.4×10^{37} He produced per second.

total # of He = $2.4 \times 10^{39} \times 5$ Gyr. $\approx 1.4 \times 10^{55}$

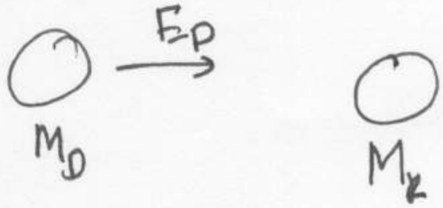
$M_{He} \approx 6.6 \times 10^{-27}$ kg.

total weight of He $\approx 9.2 \times 10^{28}$ kg

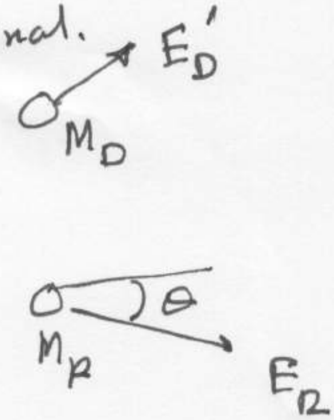
thus Helium fraction is the Sun $\approx$ 5%.

Perkins 7.3

$i \to$ initial



$f \to$ final.



then.

K.E. conservation:

$$E_D' + E_R = E_D \Rightarrow E_D' = E_D - E_R$$

Momentum conservation.

$$\vec{P}_{D_i} + \vec{P}_{R_i} = \vec{P}_{D_f} + \vec{P}_{R_f} \qquad (\vec{P}_{R_i} \to 0)$$

$$\Rightarrow \vec{P}_{D_i} - \vec{P}_{R_f} = \vec{P}_{D_f}$$

Squaring both sides:

$$|\vec{P}_{D_i}|^2 + |\vec{P}_{R_f}|^2 - 2\vec{P}_{D_i}\cdot\vec{P}_{R_f} = |P_{D_f}|^2$$

$$\Rightarrow 2M_D E_D + 2M_R E_R - 4\sqrt{M_D M_R}\sqrt{E_R E_D}\cos\theta = 2M_D(E_D - E_R)$$

$$\Rightarrow \boxed{E_R = \frac{4M_R M_D}{(M_R + M_D)^2} E_D \cos^2\theta}$$