Homework Set 6

DUE: Thursday March 13

1. (10 points) Calculate (a) the Hubble radius c/H at the time of matter-radiation equality, and (b) the mass inside the horizon then. (Use the Planck cosmological parameters $\Omega_m = 0.31$, $\Omega_\Lambda = 0.69$, $h = 0.68$.) Compare with the statements in Perkins, p. 210, and with Perkins Fig. 8.5.

2. (10 points) Perkins problem 8.5. (See Perkins Appendix C.)

3. (10 points) Perkins problem 8.8. (Perkins Table 5.1 summarizes sizes and masses of typical small galaxies and clusters.)

4. (10 points) Perkins problem 9.1.

5. (10 points) Perkins problem 9.2.

6. (10 points) Perkins problem 9.3.

7. (10 points) Perkins problem 9.10.

8. (15 points) (a) The luminosity of a typical galaxy is $\sim 2 \times 10^{10} L_\odot$, where the sun's luminosity $L_\odot = 4 \times 10^{26}$ W. The mean energy of a stellar photon is about 2 eV. Compare the photon flux from a nearby galaxy at $R \sim 1$ Mpc with that from a nearby star at $R \sim 1$ pc. (This should explain why most astronomical objects visible to the naked eye are stars.)

(b) The present luminosity density of the universe is about $10^8 L_\odot$ Mpc^{-3}. Assume that stellar light output has been fairly constant for a Hubble time, and calculate the number of photons (~ 2 eV) that have been produced by stars. Stellar energy has been produced mainly by conversion of four protons to helium, with a total release of ~ 25 MeV. Estimate the number of protons per Mpc^3 that have been converted to helium in the past Hubble time, and compare with the number of protons available $\sim \Omega_b \rho_c / m_p$.

Note: The Final Exam for Physics 129 will be Tuesday March 18 8:00-11:00 am in ISB 231. It will be an open-book, open-notes exam.

HW #6 (Problem 1)

(a) Hubble Radius = $\frac{c}{H}$

Matter radiation equality ~~$z_{eq} \approx 3403$~~

using $\Omega_r \approx 10^{-4}$ & $\Omega_m = 0.32$

we have $z_{eq} \approx 3200$.

$$H(z_{eq})^2 \simeq 2H_0^2\,\Omega_m(0)\,(1+z_{eq})^3$$

$$\Rightarrow H = \sqrt{2}\,H_0\,\sqrt{3200^3 \times 0.32}$$

$$= 1.4\times10^5\,H_0$$

(b) Thus. Hubble radius $R(z_{eq}) = 7\times10^{-6}\,\frac{c}{H_0}$

Mass inside the Horizon then;

$$M = \rho\left(\frac{4}{3}\pi R^3\right)$$

& $\rho = \frac{3H^2}{8\pi G}$

$$\Rightarrow M = \frac{H^2}{2G}R^3 = \frac{(1.4\times10^5\,H_0)^2}{2G}\left(\frac{7\times10^{-6}\,c}{H_0}\right)^3$$

$$\boxed{M = \frac{(1.4\times10^5)^2\,(7\times10^{-6}c)^3}{2GH_0}}$$

Perkins 8.5

8.38 ⟹ $$\lambda_J = v_s \left(\frac{\pi}{G\rho}\right)^{1/2}$$

Now $$v_s^2 = \frac{\partial P}{\partial \rho} = \frac{\partial}{\partial \rho}\left(w \rho c^2\right)$$ ← (5.21)

$$= w c^2 = \frac{1}{3} c^2$$

$$\Rightarrow \boxed{v_s = \sqrt{\tfrac{1}{3}}\, c}$$

Now $$\rho = \frac{3H^2}{8\pi G}$$

thus,

$$\lambda_J = \sqrt{\tfrac{1}{3}}\, c \left(\frac{8\pi^2}{3H^2}\right)^{1/2}$$

$$= \sqrt{\tfrac{1}{3}} \left(\frac{8\pi^2}{3}\right)^{1/2} \left(\frac{c}{H}\right)$$

↑ Hubble Radius.

$$\Rightarrow \boxed{\lambda_J \sim \left(\frac{c}{H}\right)}$$

Perkins 8.8

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

Plugging in values gives $\sim 2.5 \times 10^5$ m/s & 4×10^5 m/s for galaxy and cluster respectively.

Now $v_{rms} = \sqrt{\frac{3KT}{m}}$

$$= \sqrt{\frac{3KT}{mc^2}}\, c$$

Now for cmb $3KT \approx 7 \times 10^{-4}$ eV

plugging these in gives the desired answer.

Perkins 9.1

from (9.3) $\Rightarrow$ $\frac{dN}{dE} \propto E^{-(1+s)}$

$$s = -\ln P / \ln(1+\alpha)$$

Here $P = 0.8$ $\quad \alpha = 0.2$

So $s = 1.22$

Perkins 9.3

$$dx = \frac{dE}{a + bE}$$

$$\Rightarrow \quad x = \int_{1000\,\text{GeV}}^{5000\,\text{GeV}} \frac{dE}{a + bE}$$

$$= \frac{1}{b} \ln(a + bE) \Big|_{1\,\text{TeV}}^{5\,\text{TeV}}$$

~~$\approx 7.62 \times 10^5\,g\,cm^{-2}$~~ $= 7.62 \times 10^5\,g\,cm^{-2}$

~~$\approx 2.5\,km$~~

Here for muon $b \sim 10^{-6}\,gm\,cm^{-2}$

then,

$$x = \frac{7.62 \times 10^5}{3 \times 10^5}\,km \approx 2.5\,km.$$

Perkins 9.10

~~P. 2~~258: for muon neutrino.

$$\sigma \simeq 10^{-34} cm^2$$

$$m_H \simeq 1.6 \times 10^{-24} gm. \leftarrow \text{Hydrogen mass.}$$

$$d = 6.4 \times 10^5 cm$$

then, the fraction of neutrino interaction.

$$\sigma \frac{\rho}{m_H} d \times 2$$

$$= 2.8 \times 10^{-4}$$

Problem 8

(a) ~~Proton~~ photon flux ~~∝~~ $\frac{L}{4\pi R^2}$

then $\frac{F_{galaxy}}{F_{sun}} = \frac{2 \times 10^{10}}{(10^6)^2} \simeq 2 \times 10^{-2}$

(b) $L_\odot \simeq 4 \times 10^{26} W = \frac{4}{1.6} \times 10^{45} eV/s$

then, # photon $= 1.25 \times 10^{53} /s/Mpc^3$

protons $=$ ~~$\frac{4}{1.6} \times 10^{45}$~~ $4 \times \frac{4 \times 10^{53}}{1.6} \frac{1}{25 \times 10^3} \times \left(\frac{1}{H}\right) /Mpc^3$

$$= 5 \times 10^{71} /Mpc^3$$