HW # 3

Problem 1

(a) $$\frac{d\tau}{d(\ln\mu)} = b_0$$

$$\Rightarrow 8\pi^2 \frac{d}{d(\ln\mu)}\left(\frac{1}{g^2}\right) = b_0$$

$$\Rightarrow -\frac{16\pi^2}{g^3}\frac{dg}{d(\ln\mu)} = b_0$$

$$\Rightarrow \boxed{\frac{dg}{d\ln(\mu)} = -\frac{b_0 g^3}{16\pi^2}}$$

(b) $$\frac{d\tau}{d(\ln\mu)} = b_0 \Rightarrow \tau(\mu) - \tau(M) = b_0\left[\ln(\mu) - \ln(M)\right]$$

$$\Leftrightarrow \boxed{\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(M)} + \frac{b_0}{2\pi}\ln\left(\frac{\mu}{M}\right)}$$

(c) $$\alpha_1(\mu) = \alpha_2(M) \Rightarrow$$

$$\frac{1}{\alpha_1(M_Z)} + \frac{b_0^1}{2\pi}\ln\left(\frac{\mu}{M_Z}\right) = \frac{1}{\alpha_2(M_Z)} + \frac{b_0^2}{2\pi}\ln\left(\frac{\mu}{M_Z}\right)$$

$$\ln\left(\frac{\mu}{M_Z}\right) = \frac{2\pi}{b_0^2 - b_0^1}\left\{\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)}\right\}$$

$$= \frac{2\pi \times 30.5}{7.16} \Rightarrow \boxed{\mu \approx 4\times10^{13}\,\text{GeV}}$$

HW#3

~~$\alpha_3(\mu) \approx 0.029$~~

& ~~$\alpha_1(\mu) = \alpha_2(\mu) \approx 0.023$~~

$$\boxed{\begin{aligned} \frac{1}{\alpha_1(\mu)} = \frac{1}{\alpha_2(\mu)} &\approx \frac{1}{43.4} \\ \frac{1}{\alpha_3(\mu)} &\approx 34.2 \end{aligned}}$$

Long time ago I plotted these ~~lot~~ lines. It's attached at the end.

Problem 3.4

When pion π^- at rest $L=0$, $J=1$

thus.

L.H.S. $(-1)^L P_{\pi^-} P_d = (-1)^0(-1)(+1) = -1$

R.H.S.

$(-1)^L P_n P_n P_{\pi^0}$

Now since neutrons are fermions the wave function must be antisymmetric under neutron exchange. thus $L=1$

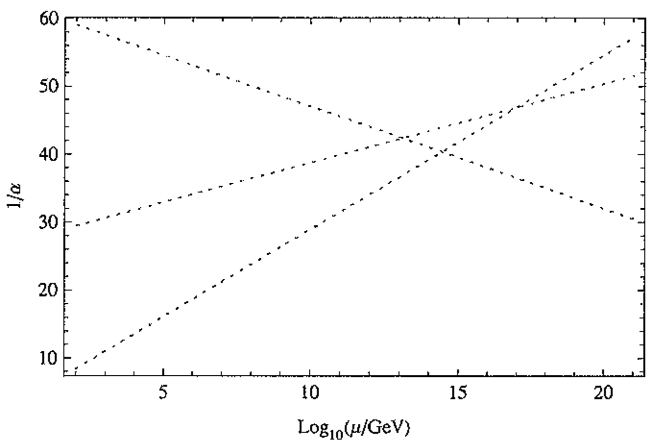
therefore

$$\text{R.H.S.} = (-1)^1(+1)(+1)(-1) = +1$$

$\boxed{\text{Parity violation}}$

In[21]:=
```
(*Standard Model *)gSM = Plot[{alphaInv1[x, alpha3Mz, b3sm],
   alphaInv1[x, alpha2Mz, b2sm], alphaInv1[x, 5 / 3 * alpha1Mz, b1sm]},
  {x, 2, 21}, Axes → False, Frame → True, FrameLabel → {"Log10 (μ/GeV)", "1/α"},
  PlotStyle → {{Gray, Dotted}, {Gray, Dotted}, {Gray, Dotted}}]
```

Out[21]=



In[22]:=
```
stichedAlpha1[alphaMz_, b_List, thr_List] :=
 Module[{sm, alphaMSSM, mssm, xThr},
  xThr = Log[10, thr];
  sm = alphaInv1[x, alphaMz, b[[1]], thr[[1]]];
  alphaMSSM = 1 / alphaInv1[xThr[[2]], alphaMz, b[[1]], thr[[1]]];
  mssm = alphaInv1[x,
    alphaMSSM, b[[2]], thr[[2]]];
  Piecewise[{
    {sm, x < xThr[[2]]},
    {mssm, xThr[[2]] ≤ x}
   }]
 ]
```

In[23]:=
```
stichedAlpha2[alphaMz_, b_List, thr_List] :=
 Module[{sm, alphaMSSM, mssm, alphaMSSM4hd, mssm4hd, xThr},
  xThr = Log[10, thr];
  sm = alphaInv1[x, alphaMz, b[[1]], thr[[1]]];
  alphaMSSM = 1 / alphaInv1[xThr[[2]], alphaMz, b[[1]], thr[[1]]];
  mssm = alphaInv1[x,
    alphaMSSM, b[[2]], thr[[2]]];
  alphaMSSM4hd = 1 / alphaInv1[xThr[[3]],
     alphaMSSM, b[[2]], thr[[2]]];
  mssm4hd = alphaInv1[x,
    alphaMSSM4hd, b[[3]], thr[[3]]];
  Piecewise[{
    {sm, x < xThr[[2]]},
    {mssm, xThr[[2]] ≤ x ≤ xThr[[3]]},
    {mssm4hd, xThr[[3]] ≤ x}
   }]
 ]
```

HW # 3

Perkins 3.9

Back of the book!

Problem 6

(a) Quarks have spin $\frac{1}{2}$ thus.

Meson spins are $= \frac{1}{2} + \frac{1}{2} = 1 \leftarrow$ vector meson

& $\frac{1}{2} - \frac{1}{2} = 0 \leftarrow$ scalar meson.

(b) Adding the first two quarks get me,

$|\frac{1}{2}\rangle|\frac{1}{2}\rangle \Rightarrow |1\rangle$ & $|0\rangle$

& then, $|1\rangle|\frac{1}{2}\rangle \longrightarrow |\frac{3}{2}\rangle$ & $|1/2\rangle$ //

also $|0\rangle|1/2\rangle \longrightarrow |1/2\rangle$ ✓

Problem 7

$$\frac{I_\mu(t)}{I_\mu(0)} = 1 - \sin^2 2\theta \sin^2\left[\frac{(E_3 - E_2)t}{2}\right] \quad \text{----} \ (1)$$

Now $E_i = p + \frac{m_i^2}{2p}$ from eq. (4.9)

then, $E_3 - E_2 = \frac{1}{2|\vec{p}|}\left[m_3^2 - m_2^2\right]$

Since $m^2 \ll E^2$ thus $E^2 \approx |\vec{p}|^2 c^2$

then.

$$\frac{I_\mu(t)}{I_\mu(0)} \simeq 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E} t\right)$$

Now $t = \frac{L}{c}$ also notice to fix the unit in (1) we need an $\hbar$ in the denominator.

thus.

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \cdot \sin^2 \theta \left(\frac{1}{4\hbar c}\frac{\Delta m^2 L}{E}\right)$$

Now $\hbar c = 0.197$ GeV fm $=$ ~~0.197 MeV km~~

~~$= 0.197 \times 10^{+9}$ eV fm~~

$\hbar c = 0.197 \text{ GeV fm}$

$= 0.197 \times 10^{-6} \text{ eV m}$

$= 0.197 \times 10^{-9} \text{ eV Km}$

then. $4\hbar c = 0.788 \times 10^{-9} \text{ eV km}.$

then. $\frac{1}{4\hbar c} = 1.27 \times 10^{9} \text{ eV}^{-1} \text{ Km}^{-1}$

$\Delta m^2 \rightarrow \text{eV}^2$ & $E \rightarrow \text{GeV}$

$\rightarrow \text{eV}(10^{-9} \text{GeV})$

Therefore

we have.

$$P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - \sin^2 2\theta \sin^2 \left\{ 1.27 \frac{(\Delta m/\text{eV})^2 (L/\text{km})}{(E/\text{GeV})} \right\}$$

<u>Perkins 4.1</u>

$\phi_\mu = 250\ m^{-2} s^{-1}$

Rate of ionization $R = 2.5\ MeV \frac{cm^2}{gm}$

$= 0.25\ MeV\ kg^{-1}\ m^2$

then

total dose $= \phi_\mu \times R \times Year.$

$= 3.2 \times 10^{-4}$ gray

$= 0.032$ rad.

One tenth of the natural dose.

Now $1\ MeV \approx 2 \times 10^{-30}\ kg$

~~So we have about 60×10^{30} protons.~~

~~let the surface area $\approx 2\ m^2$~~

~~Then # of years~~

~~then. $R_p = 2 \times 10^{33}\ MeV \frac{m^2}{kg}$~~

thus there's about 10^{30} MeV/kg radiation from proton decay. Now

Then the proton lifetime $= \frac{10^{30} MeV/Kg}{300\ rad} \times 80 yrs.$

$>$ 10 age of the universe.

You can do even better by considering the universe.