Homework Set 1

DUE: Thursday April 24

1. For a flat universe with $\Omega_{m,0} < 1$ and positive cosmological constant $\Omega_{\Lambda,0} = 1 - \Omega_{m,0}$, the density contributions of the matter and cosmological constant are equal when the scale factor has the value $a_{m\Lambda} = (\Omega_{m,0}/\Omega_{\Lambda,0})^{1/3}$. This equals 0.75 for the Benchmark Model: $\Omega_{m,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$. Show that for this case the Friedmann equation can be integrated to give the expression

$$H_0 t = \frac{2}{3\sqrt{1-\Omega_{m,0}}} \ln[y^{3/2} + \sqrt{1+y^3}] \,,$$

where $y \equiv a/a_{m\Lambda}$. Show that for $a \ll a_{m\Lambda}$, this reduces to

$$a(t) \approx (\frac{3}{2}\sqrt{\Omega_{m,0}} H_0 t)^{2/3} \,,$$

and for $a \gg a_{m\Lambda}$, it reduces to

$$a(t) \approx a_{m\Lambda} \exp(\sqrt{1-\Omega_{m,o}} H_0 t) \,.$$

Show finally that the age of the universe today in this case is

$$t_0 = \frac{2}{3H_0\sqrt{1-\Omega_{m,o}}} \ln\left[\frac{\sqrt{1-\Omega_{m,o}}+1}{\sqrt{\Omega_{m,0}}}\right] \,,$$

and that for the Benchmark Model this is $t_0 = 0.964 H_0^{-1}$.

2. Geometry. (a) Show that if $k = 0$ and the scale factor a grows as $t^{2/3}$, the apparent angular sizes of distant objects of the same linear size have a minimum at $z = 1.25$. (b) Consider a galaxy of physical (visible) size 5 kpc. What angle would this galaxy subtend if situated at redshift 0.1? 1? 5? Do the calculation in a flat universe, first with zero cosmological constant, and then in the Benchmark Model with $\Omega_{m,0} = 0.3$. You are welcome to use Ned Wright's Cosmology Calculator, at http://www.astro.ucla.edu/∼wright/CosmoCalc.html

3. Short problems:

(a) If a neutrino has mass m_ν and decouples at $T_{\nu d} \sim 1$ MeV, show that the contribution of this neutrino and its antiparticle to the cosmic density today is (Dodelson Eq. 2.80)

$$\Omega_\nu = \frac{m_\nu}{94 h^2 \mathrm{eV}} \,.$$

(b) Verify that $\eta_b \equiv n_b/n_\gamma$ is given by (Dodelson Eq. 3.11)

$$\eta_b = 5.5 \times 10^{-10} \left(\frac{\Omega_b h^2}{0.020} \right) .$$

(c) Verify the time-temperature relation (Dodelson Eq. 3.30)

$$t = 132 \, \mathrm{sec} \, (0.1 \mathrm{MeV}/T)^2 \, .$$

4. Suppose that the neutron decay time were $\tau_n = 89$ s instead of $\tau_n = 890$ s, with all other physical parameters unchanged. Estimate Y_p, the primordial mass fraction of nucleons in ^{4}He, assuming that all available neutrons are incorporated into ^{4}He.

5. Suppose that there were no baryon asymmetry so that the number density of baryons exactly equaled that of anti-baryons. Determine the final relic density of (baryons + anti-baryons). At what temperature is this relic density reached?

6. Give the tentative title of your term project, and list the key references you intend to consult. (I'll be glad to help with this. I can meet with you after class on Tuesday or at 2 pm Wednesday April 16 and 23. But note that I will be away Thursday afternoon April 17 after class.)