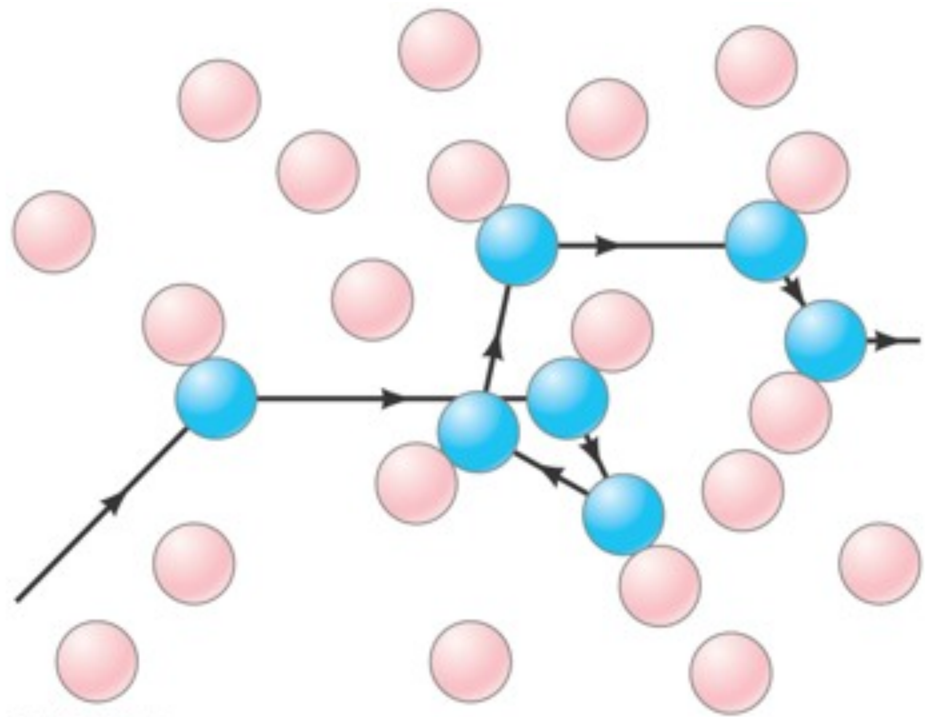
Physics 5D - Lecture 3

Mean Free Path, Internal Energy, Heat

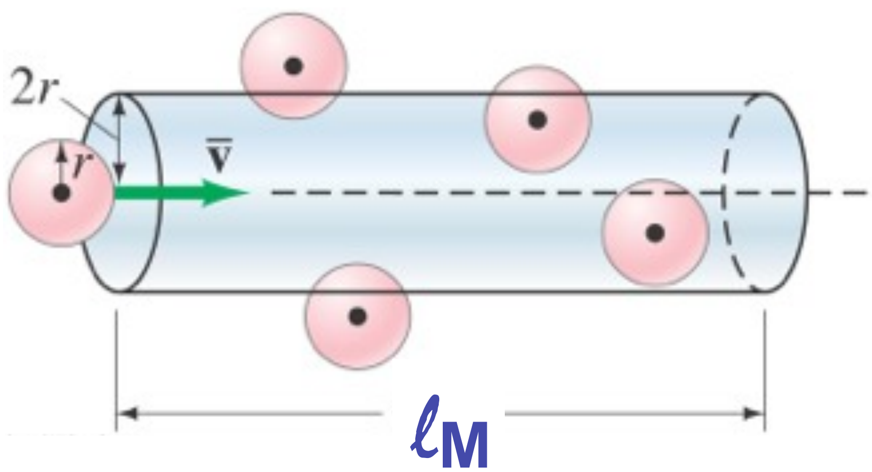
18-6 Mean Free Path

Because of their finite size, molecules in a gas undergo frequent collisions. The average distance a molecule travels between collisions is called the **mean free path**.



18-6 Mean Free Path

The mean free path can be calculated easily, assuming that only one of the molecules is moving. The mean distance ℓ_M before collision is then the distance such that the volume of the cylinder that the moving particle sweeps out = $\pi (2r)^2 \ell_M$ = V/N = the average volume per molecule.



$$\ell_M = \frac{1}{4\pi r^2 (N/V)}$$

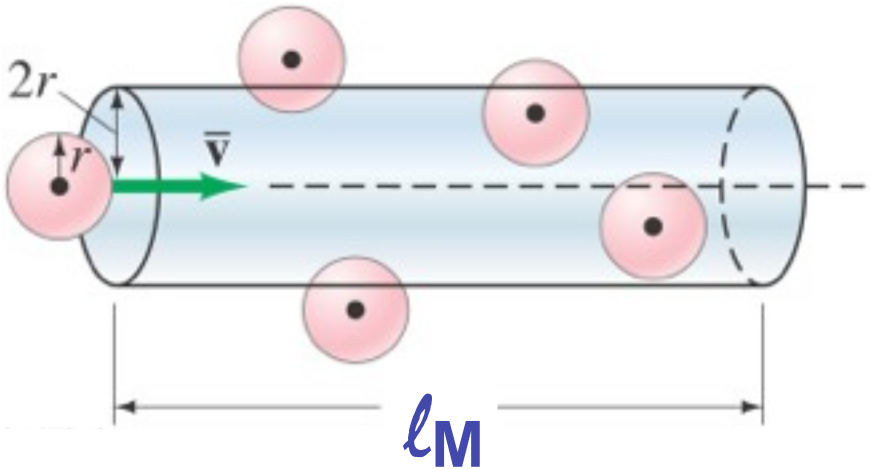
The mean free time is then

$$t_M = \ell_M / \bar{v}$$

18-6 Mean Free Path

The mean free path can be calculated, given the average speed, the density of the gas, the size of the molecules, and the relative speed of the colliding molecules. The result, now including the motion of all the particles, is changed by $\sqrt{2}$:

$$\ell_M = \frac{1}{4\pi\sqrt{2}r^2(N/V)}.$$



Question: Estimate the mean free path of air molecules at standard temperature and pressure (STP: 0°C, 1 atm). The diameter of O_2 and N_2 molecules is about 3 x 10^{-10} m.

Answer: $$\ell_M = \frac{1}{4\pi\sqrt{2}r^2(N/V)},$$ **and**

$$N/V = \frac{6.02\times10^{23}}{22.4\times10^{-3}\ m^3} = 2.69\times10^{25}\ m^{-3}$$

$$\text{so}\ \ell_M = \frac{1}{17.7\ (1.5\times10^{-10}\ m)^2\ 2.69\times10^{25}\ m^{-3}}$$

$$= 0.9\times10^{-7}\ m \approx 10^{-6}\ m = 1\ \mu m = 1\ \text{micron}$$

How can you increase the mean free path of air molecules in a closed container?

A. Increase the volume V

B. Decrease the temperature T

C. Both A and B

How can you increase the mean free path of air molecules in a closed container?

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The mean free path is $\ell_M = \dfrac{1}{4\pi\sqrt{2}r^2(N/V)}.$

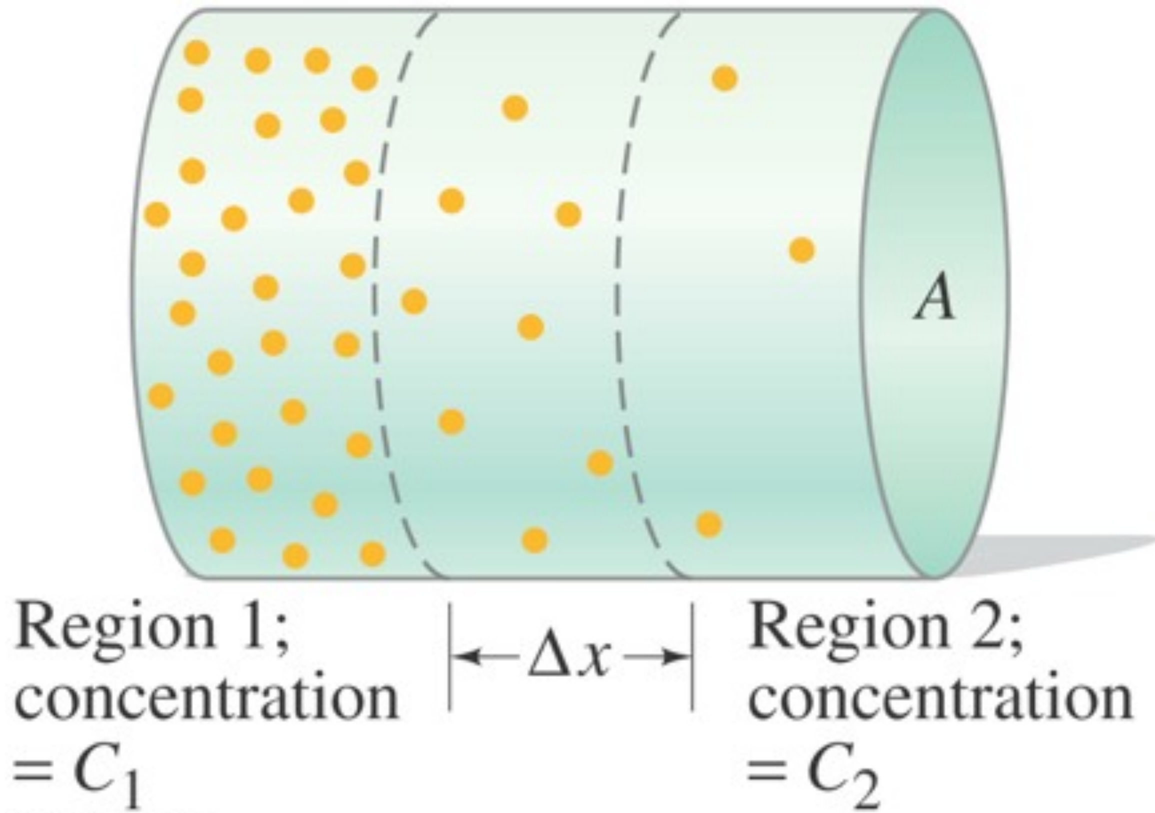
It doesn't depend on temperature, but increasing V increases ℓ_M.

18-7 Diffusion

Even without stirring, a few drops of dye in water will gradually **spread** throughout. This process is called **diffusion**.

18-7 Diffusion

Diffusion occurs from a region of **high** concentration to a region of **lower** concentration.



18-7 Diffusion

The **rate** of diffusion is given by:

$$J = DA\frac{dC}{dx}.$$

In this equation, D is the **diffusion constant**.

TABLE 18–3 Diffusion Constants, D (20°C, 1 atm)

Diffusing Molecules	Medium	D (m^2/s)
H_2	Air	6.3×10^{-5}
O_2	Air	1.8×10^{-5}
O_2	Water	100×10^{-11}
Blood hemoglobin	Water	6.9×10^{-11}
Glycine (an amino acid)	Water	95×10^{-11}
DNA (mass 6×10^6 u)	Water	0.13×10^{-11}

18-7 Diffusion

Guess how long it might take for ammonia (NH_3) to be detected 10 cm from a bottle after it is opened, assuming only diffusion is occurring.

A. 1 s

B. 10 s

C. 100 s

D. 1000 s

E. 10,000 s

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18-7 Diffusion

Example 18-9: Diffusion of ammonia in air.

Estimate how long it takes for ammonia (NH_3) to be detected 10 cm from a bottle after it is opened, assuming only diffusion is occurring.

Answer: The diffusion rate J = # molecules N crossing area A in time t, i.e. $J = N/t$, so $t = N/J$.

Using $J = DA\,\Delta C/\Delta x$, $t = (N/DA)(\Delta x/\Delta C)$. Ammonia is between H_2 and O_2 in size, so $D \approx 4\text{x}10^{-5}\ \text{m}^2/\text{s}$. Here N = (average concentration $\bar{C}$)/V = $\bar{C}/A\,\Delta x$. $\bar{C} = ½C$ and $\Delta C = C$. Then $t = (C/\Delta C)(\Delta x)^2/D$ or

$t = ½(\Delta x)^2/D = ½(0.1\text{m})^2/(4\text{x}10^{-5}\ \text{m}^2/\text{s}) = 125\ \text{s}$.

18-7 Diffusion

We just found that the time for diffusion is related to the distance by $t = \frac{1}{2}(\Delta x)^2/D$, or equivalently $\Delta x \propto t^{1/2}$. Why should diffusion work this way?

Consider the 1-dimensional example of a particle that can move one step right or left per unit time:

After t time intervals, it has gone

$$\Delta x = \overbrace{(\pm 1 \pm 1 \pm 1 \ldots \pm 1)}^{t} \text{ steps, with } \overline{\Delta x} = 0. \text{ But}$$

$$(\overline{\Delta x})^2 = (\pm 1 \pm 1 \pm 1 \ldots \pm 1)^2 = (\pm 1)^2 + (\pm 1)^2 + (\pm 1)^2 \ldots + (\pm 1)^2$$

$= t$ steps (cross-terms vanish!)

19-1 Heat as Energy Transfer

We often speak of **heat** as though it were a **material** that flows from one object to another; it is not. Rather, it is a form of **energy transfer**.

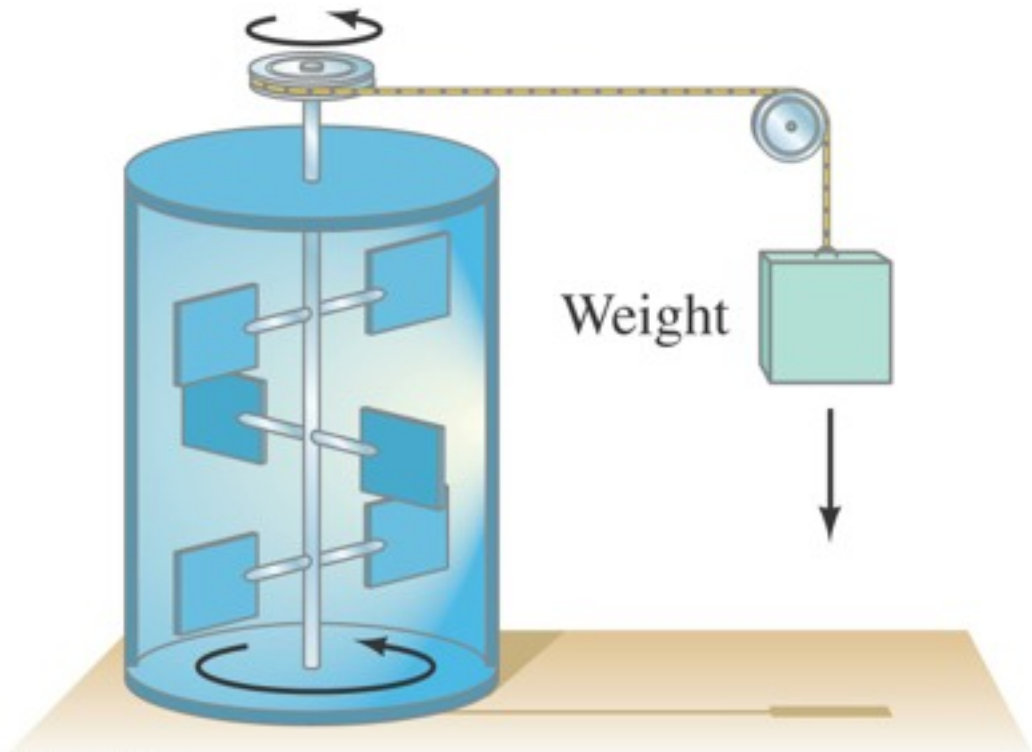
Unit of heat: **calorie** (cal)

1 **cal** is the amount of **heat** necessary to raise the **temperature** of 1 g of water by **1 Celsius degree**.

Don't be fooled—the calories on our food labels are really **kilocalories** (kcal or Calories), the heat necessary to raise 1 kg of water by 1 Celsius degree.

19-1 Heat as Energy Transfer

If **heat** is a form of **energy**, it ought to be possible to equate it to other forms. The experiment below found the **mechanical equivalent of heat** by using the falling weight to heat the water:



4.186 J = 1 cal

4.186 kJ = 1 kcal

James Prescott Joule
1818-1889

19-1 Heat as Energy Transfer

Definition of **heat**:

Heat is energy transferred from one object to another because of a difference in temperature.

The realization that heat is a form of energy, and that energy is conserved, is largely due to Joule and two Germans who trained as physicians, Julius von Mayer and Herman von Helmholz, 1841-7.

JULIUS VON MAYER

Suppose you throw caution to the wind and eat too much ice cream and cake on the order of 500 Calories. To compensate, you want to do an equivalent amount of work climbing stairs or a mountain. How much total height must you climb?

A. 30 m

B. 300 m

C. 3 km

D. 30 km

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A. 30 m

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D. 30 km

500 Calories = 500 (4186 J)

$\approx 2 \times 10^6$ J = mgh

= (70 kg)(10 m/s^2) h

h = 2×10^6 J/ (700 kg m/s^2)

$\approx 3 \times 10^3$ m = 3 km

Note: Your brain, 2% of your body weight, uses about 20% of your energy, ~ 500 Calories per day.

19-2 Internal Energy

The **sum total** of all the energy of all the molecules in a substance is its **internal** (or thermal) **energy**.

Temperature: measures molecules' average **kinetic energy**

Internal energy: **total** energy of all molecules

Heat: **transfer** of energy due to difference in temperature

19-2 Internal Energy

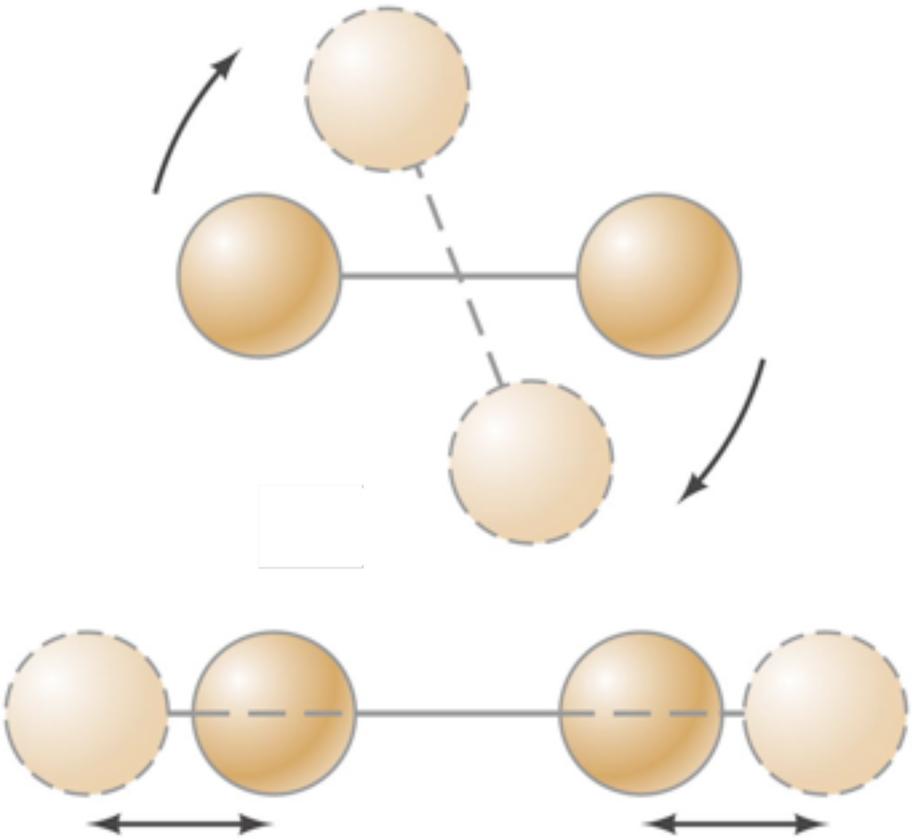
Internal energy of an **ideal** monatomic gas:

$$E_{\text{int}} = N\left(\tfrac{1}{2}m\overline{v^2}\right).$$

But since we know the average kinetic energy in terms of the temperature, we can write:

$$E_{\text{int}} = \tfrac{3}{2}NkT.$$

19-2 Internal Energy



If the gas is **molecular** rather than **atomic**, then **rotational** and **vibrational** kinetic energy need to be taken into account as well. (We'll come back to this next week.)

19-3 Specific Heat

TABLE 19–1 Specific Heats
(at 1 atm constant pressure and 20°C unless otherwise stated)

Substance	Specific Heat, c kcal/kg · C° (= cal/g · C°)	Specific Heat, c J/kg · C°
Aluminum	0.22	900
Alcohol (ethyl)	0.58	2400
Copper	0.093	390
Glass	0.20	840
Iron or steel	0.11	450
Lead	0.031	130
Marble	0.21	860
Mercury	0.033	140
Silver	0.056	230
Wood	0.4	1700
Water		
Ice (−5°C)	0.50	2100
Liquid (15°C)	1.00	4186
Steam (110°C)	0.48	2010
Human body (average)	0.83	3470
Protein	0.4	1700

The amount of heat required to change the temperature of a material is proportional to the **mass** and to the **temperature change**:

$$Q = mc\,\Delta T.$$

The **specific heat**, c, is characteristic of the material. Some values are listed at left. Liquid water's specific heat is the highest in the table.

Water has one of the highest specific heats of common substances. That means for a given input of heat, the temperature of a certain amount of water changes

A. more than

B. less than

C. the same as

the same amount of most other substances.

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The specific heat of concrete is greater than that of soil. A baseball field (with real soil) and the surrounding concrete parking lot are warmed up during a sunny day. Which would you expect to cool off faster in the evening when the sun goes down?

A. the concrete parking lot
B. the baseball field
C. both cool off equally fast

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$$Q = mc\,\Delta T.$$

The baseball field, with the lower specific heat, will change temperature more readily, so it will cool off faster. The high specific heat of concrete allows it to "retain heat" better and so it will not cool off so quickly – it has a higher "thermal inertia."

Water **has a higher specific heat than *sand*. Therefore, on the beach at daytime, breezes would blow:**

A. from the ocean to the beach

B. from the beach to the ocean

C. either way, makes no difference

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The sun heats both the beach and the water

» beach heats up faster
» warmer air above beach rises
» cooler air from ocean moves in underneath
» breeze blows ocean → land

How much heat is needed to raise the temperature of an empty 20-kg iron vat (c=0.11 kcal/°C/kg) from 10°C to 90°C?

Answer:

$Q = mc\,\Delta T$ = (20 kg)(0.11 kcal/°C/kg)(80°C)

= 176 kcal

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$= 176\text{ kcal}$

What if the vat is filled with 20 kg of water?

Additional heat required

$\Delta Q = mc\,\Delta T = (20\text{ kg})(1.0\text{ kcal/°C/kg})(80\text{°C})$

$= 1600\text{ kcal}$, so total heat needed is

$Q = 1776\text{ kcal}$

Coming up next week:

- Calorimetry—Measuring Specific Heats
- Latent Heat
- The First Law of Thermodynamics
- Calculating the Work Done by a Gas
- Specific Heats of Real Gases
- Adiabatic Expansion of Gases