Physics 5I   Introduction to Physics I – HONORS SECTION   Fall 2011

Homework #3 – Due Friday October 28

Before doing the following homework problems, work through the Einstein's Rocket thought experiments at http://physics.ucsc.edu/~snof/er.html to understand that if an observer at rest in an inertial reference frame sees a clock moving in the *x*-direction at speed *v*, the clock appears to run slow by the factor $1/\gamma$ where $\gamma = (1\text{-}v^2/c^2)^{-1/2}$ and its length in the *x*-direction appears shorter by the same factor. These special relativistic effects are known as time dilation and length contraction.

1. An atomic clock is at the North Pole (where is it at rest). Another is at the equator, where it moves with to the rotation of the earth. Taking into account the effects of special relativity, how far will the clocks be out of synchronization after 2 years?

2. Giancoli, Chapter 6, Problem 58. (Speed and period of GPS satellites.)

3. (a) From the results of Problem 2, use special relativity to calculate the difference in seconds per day between the time on the atomic clock on a GPS satellite and the time on an atomic clock at rest.

(b) The time signals from the GPS satellites are corrected to be the same as if each satellite were at rest, so that the location of a GPS receiver can be determined by simple triangulation. Estimate the error in the location if the time of the GPS signals did not include the special relativistic correction. (This problem doesn't reflect the complete situation, since GPS satellite signals are also corrected for general relativistic effects; that is, they are also corrected for the fact that gravity is weaker at their orbits than at the surface of the earth. The general relativistic GPS correction is ~6 times larger than the special relativistic one, so the error would be even greater than you just calculated if general relativity weren't also taken into account.)

4. Giancoli, Chapter 6, Problem 60.

5. (a) From the information given in the previous problem, calculate the speed of the sun in its orbit, in km/s.

(b) From the fact that the stars in the disk of the Milky Way all orbit roughly in circles about the center of the galaxy with about the same speed as the sun, show that the mass as a function of distance r from the center of the galaxy grows as $M(r) = Kr$ and determine the constant K. (For this and the next part of this problem, assume for simplicity that this mass is distributed spherically.)

(c) If satellite galaxies orbiting about the Milky Way are observed to orbit with about the same circular velocity as you calculated in part (a) out to a distance of about 300,000 light years, determine the total mass of the Milky Way out to that distance.

6. When it is stationary, the half-life of a species of elementary particle is t_0. (This means that if there are N of these particles at time $t = 0$ then there are half as many (N/2) at time $t = t_0$, $N/4$ at time $2t_0$, $N/8$ at time $3t_0$, and so on.) We observe a beam of these particles moving with speed v, and find that the number has decreased by a factor of two when it has traveled a distance $x = 2ct_0$. Determine the speed v, expressing your answer as a fraction of the speed of light c.

7. The relativistic transformation formula for velocities is $u = (u' + v)/(1 + u'v/c^2)$. That is, if an object is moving in the primed reference frame (x', t') with velocity u' and the primed reference frame is moving in the (x,t) reference frame with velocity v, then the object's velocity in reference frame (x,t) is u.

(a) Suppose a rocket ship moving with speed $0.75c$ respect to the earth launches a small rocket in the same direction with speed (as seen from the rocket) of $0.5c$. What is the speed of the small rocket with respect to the earth?

(b) Suppose that on the front of the rocket ship moving with speed $0.75c$ with respect to the earth there is a headlight. What is the speed of the light from this headlight with respect to the earth?

8. The relativistic time dilation and length contraction in the direction of motion imply that the special relativistic "Lorentz" transformation between inertial reference frame (x,t) and a frame (x',t') moving at speed v in the x-direction is $x = \gamma\,(x'+vt')$, $t = \gamma\,(t' + vx'/c^2)$, where as usual $\gamma = (1-v^2/c^2)^{-1/2}$, and the origins of the reference frames coincide at $t = t' = 0$.

(a) Show that $c^2t^2 - x^2 = c^2t'^2 - x'^2$, and show that this means that the lightcones in the two reference frames agree and that observers in both reference frames will agree on whether an event is inside the forward or backward lightcone or outside the lightcone.

(b) In the Lightcone of Past and Future (the 7th slide in the October 21 lecture), there is a symbol labeled "Supernova remnants, one year from today." By measuring the figure, determine how many years from today those "Supernova remnants one year from today" are seen at Earth.

9. Play the Einstein's Rocket "Round Trip" game. The score is the elapsed time on the rocket clock, and the goal is to make the trip around Jupiter and back to Earth in as short a time as possible. (a) Record your scores for several attempts where you actually make it back to Earth. See how low a score you can get.

(b) Now try doing this trip in Newton's Rocket, and record your scores for several attempts when you make it back to Earth.

(c) Describe the differences you feel between Einstein's and Newton's Rocket, and see if you can explain how they arise from the differences in the underlying physics.