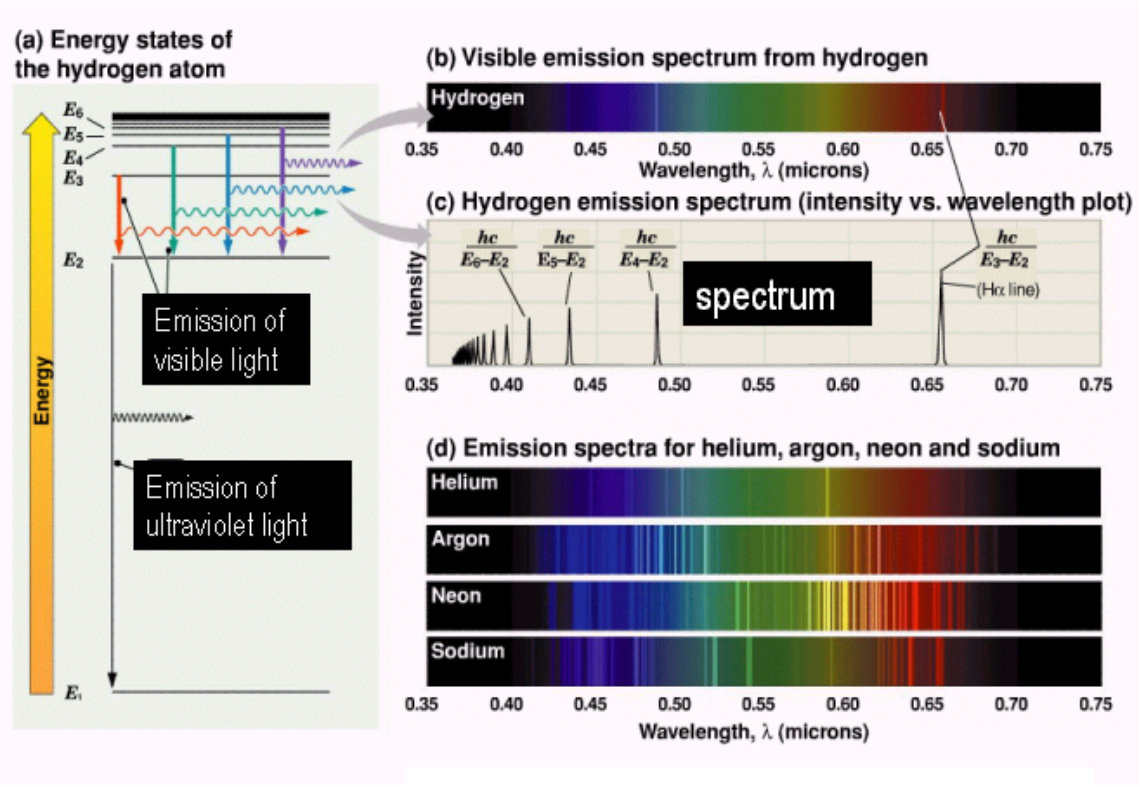
Physics 5K INTRODUCTORY PHYSICS – HONORS SECTION Spring 2012

Homework 2

Due Friday May 11 in Class



The following five problems concern the hydrogen atom, and the sixth problem concerns a hydrogen-like atom. Find the energy levels of the hydrogen atom by following Niels Bohr's derivation. Bohr assumed in 1913 that the electron goes around the proton in a circular orbit with angular momentum L having quantized values $L = n\hbar$, where $n = 1, 2, 3, \ldots$ This equation plus $F = ma$ (where F is the electrical attractive force between the proton and electron) are two equations for the two unknowns, the radius r and velocity v. (Note that this is a highly simplified treatment, which however does give the correct quantum mechanical energies. Note also that the "fine structure constant" α is defined in terms of fundamental constants as $\alpha \equiv e^2/(4\pi\varepsilon_0\hbar c)$ and has the numerical value $\alpha = 1/137.036$.)

1. (15 points) (a) Find the radius a_0 of the $n = 1$ orbit, and in terms of a_0 find the radii r_n of the orbits for the higher values of n.

$F = ma$ here sets the Coulomb force equal to the centripetal force: $\dfrac{e^2}{4\pi\varepsilon_0 r^2} = m_e \dfrac{v^2}{r}$

Bohr's assumption: $mrv = nh$

(b) Show that the ground state (i.e. $n = 1$) energy is $E_1 = \frac{1}{2}\,\alpha^2\, m_e c^2$ and express E_1 in units of electron volts (eV).

(c) Find an expression for the energy levels E_n of the states of the hydrogen atom labeled by Bohr's quantum number n.

2. (15 points) The light emitted by the hydrogen atom's transition from the $n = 3$ to the $n = 2$ energy level is called Hα.

(a) Find the energy E of the photons of this light in eV.

(b) Find the frequency ν of this light by using Planck's formula $E = h\nu$.

(c) Find the wavelength λ. What is the color of this light?

3. (10 points) (a) What is the energy required to reach the $n = 3$ energy level from the ground state (the $n = 1$ energy level)? What sort of light has this much energy?

(b) What is the minimum energy required to ionize a hydrogen atom (i.e. free the electron)?

4. (5 points) Suppose that, on average, a hydrogen atom will exist in the $n = 2$ state for about 10^{-8} second. How many revolutions does the electron make in this time.

5. (10 points) Giancoli, problem 27-72 on the **Zeeman effect**.

6. (15 points) It is possible for a muon to be captured by a proton to form a *muonic atom*. A muon is identical to an electron except for its mass, which is 105.7 MeV/c^2, and the fact that it is unstable, with a lifetime of about 2.2 microseconds.

(a) Calculate the radius of the first Bohr orbit of a muonic atom.

(b) Calculate the magnitude of the lowest energy state in eV.

(c) What is the wavelength of the $n = 2$ to $n = 1$ transition in a muonic atom?