Physics 5K INTRODUCTORY PHYSICS – HONORS SECTION Spring 2012

Homework 3 - SOLUTIONS
Due Friday May 18 in Class

The following three problems concern radioactivity.

1. (10 points) You measure 50,000 decays of a radioactive sample in 20 seconds. You measure the same sample 50 days later and observe only 10,000 decays in 20 seconds. What is the half life of the radioactivity?

Answer: Since the number of decays in a given time interval is proportional to the number of radioactive nuclei that remain, one knows that one fifth of the original nuclei still exist after 50 days. Thus

$2^{-t/T1/2} = 1/5$

where t = 50 days. Then, taking the logarithm of both sides of the equation,

$(t / T_{1/2}) \log 2 = - \log 5$, or $t / T_{1/2} = - (\log 5)/(\log 2) = 2.32$, so $T_{1/2} = 50/2.32 =$ <u>21.5 days</u>.

Check: 50/21.5 = 2.32, and indeed $2^{-2.32} = 0.200 = 1/5$.

2. (10 points) Americium is the only synthetic element to have found its way into the household, where one common type of smoke detector contains about 0.2 microgram of $^{241}Am^{95}$ as a source of α particles. The α particles ionize the air in the space between two electrodes and this permits a small constant current to flow between the electrodes. Smoke that enters the chamber absorbs the alpha particles, which reduces the ionization and interrupts this current, setting off the alarm. The half-life of $^{241}Am^{95}$ is 432 years. How many α particles are emitted per second by 0.2 microgram of $^{241}Am^{95}$?

Answer: The decay rate $R = N \lambda = N / \tau$, where the decay constant $\lambda = 1 / \tau$, and τ is the lifetime, related to the half-life $T_{1/2}$ by $T_{1/2} = \tau \ln 2 = 0.693\, \tau$. Thus for $^{241}Am^{95}$ $\tau = 432$ yr / 0.693 = 623 yr (3.16×10^7 s/yr) = 1.97×10^{10} s. Since (see next problem) 6.02×10^{23} atoms of $^{241}Am^{95}$ is 241 g, the number N of atoms in 0.2 microgram is $N = (6.02 \times 10^{23})(0.2 \times 10^{-6}\ g)/(241\ g) = 5.00 \times 10^{14}$. Then the number of particle emitted per second by 0.2 μg of $^{241}Am^{95}$ is $R = N / \tau = (5.00 \times 10^{14}) / (1.97 \times 10^{10}\ s) =$ <u>2.5×10^4 per s</u>.

3. (10 points) Supose we have a mole of $^{241}Am^{95}$.
(a) What is the mass of a mole of $^{241}Am^{95}$?

Answer: The atomic weight is 241, so the mass is <u>241 grams</u>.

(b) After 32,400 years, what is the number of atoms that remain?

Answer: The half-life of 32,400 years divided by the half-life 432 years is 32400/432 = 75 half-lives. Then the number of atoms left is N = (Avogadro's number) (2^{-75}) = ($6.02\text{x}10^{23}$)($2.6\text{x}10^{-23}$) = 16 .

The next problem concerns the scanning tunneling microscope.

4. (15 points) Suppose that we have a scanning tunneling microscope tip made of a metal with a work function of 4 eV moving at distance *x* Angstroms above another metal with the same work function. (1 Angstrom = 10^{-10} m.) Calculate the probability that an electron will cross the barrier between the microscope tip and the metal as a function of their separation *x*.

Answer: Since the work function is 4 eV, that means that 4 eV is required to remove an electron from the surface of each metal. The wavefunction corresponding to penetration by an electron ($m_e c^2 =$ 511,000 eV) of a barrier V = 4 eV high and *x* Angstroms thick is

$\psi = \exp[-(1/\hbar c)\int(2m_e c^2 V)^{1/2}\,dx] = \exp[-(1/\hbar c)\,(2m_e c^2 V)^{1/2}\,x]$. In the exponent is

$(1/\hbar c)\,(2m_e c^2 V)^{1/2} = [(6.58\text{x}10^{-16}\text{ eV-s})(3\text{x}10^{8}\text{ m/s})]^{-1}\,[(1.22\text{x}10^{6}\text{ eV})(4\text{ eV})]^{1/2}\,(x\ 10^{-10}\text{ m}) =$
$= (1.97\text{x}10^{-7}\text{ eV-m})^{-1}\,(2.21\text{x}10^{-7}\,x\text{ eV-m}) = 1.12\,x$.

The probability that the electron will cross this barrier is then

$P = |\psi|^2 = \underline{\exp(-2.24\,x)}$

where *x* is in Angstroms. For example, if x = 2 Angstroms, then P = 0.011, while if x = 4 Angstroms, then P = 0.000128. Thus, as explained in class, the current flow between the scanning tunneling microscope tip and the metal surface being scanned is exponentially sensitive to the height of the tip above the metal.