Physics 5K Lecture 1 - Friday April 6, 2012

Some Key Ideas of Quantum Mechanics Illustrated by the 2-Slit Experiment

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Topics to be covered in Physics 5K include the following:

Some key ideas of quantum mechanics, illustrated by the two-slit experiment. (See *Feynman Lectures on Physics*, Vol. 1, Chapter 37: “Quantum Behavior.”)

How the laws of classical mechanics follow from extremizing the action, and how this generalizes to quantum mechanics: the path integral formulation of quantum theory. (See *Feynman Lectures on Physics*, Vol. 2, Chapter 19: “The Principle of Least Action.”)

How quantum mechanical particles can travel through “barriers” – regions that are forbidden according to classical physics. (Advanced discussion: R. Shankar, *Principles of Quantum Mechanics*, 2nd Edition, pp. 441-445.) Connections with radioactivity, nuclear fusion, and the scanning tunneling microscope.

The piezoelectric effect and ferroelecticity.

The photoelectric effect and the band theory of solids. Why metals are shiny and gold is golden. How a light emitting diode (LED) works.

Superconductivity. Fermions and bosons. Flux quantization. Applications of superconductivity.

Relativity and motion of particles at high speeds. Cosmic rays and magnetic fields.

Relativity and electromagnetism: why electrical phenomena in one reference frame are a combination of electric and magnetic phenomena in another frame.

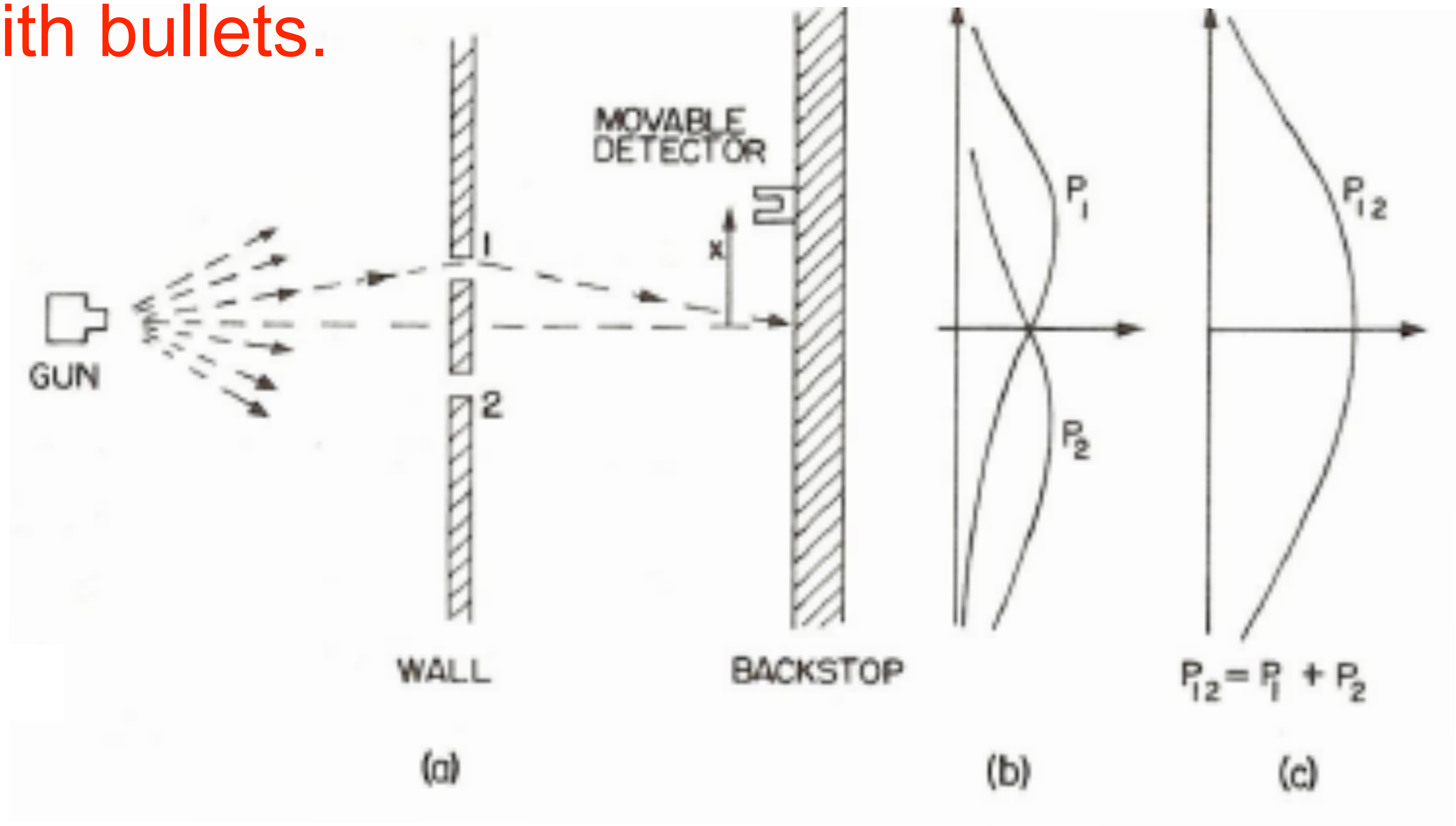
Electromagnetic waves in the universe, including radiation from matter falling into giant black holes in the centers of galaxies.

Some key ideas of quantum mechanics, illustrated by the two-slit experiment.

(See *Feynman Lectures on Physics*, Volume 1, Chapter 37: “Quantum Behavior.”)

We’re going to describe the results of a two-slit experiment involving bullets, water waves, and electrons. The bullets and water waves examples are easy to realize with actual apparatus. The electron example is a thought experiment, but it describes how electrons have been seen to behave experimentally.

Two-slit experiment with bullets.



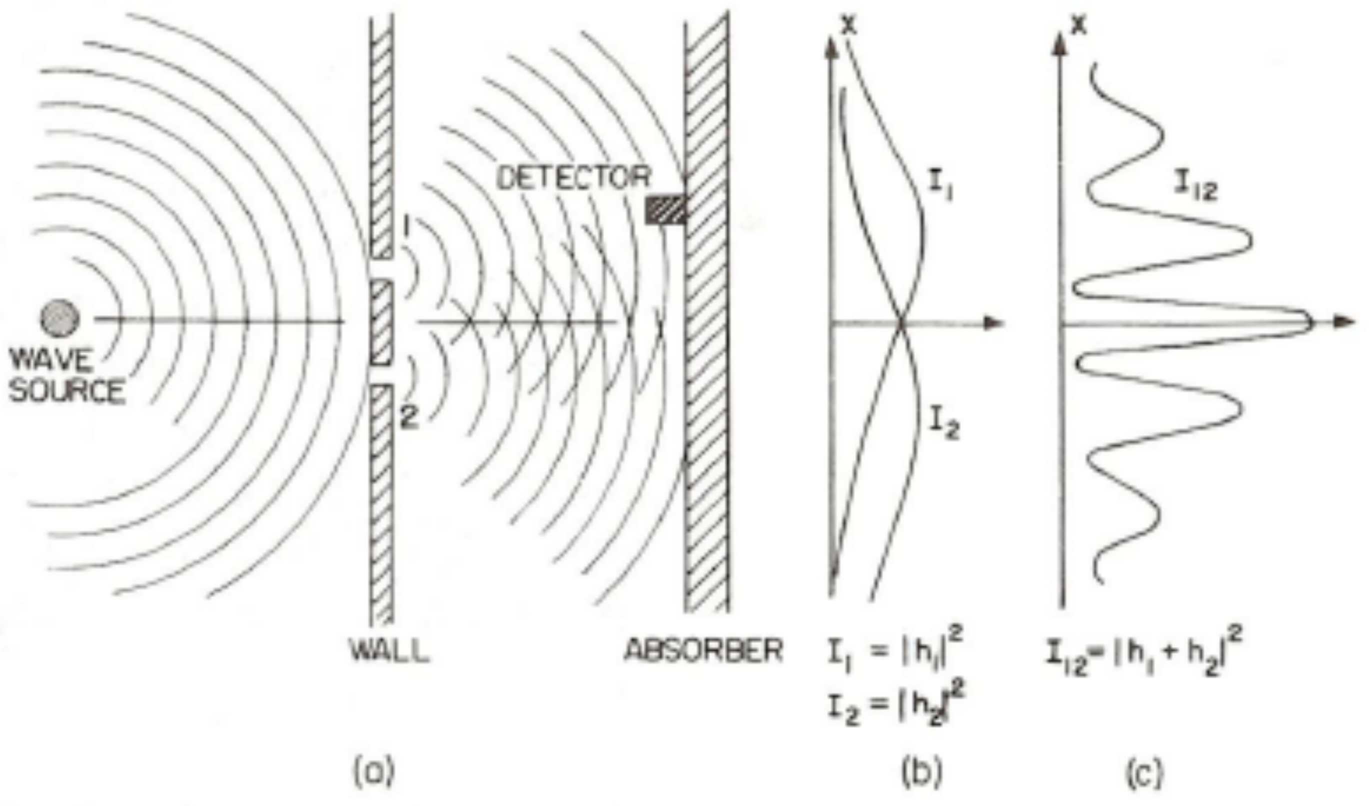
With slit 1 open, the bullets are detected at various points on the backstop with probability P_1.

With slit 2 open, the bullets are detected at various points on the backstop with probability P_2.

With both slits open, the bullets are detected at various points on the backstop with probability $P_{12} = P_1 + P_2$. This is not surprising.

Two-slit experiment with water waves.

The detector measures the wave intensity I, which is proportional to the wave height h: $I = |h|^2$.

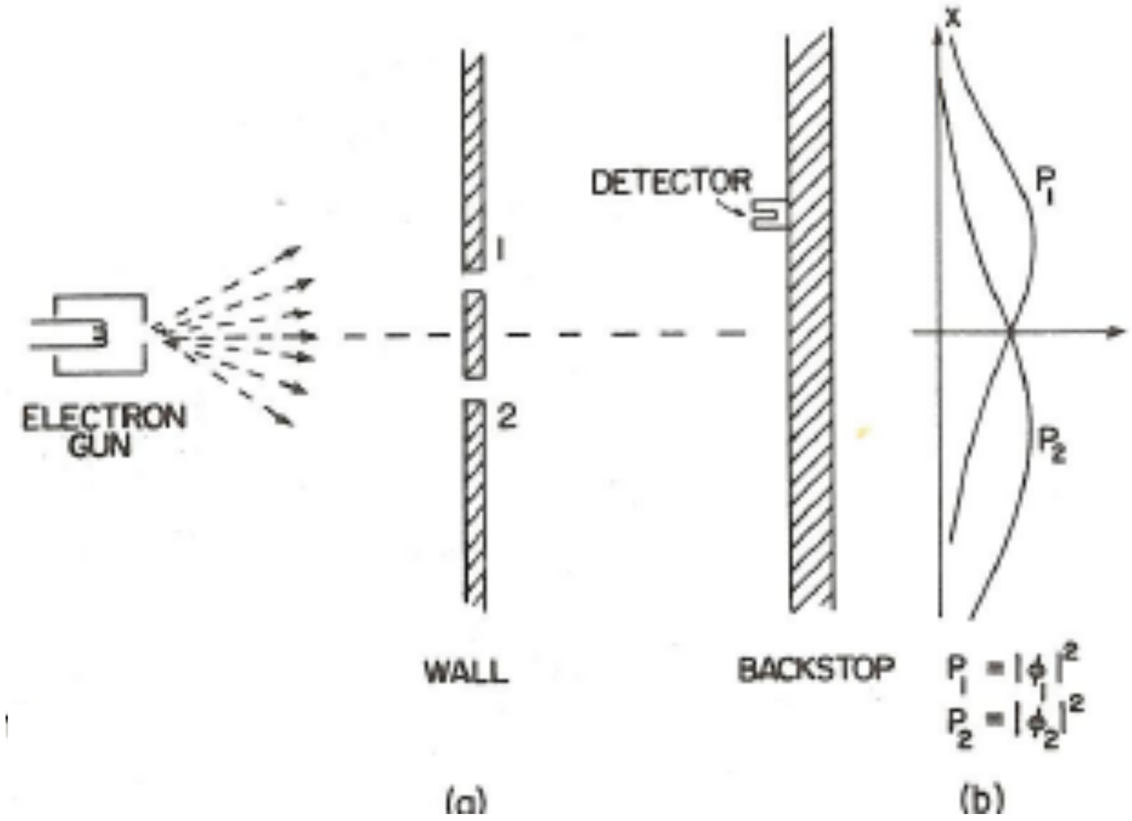


With slit 1 open, the waves are detected at various points on the backstop with intensity $I_1 = |h_1|^2$.

With slit 2 open, the waves are detected at various points on the backstop with probability $I_2 = |h_2|^2$.

With both slits open, the waves are detected at various points on the backstop with intensity $I_{12} = |h_1 + h_2|^2 \neq I_1 + I_2$. The waves interfere, as waves always do, so the wiggles are not surprising.

Two-slit experiment with electrons.



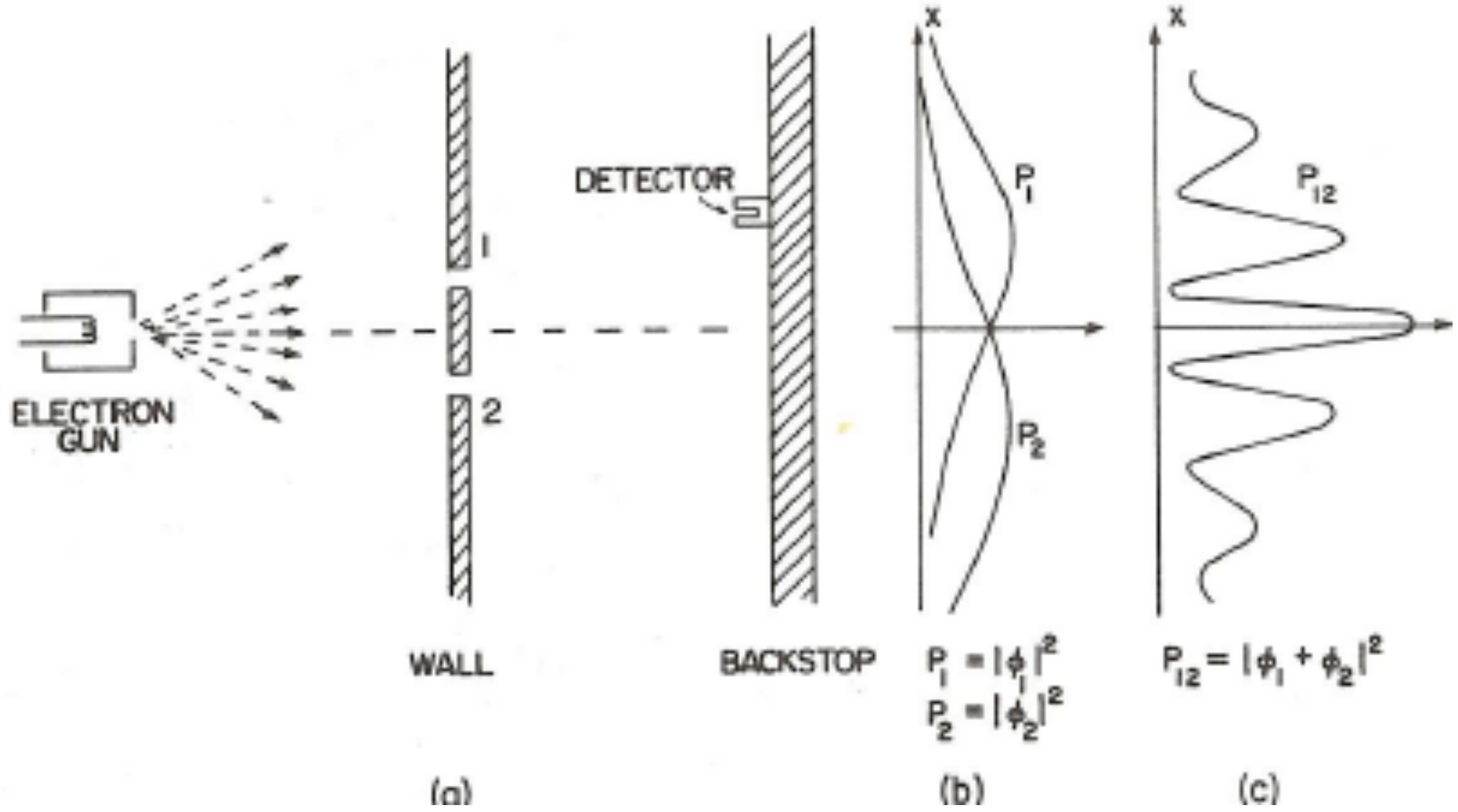
The detector measures the probability P of finding an electron at a given position on the backstop.

With slit 1 open, the electrons are detected at various points on the backstop with probability P_1.

With slit 2 open, the electrons are detected at various points on the backstop with probability P_2. Thus far, just like bullets or waves.

Two-slit experiment with electrons.

The detector measures the probability P of finding an electron at a given position on the backstop.

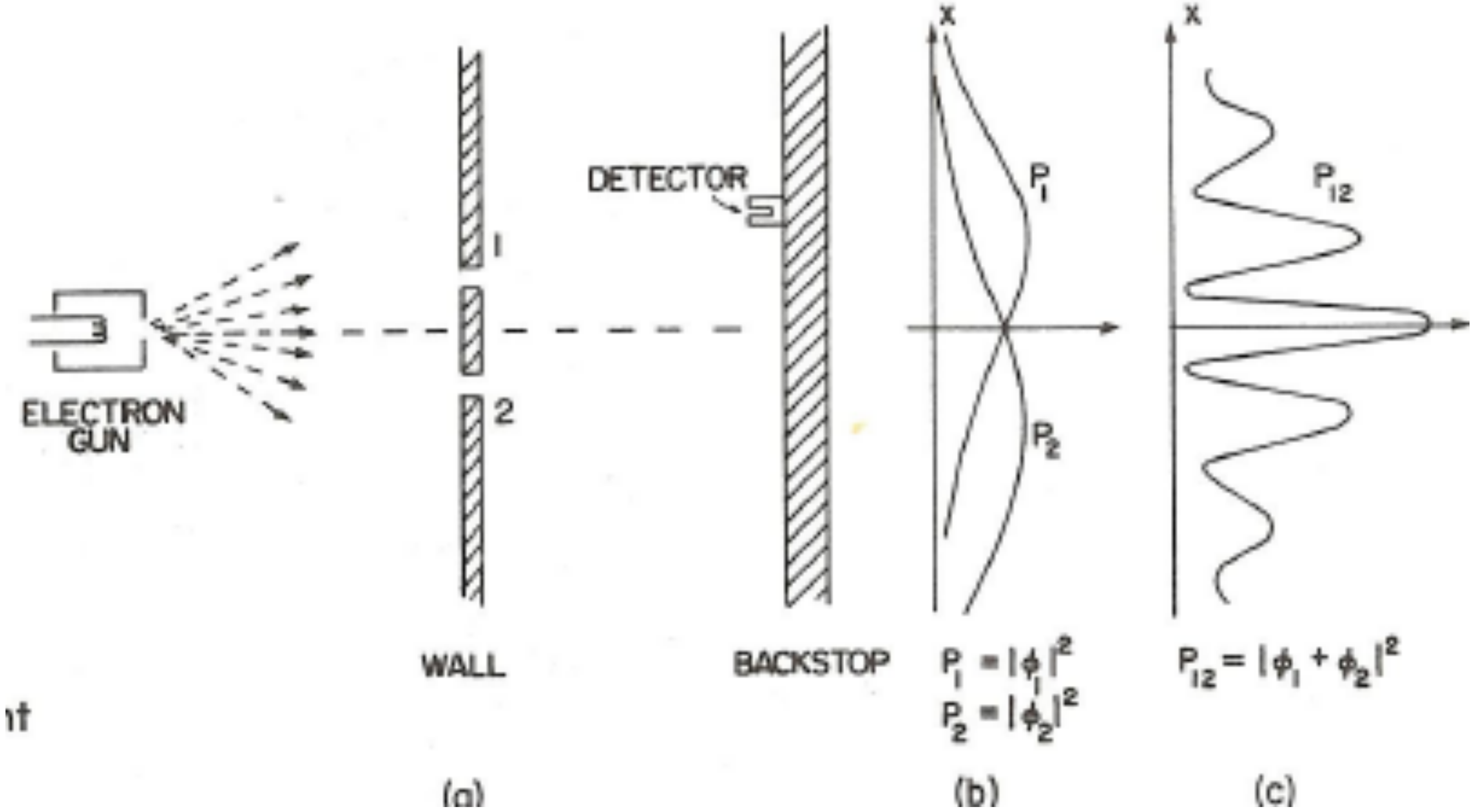


With slit 1 open, the electrons are detected at various points on the backstop with probability P_1.

With slit 2 open, the electrons are detected at various points on the backstop with probability P_2. Thus far, just like bullets or waves.

With both slits open, the electrons are detected at various points on the backstop with probability $P_{12} = |\phi_1 + \phi_2|^2 \neq P_1 + P_2$. The electrons interfere as waves do. But electrons, like bullets, come in lumps, so this is surprising!

Electrons come in lumps -- that is, an electron is either detected or not, but never half an electron with charge e/2. So it is natural to assume that each electron either went through slit 1 or through slit 2.



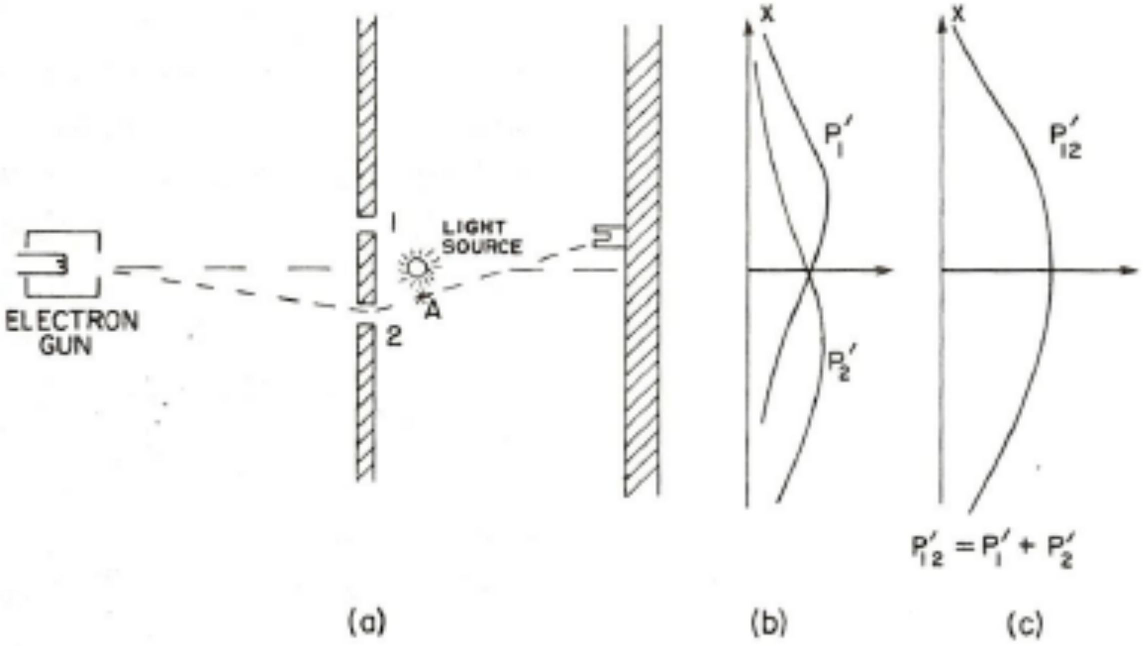
However, if that were true, how could it be that there are fewer electrons detected at some positions with both slits open than with just slit 1 open?

One can do the experiment with the electron gun turned way down, so that at any given time there is at most one electron in the apparatus. Yet with both slits open, the pattern is (c). A single electron interferes with itself!

The mathematics of electron interference: the electron "wave function" ϕ is a complex number: $\phi = \phi_r + i\phi_i$, where $i = \sqrt{(-1)}$, $\phi_r = \mathrm{Re}(\phi)$ and $\phi_i = \mathrm{Im}(\phi)$.

$$P_{12} = |\phi_1 + \phi_2|^2 = P_1 + P_2 + \underbrace{2(\cos\theta_{12})\,|\phi_1||\phi_2|}_{\text{interference term}}$$

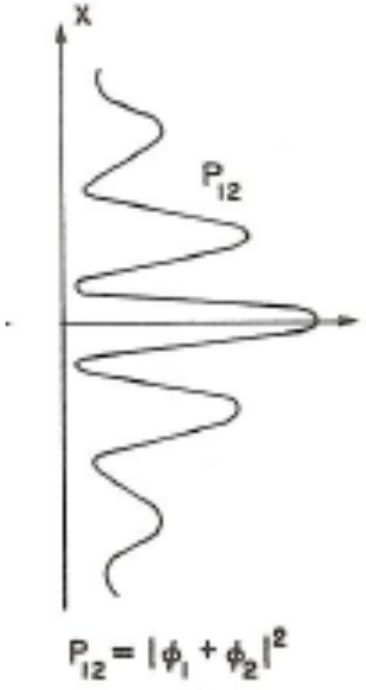
Watching the electrons.



Let’s suppose that we illuminate the slits so that we can see which slit each electron goes through.

In that case, the electrons behave the same way as bullets -- no interference!

Now we turn the light down, so that we can only see which slit about half the electrons went through. It turns out that for those, the pattern is just like the one above. But the electrons that are not illuminated as they go through the slit are distributed just like those with no light at all!



The electrons that *are not seen* going through one or the other slit exhibit interference, while those that *are seen* do not!

The electrons that *are not seen* going through one or the other slit exhibit interference, while those that *are seen* do not!

When one disturbs the electrons by illuminating them (hitting them with one or more photons of light), then one can say that they go through one or the other slit.

When one does not disturb the electrons by illuminating them, then one *cannot* say that they go through one or the other slit.

How can electrons behave like that? No one knows! One can't make a model that behaves like that, based on classical physics. But that's how electrons do behave. So do photons and other tiny particles.

We see that electrons behave sometimes like particles (they come in lumps) and sometimes like waves (they interfere if they are not disturbed).

The principles of quantum mechanics.

(1) The probability of an event in an experiment is given by the square of the absolute value of a complex number ϕ which is called the probability amplitude.

P = probability

ϕ = probability amplitude

$$P = |\phi|^2$$

(2) When an event can occur in several alternative ways, the probability amplitude for the event is the sum of the probability amplitudes for each way considered separately. There is interference.

$$\phi_{12} = \phi_1 + \phi_2$$

$$P_{12} = |\phi_{12}|^2 = |\phi_1 + \phi_2|^2$$

(3) If an experiment is capable of determining whether one or another alternative is actually taken, the probability of the event is the sum of the probabilities for each alternative. The interference is lost.

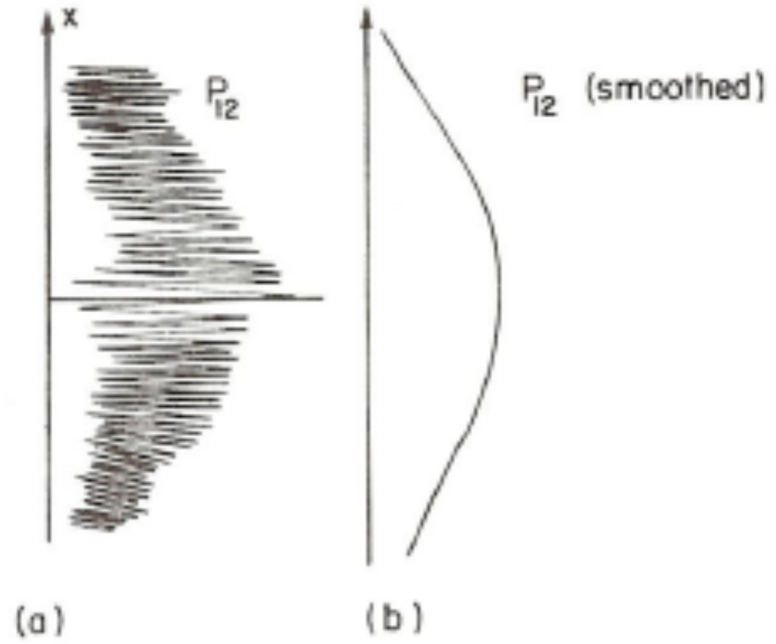
$$P_{12} = P_1 + P_2$$

One might still ask, “Why does it work that way?”

No one can explain any more than we have just explained. No one can give you any deeper representation of the situation.

Note an important difference between classical and quantum mechanics. In talking about the probability that an electron will arrive at a certain point, we have implied that, in the best possible experiment, it is impossible to predict exactly what will happen -- we can only predict the odds.

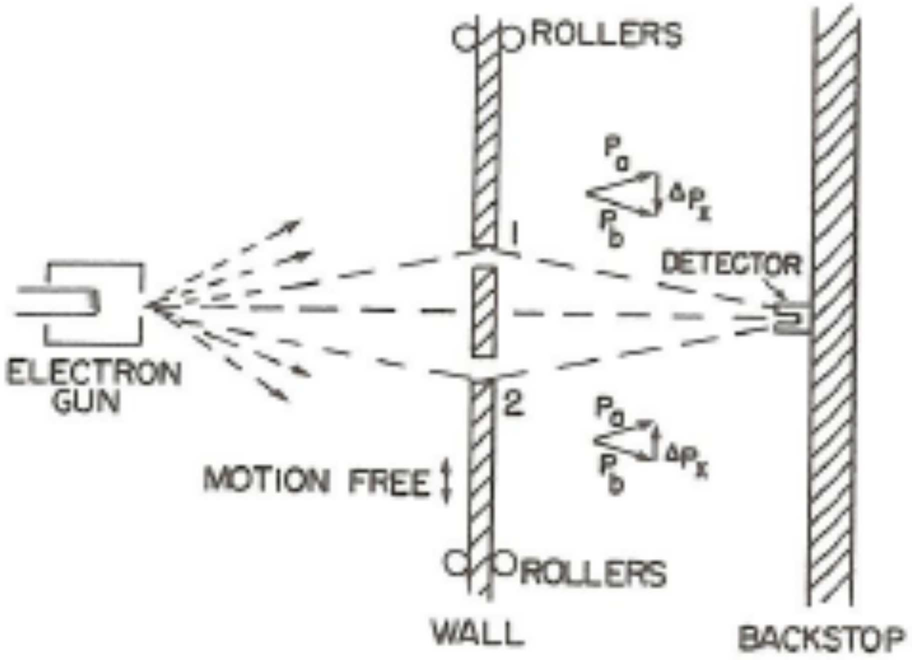
If quantum mechanics applies to everything, why don’t bullets behave like electrons, and interfere. It turns out that for big things like bullets, the wavelengths are so tiny that the interference patterns become very thin. Any physical detector straddles many waves, and gives the smooth curve at right.



The uncertainty principle.

Suppose we try to tell which slit an electron goes through, not by illuminating it, but by noticing the tiny deflection of the wall with the slits when the electron bounces off a slit.

In order to do this, it is necessary to know the initial momentum of the wall.



Werner Heisenberg showed that if you know p_x, the x-component of the momentum, with a certain accuracy Δp_x, you can’t at the same time know its position x more accurately than $\Delta x = h/4\pi\Delta p_x$, where Planck’s constant $h = 6.6 \times 10^{-34}$ m^2 kg/s.

It can be shown that, in order to measure the momentum of the wall sufficiently accurately to determine which slit the electron went through, the vertical position of the wall itself will be sufficiently uncertain to shift the position of the pattern observed at the backstop up or down enough to smear out the interference. Thus, the uncertainty principle protects quantum mechanics.