Barrier Penetration, Radioactivity, and the Scanning Tunneling Microscope

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Topics to be covered in Physics 5K include the following:

Some key ideas of quantum mechanics, illustrated by the two-slit experiment. (See *Feynman Lectures on Physics*, Vol. 1, Chapter 37: "Quantum Behavior.")

How the laws of classical mechanics follow from extremizing the action, and how this generalizes to quantum mechanics: the path integral formulation of quantum theory. (See *Feynman Lectures on Physics*, Vol. 2, Chapter 19: "The Principle of Least Action.")

How quantum mechanical particles can travel through "barriers" – regions that are forbidden according to classical physics. (Advanced discussion: R. Shankar, *Principles of Quantum Mechanics*, 2nd Edition, pp. 441-445.) Connections with radioactivity, nuclear fusion, and the scanning tunneling microscope.

The piezoelectric effect and ferroelecticity.

The photoelectric effect and the band theory of solids. Why metals are shiny and gold is golden. How a light emitting diode (LED) works.

Superconductivity. Fermions and bosons. Flux quantization. Applications of superconductivity.

Relativity thought experiments and computer games. Cosmic rays and magnetic fields.

Relativity and electromagnetism: why electrical phenomena in one reference frame are a combination of electric and magnetic phenomena in another frame.

Electromagnetic waves in the universe, including radiation from matter falling into giant black holes in the centers of galaxies.

According to quantum mechanics, the probability that a particle starting at point x_1 and time t_1 will arrive at point x_2 at time t_2 is the square $|\Psi|^2$ of a probability amplitude Ψ. The total amplitude is the sum of the amplitudes for each possible path. The amplitude Ψ for each path is proportional to $\exp(iS/\hbar)$ where "h-bar" $\hbar = h/2\pi = 1.054 \times 10^{-34}$ m^2 kg/s, and Planck's constant $h = 6.626 \times 10^{-34}$ m^2 kg/s. The Action S has dimensions of energy x time, and h has the same dimensions.

A particularly striking use of the path-integral formulation of quantum mechanics is its implications for "barrier penetration," particles moving through places that they are forbidden to be, according to classical mechanics. Recall this approach from the last lecture.

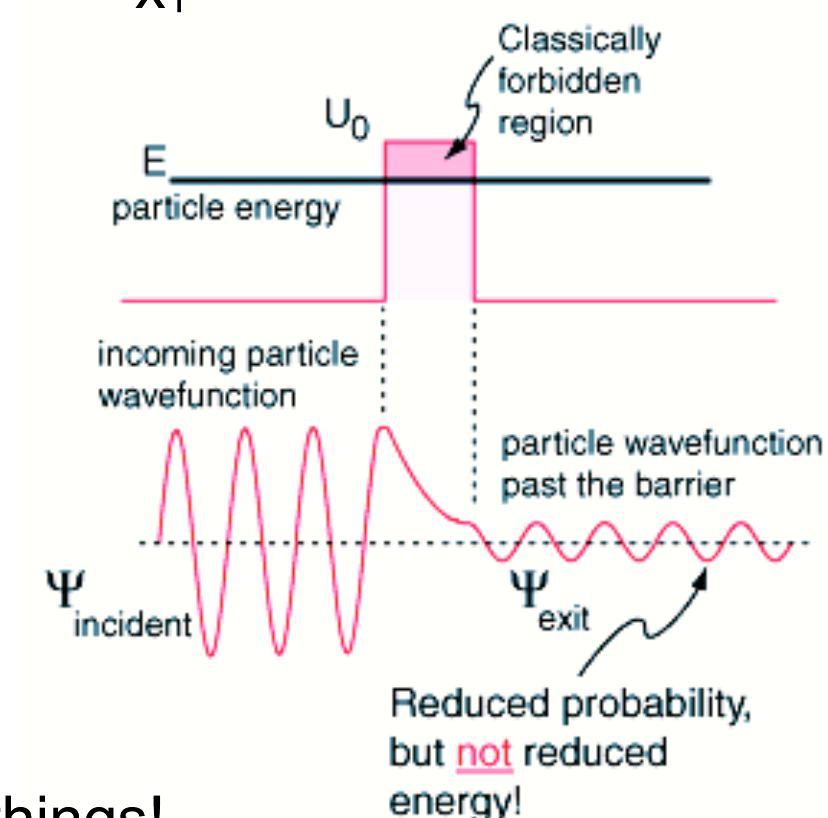
According to classical mechanics, $ma = F = -V'(x)$ and $E = \frac{1}{2} mv^2 + V(x)$.
Note that $L \equiv KE - PE = \frac{1}{2} mv^2 - V(x) = mv^2 - E$, so

$$\text{Action} = S = \int_{t_1}^{t_2} (KE - PE)\, dt = \int_{t_1}^{t_2} (mv^2 - E)\, dt = \int_{t_1}^{t_2} mv \frac{dx}{dt}\, dt - \int_{t_1}^{t_2} E\, dt = \int_{x_1}^{x_2} p\, dx - E(t_2 - t_1),$$

where $p = mv = [2m(E-V)]^{1/2}$. Let's apply this to the quantum mechanical formalism where $\Psi = A \exp(iS/\hbar)$.

Consider a situation where V is greater than E, so that $p = [2m(E-V)]^{1/2}$ is imaginary. This describes a situation where the particle is forbidden to be, according to classical physics. But the wavefunction Ψ doesn't vanish, rather it includes a term

$$\Psi = A \exp(iS/\hbar) = A \exp\left(- \int [2m(V-E)]^{1/2}\, dx/\hbar \right).$$



This week we will show how to use this to calculate important things!

Barrier Penetration

In class we started from the path integral expression for the wavefunction

$$\psi = A \exp[\frac{i}{\hbar}\int L\, dt\,]$$

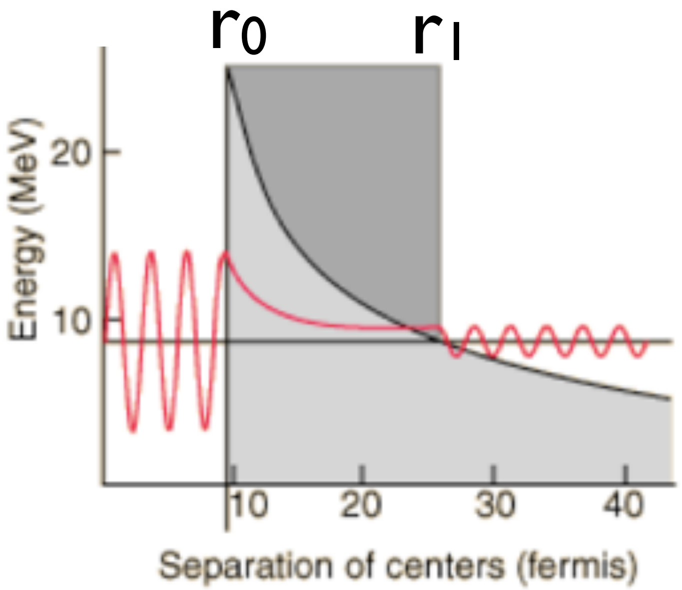
For a barrier penetration situation like α radioactivity, as we discussed in class we can replace the integral of L dt by p dx, where momentum p is given by

$$p = \sqrt{2m(E\text{-}V)} = i\,\sqrt{2m(V\text{-}E)}$$

Then

$$\psi = A \exp\left[-\frac{1}{\hbar}\int_{r_0}^{r_1}\sqrt{2m(V\text{-}E)}\, dx\right]$$

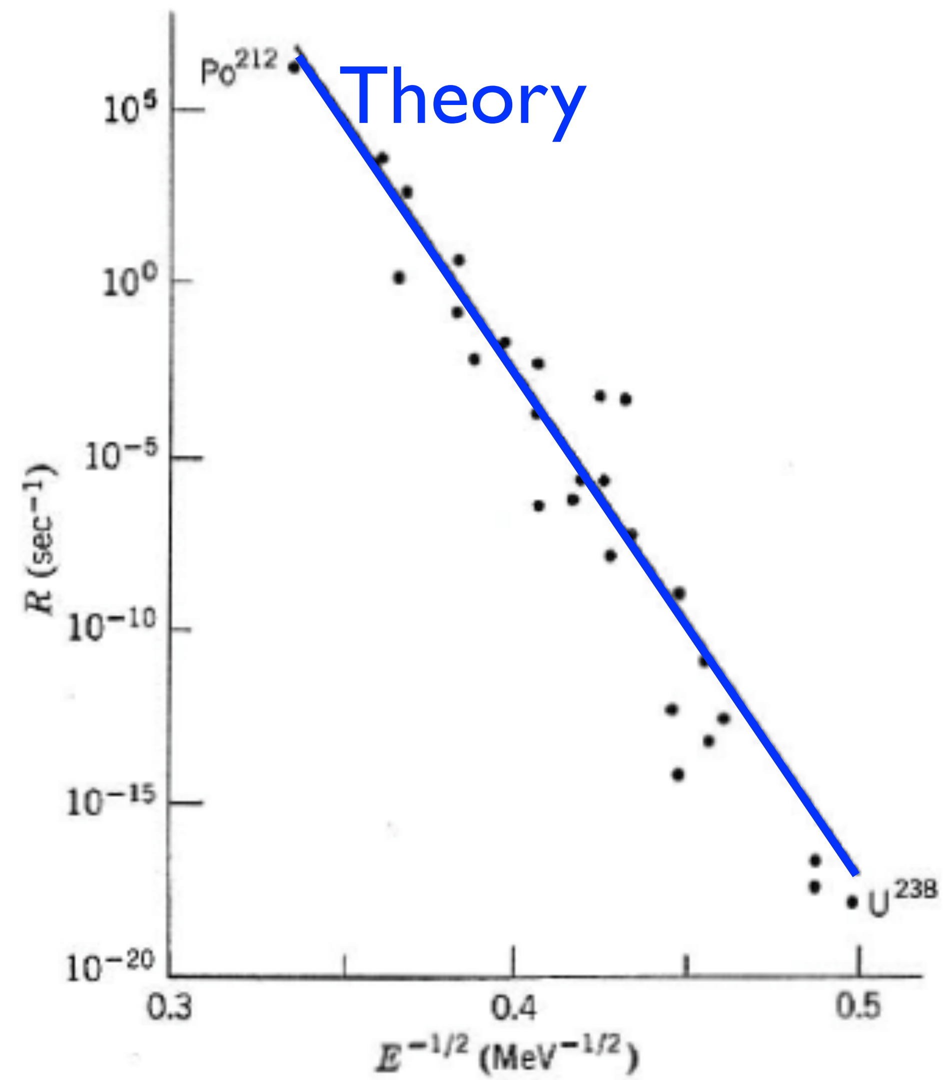
and the probability P of barrier penetration is $|\,\psi\,|^2$ each time the α particle hits the barrier. This happens $v/2r_0$ times per second, where we can calculate the particle's velocity v from $E = ½\, mv^2$. Thus the α decay rate is $R = (v/2r_0)\,|\,\psi\,|^2$.



This barrier penetration theory was applied to α decay in 1928 by George Gamow and others. The potential energy V for this case is just the electro-static potential energy between the α particle (charge +2e) and the nucleus without the α particle (charge +Ze):

$$V(r) = \frac{2Ze^2}{4\pi\epsilon_0 r}$$

The limits of integration are r_0, the radius of the nucleus, and r_1, the outer edge of the barrier, where $V(r_1) = E$. At $r > r_1$, $E > V(r)$ and the particle is in a classically allowed region. In classical physics the particle could not escape the barrier, but in quantum mechanics there is a small but non-zero probability of barrier penetration. Experimentally, there was a tremendous range in measured α decay rates, from $R = 5 \times 10^{-18}$/sec for ${}^{92}U^{238}$, with α particle energy $E_\alpha = 4.2$ MeV, to $R = 2 \times 10^{6}$/sec for ${}^{84}Po^{212}$, with $E_\alpha = 8.9$ MeV. Quantum barrier penetration theory explained this, and also correctly predicted the decay rates measured for other α decays, shown as dots in the diagram at right.



Barrier penetration is important in other nuclear processes, including nuclear fission and fusion -- which is practically important, since we on earth derive almost all of our energy directly or indirectly from fusion in the sun, our only other important source of energy being fission. In both fission and fusion, the barrier is caused by electrical repulsion.

In fusion, nuclei with charges Z_1e and Z_2e must hit each other hard enough (which requires high temperature T) and often enough (which requires high density, that is, a large number of nuclei per unit volume); both requirements are met in stellar interiors and a few minutes after the Big Bang.

In fission, a massive nucleus like that of uranium splits into two large fragments, which we can again regard as having charges Z_1e and Z_2e, but this time Z_1 and Z_2 are larger. Usually $Z_1 \neq Z_2$, with $Z_2 \approx (2/3)\, Z_1$.

In each case, we can calculate the rate at which a given nuclear process can occur, but there is no way to tell whether any particular nucleus will undergo that process. Radioactivity is random, but the rate at which such processes occur is not random at all.

Proton-proton chain reaction

From Wikipedia

The proton-proton chain reaction is one of several fusion reactions by which stars convert hydrogen to helium, the primary alternative being the CNO cycle. The proton-proton chain dominates in stars the size of the Sun or smaller.

Overcoming electrostatic repulsion between two hydrogen nuclei requires a large amount of energy, and this reaction takes an average of 10^9 years to complete at the temperature of the Sun's core. Because of the slowness of this reaction the Sun is still shining; if it were faster, the Sun would have exhausted its hydrogen long ago.

In general, proton-proton fusion can occur only if the temperature (i.e. kinetic energy) of the protons is high enough to overcome their mutual Coulomb repulsion. The theory that proton-proton reactions were the basic principle by which the Sun and other stars burn was advocated by Arthur Stanley Eddington in the 1920s. At the time, the temperature of the Sun was considered too low to overcome the Coulomb barrier. After the development of quantum mechanics, it was discovered that tunneling of the wavefunctions of the protons through the repulsive barrier allows for fusion at a lower temperature than the classical prediction.

The first step involves the fusion of two hydrogen nuclei ^1H (protons) into deuterium, releasing a positron and a neutrino as one proton changes into a neutron (a weak interaction).

$$^1H + {^1H} \rightarrow {^2D} + e^+ + \nu_e + 0.42\ \text{MeV}$$

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$$^{1}H \; + \; ^{1}H \; \rightarrow \; ^{2}D \; + \; e^{+} \; + \; \nu_e \; + \; 0.42 \text{ MeV}$$

This first step is extremely slow, both because the protons have to tunnel through the Coulomb barrier and because it depends on weak interactions.

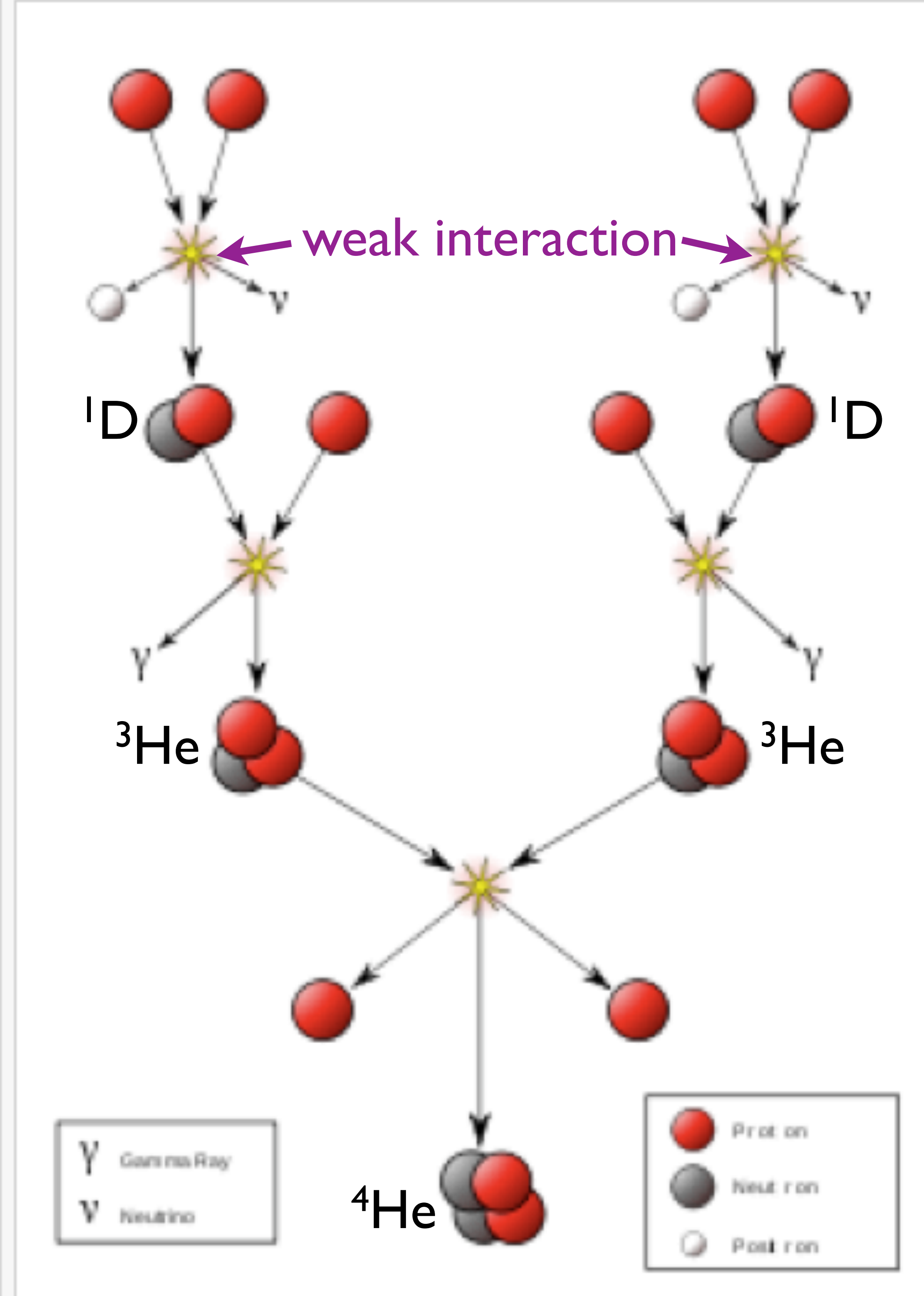
The positron immediately annihilates with an electron, and their mass energy is carried off by two gamma ray photons.

$$e- \; + \; e+ \; \rightarrow \; 2\,\gamma \; + \; 1.02 \text{ MeV}$$

After this, the deuterium produced in the first stage can fuse with another hydrogen to produce a light isotope of helium, 3He:

$$^{1}D \; + \; ^{1}H \; \rightarrow \; ^{3}He \; + \; \gamma \; + \; 5.49 \text{ MeV}$$

From here there are three possible paths to generate helium isotope ^{4}He. In pp I helium-4 comes from fusing two of the helium-3 nuclei produced; the pp II and pp III branches fuse ^{3}He with a pre-existing ^{4}He to make Beryllium. In the Sun, branch pp I takes place with a frequency of 86%, pp II with 14% and pp III with 0.11%. There is also an extremely rare pp IV branch.

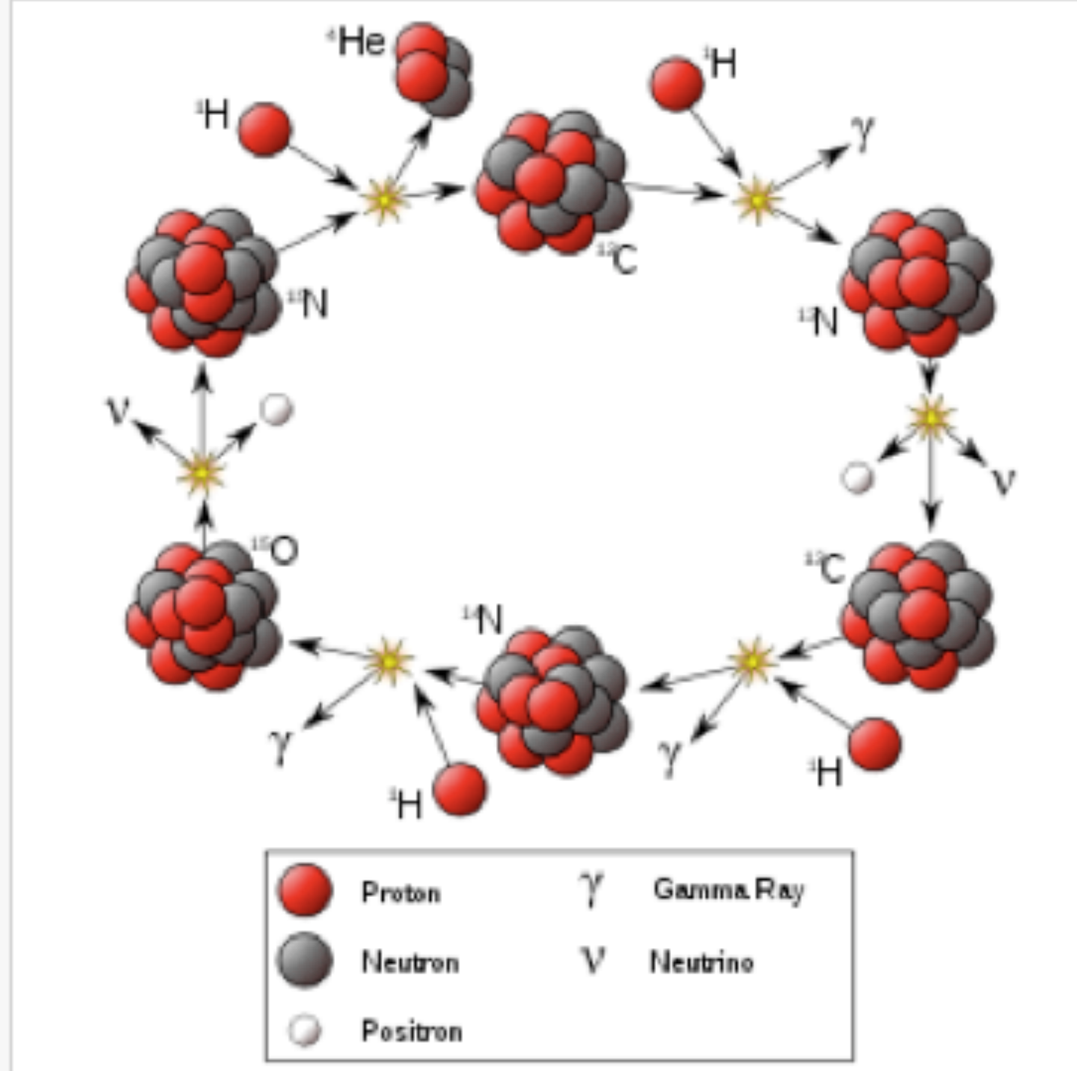


The Proton-proton chain dominates in stars the size of the Sun or smaller.

CNO cycle

From Wikipedia, the free encyclopedia

The **CNO cycle** (for carbon-nitrogen-oxygen), or sometimes **Bethe-Weizsäcker-cycle**, is one of two sets of fusion reactions by which stars convert hydrogen to helium, the other being the proton-proton chain. Theoretical models show that the CNO cycle is the dominant source of energy in stars heavier than about 1.5 times the mass of the sun. The proton-proton chain is more important in stars the mass of the sun or less. This difference stems from temperature dependency differences between the two reactions; pp-chain reactions start occurring at temperatures around 4×10^6 K, making it the dominant force in smaller stars. The CNO chain starts occurring at approximately 13×10^6 K[citation needed], but its energy output rises much faster with increasing temperatures. At approximately 17×10^6 K[citation needed], the CNO cycle starts becoming the dominant source of energy. The Sun has a core temperature of around -15.7×10^6 K and only 1.7% of ^{4}He nuclei being produced in the Sun are born in the CNO cycle. The CNO process was proposed by Carl von Weizsäcker[1] and Hans Bethe[2] independently in 1938 and 1939, respectively.



Overview of the CNO-I Cycle.

In the CNO cycle, four protons fuse using carbon, nitrogen and oxygen isotopes as a catalyst to produce one alpha particle, two positrons and two electron neutrinos . The positrons will almost instantly annihilate with electrons, releasing energy in the form of gamma rays. The neutrinos escape from the star carrying away some energy. The carbon, nitrogen, and oxygen isotopes are in effect one nucleus that goes through a number of transformations in an endless loop.

CNO cycle

Homework 1, Problems 4-7

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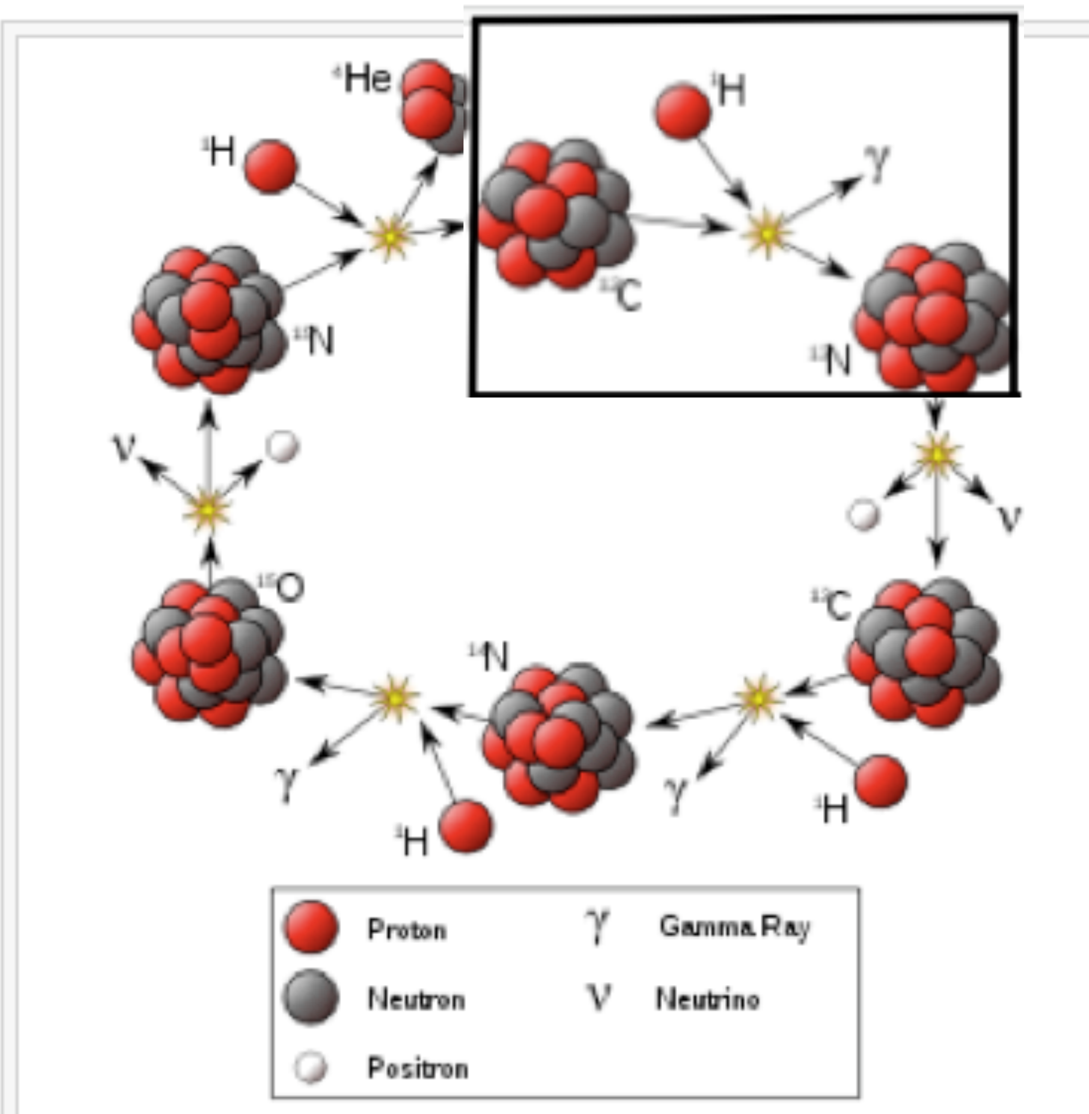

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Radioactive Decay

- The rate at which a decay process occurs (the number of decays per second) is proportional to the number N of radioactive nuclei present
- If N nuclei are present at some instant, the rate of change of N is given by

$$\frac{dN}{dt} = -\lambda N$$

- Here λ is called the "decay constant", and the minus sign indicates that the number is falling.

- This formula is called a differential equation and has the solution

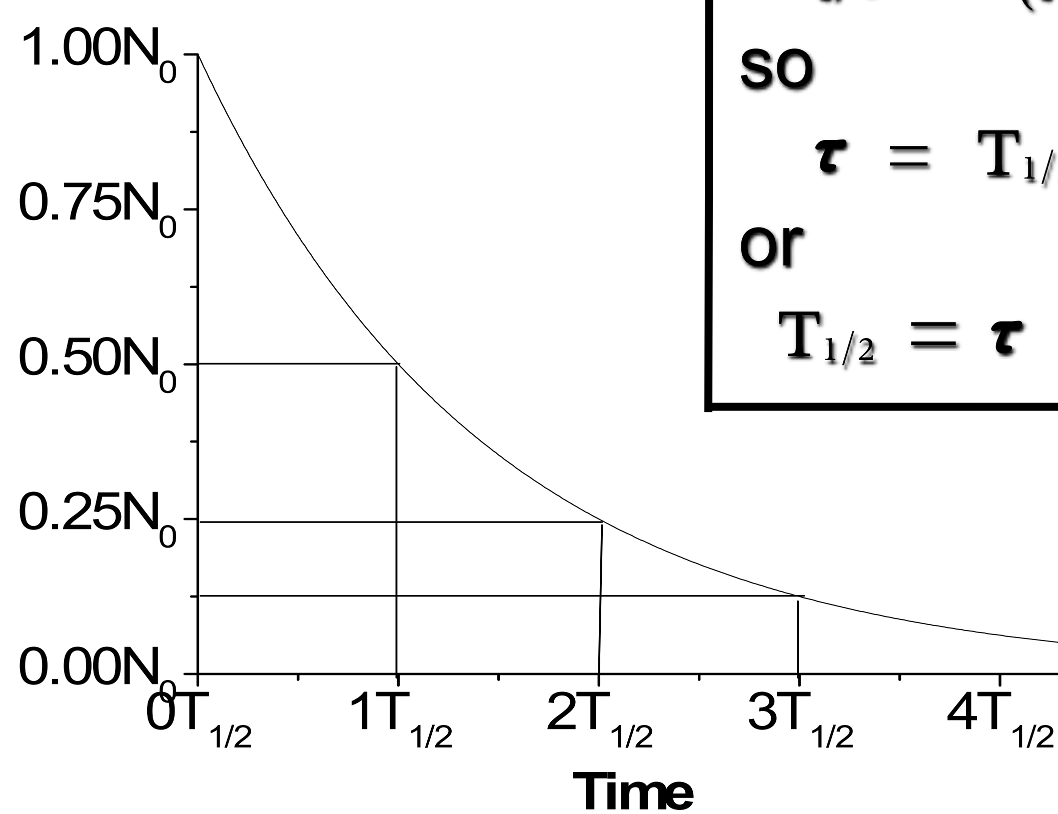
$$N = N_0 e^{-\lambda t}$$

- The constant N_0 represents the initial number of particles.

- From these formulae we can determine the decay rate (the number of emissions per second)

$$R = \left|\frac{dN}{dt}\right| = N_0 \lambda e^{-\lambda t} = R_0 e^{-\lambda t}$$

- The rate, R, is called the activity of the sample
- A sample's reactivity therefore reduces exponentially with time
- The half life, $T_{1/2}$, of a radioactive substance is the amount of time it takes to halve the amount of radioactive material in a particular sample



Note:

$$e^{-\lambda t} = e^{-t/\tau}$$

implies that the "lifetime" $\tau = 1/\lambda$.

The relation between lifetime τ and half-life $T_{1/2}$ follows from

$$e^{-t/\tau} = 2^{-t/T_{1/2}}$$

Taking natural logs,

$$t/\tau = (t/T_{1/2}) \ln 2$$

so

$$\tau = T_{1/2} / \ln 2$$

or

$$T_{1/2} = \tau \ln 2 = 0.693\, \tau$$

Radioactive Decay

Lifetime τ and half-life $T_{1/2}$

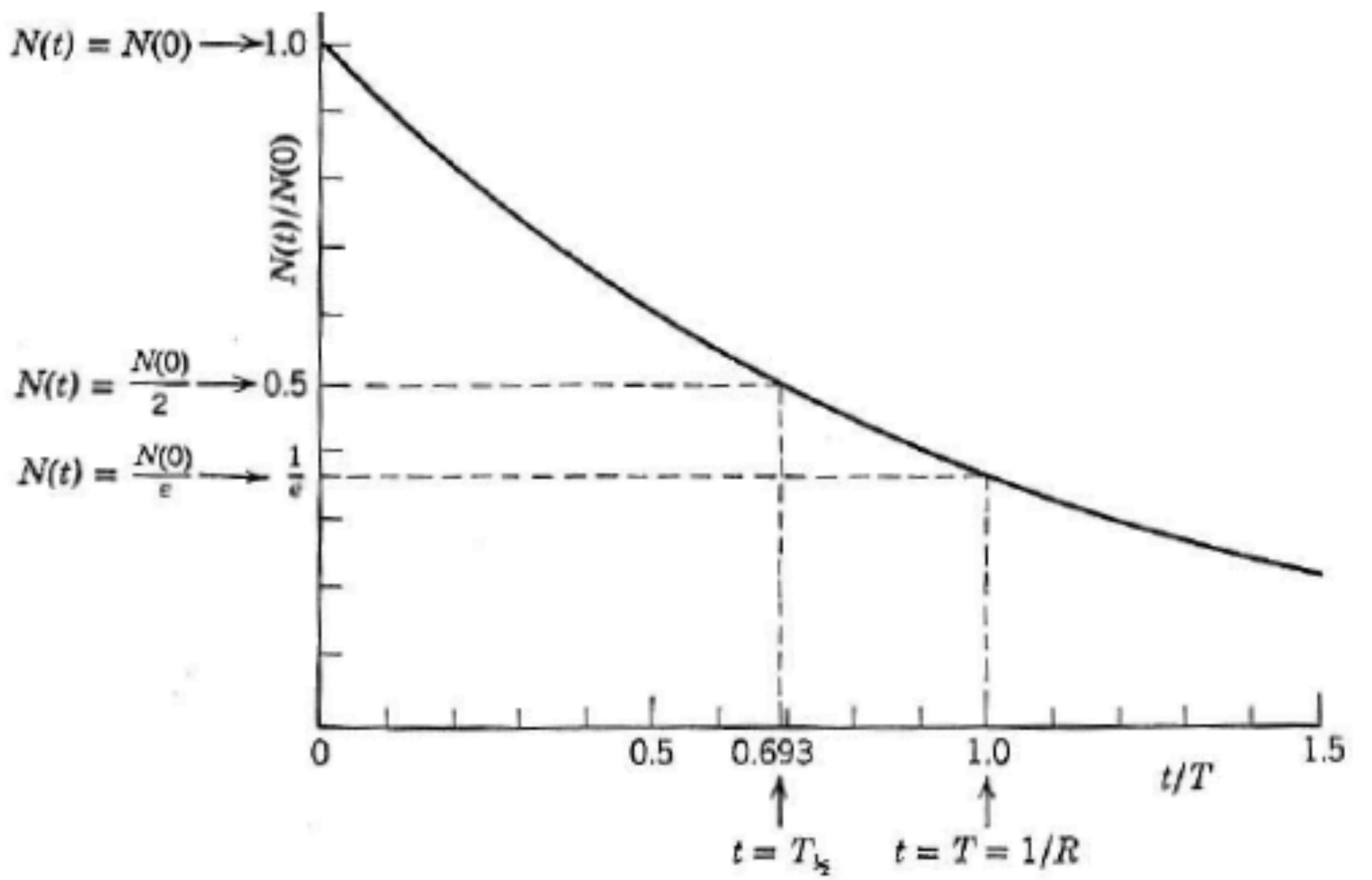


Figure 16-3 The exponential decay law for $N(t)$, the number of nuclei surviving at time t. Also shown are the lifetime T and half-life $T_{1/2}$. Note that $N(t)$ is expressed in units of the original number of nuclei $N(0)$, while time is expressed in units of the lifetime T.

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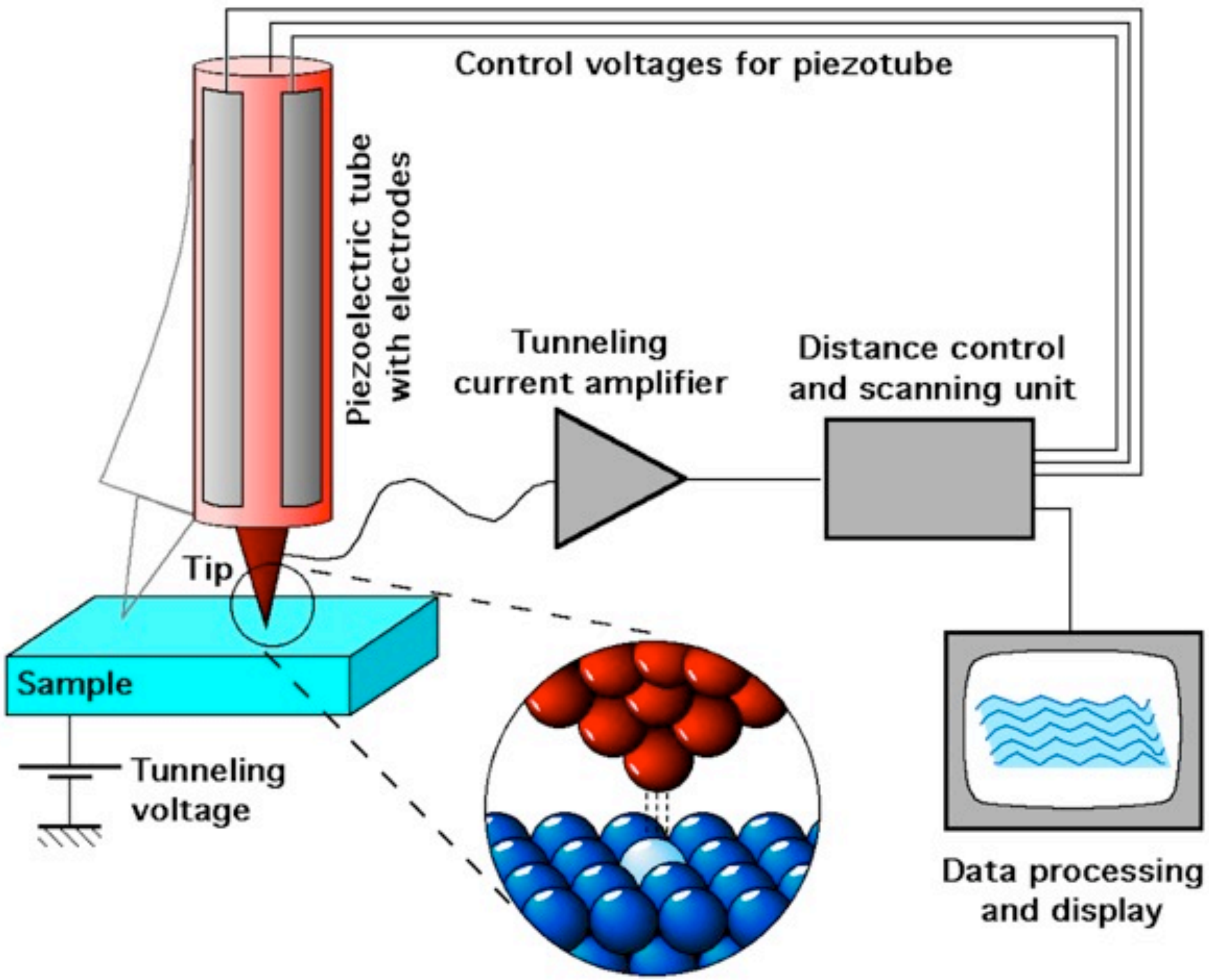
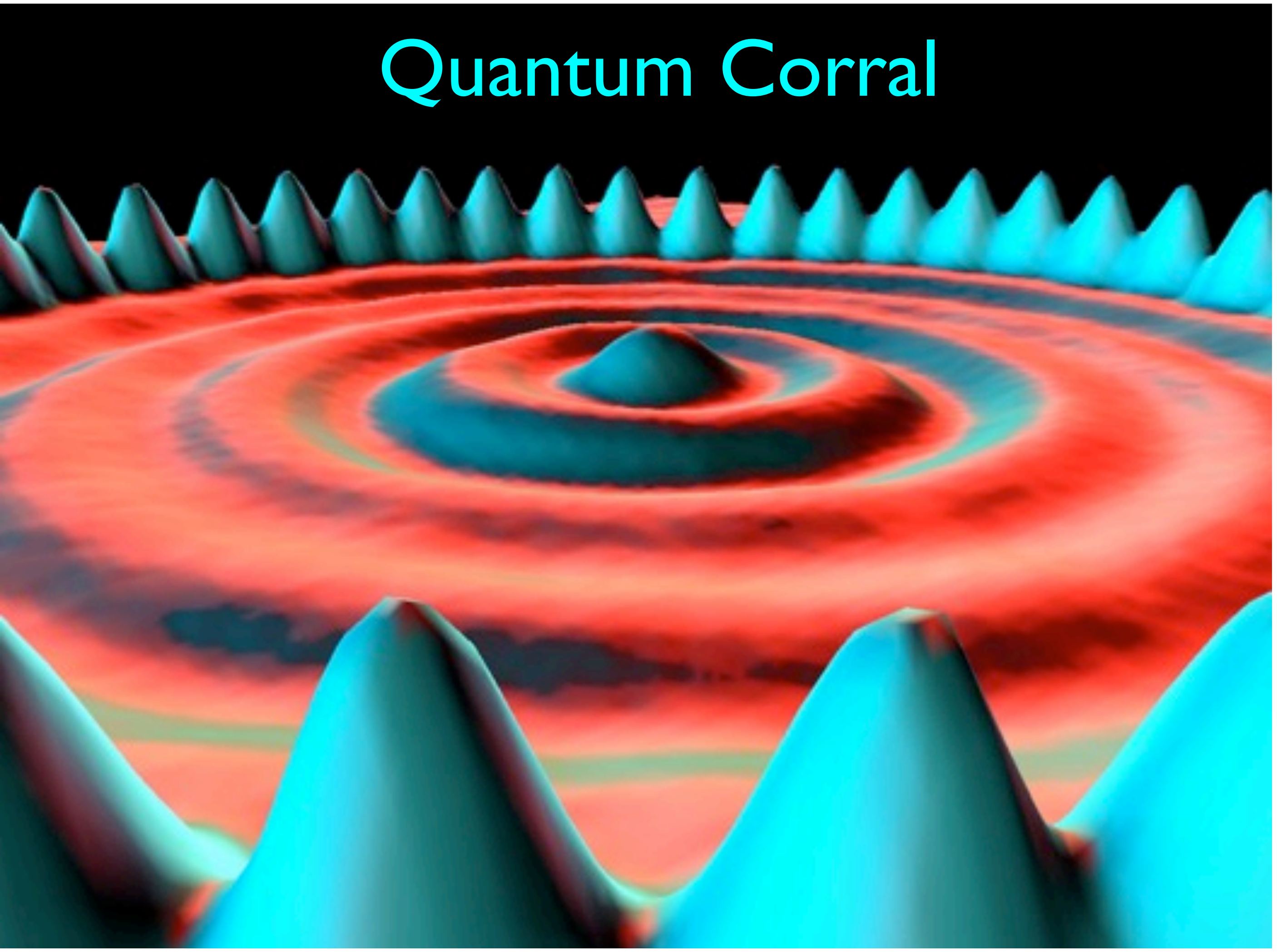
Figures are from Robert Eisberg and Robert Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles* (Wiley, 1985).

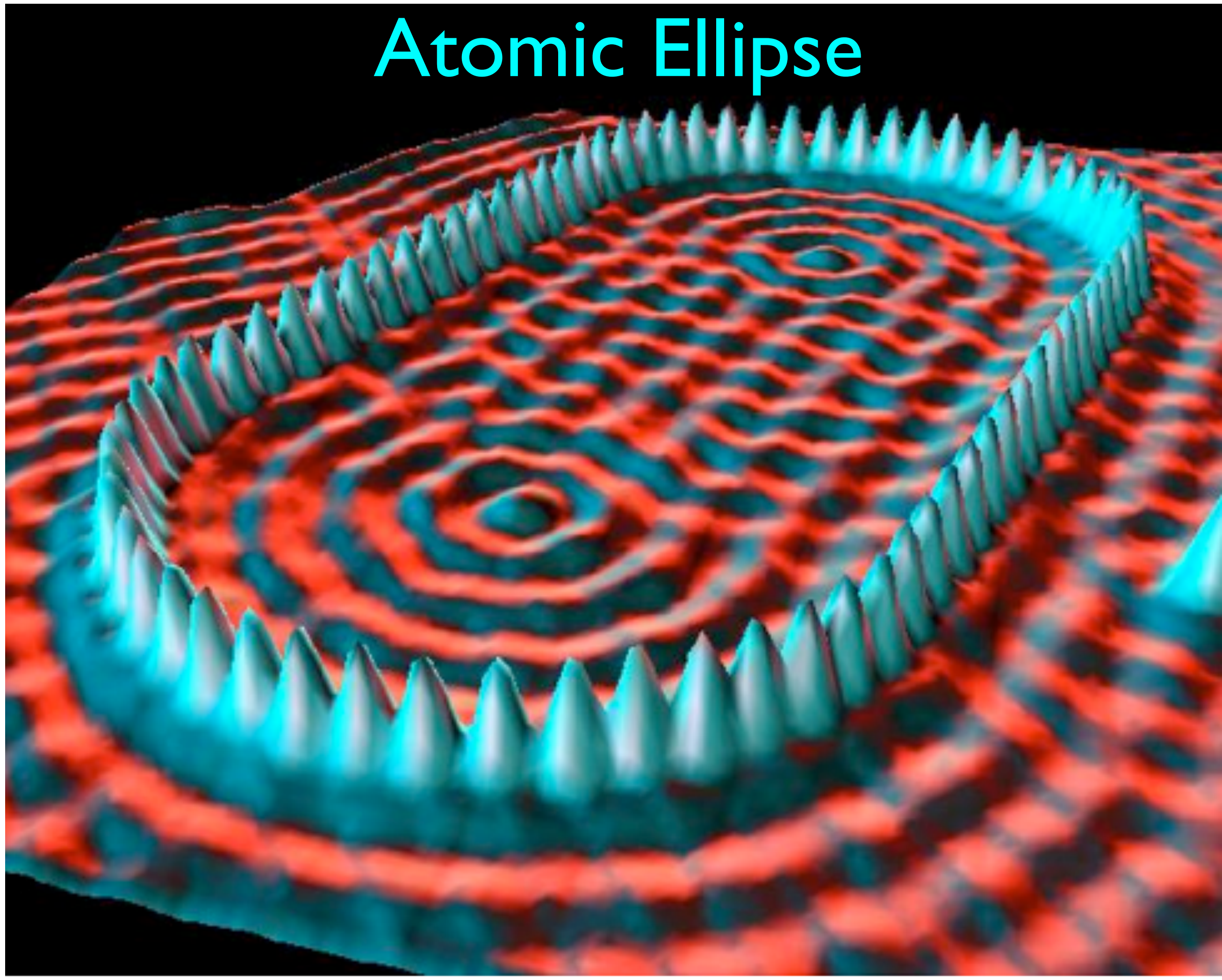
Scanning Tunneling Microscope

Barrier penetration is important in lots of scientific and practical devices. We'll discuss the scientific application to atomic-scale microscopy, and perhaps later discuss other applications to solid state devices. In a scanning tunneling microscope (STM), the tip of a probe scans across the sample using the piezo-electric effect. The current that flows between the tip and the sample must cross the electric potential barrier between them, and exponential dependence of barrier penetration insures that this current is exquisitely sensitive to the spacing between the tip and the surface.



Scanning tunneling microscopes normally generate images by holding the current between the tip of the electrode and the specimen at some constant (set-point) value by using a piezoelectric crystal to adjust the distance between the tip and the specimen surface, while the tip is piezoelectrically scanned in a raster pattern over the region of specimen surface being imaged. By holding the force, rather than the electric current, between tip and specimen at a set-point value, atomic force microscopes similarly allow the exploration of nonconducting specimens. In either case, when the height of the tip is plotted as a function of its lateral position over the specimen, an image that looks very much like the surface topography results.





The invention of the scanning tunneling microscope (STM) in 1981 allowed scientists to view the world from an atomic perspective for the first time. The revolutionary microscope, for which two IBM researchers Gerd Binnig and Heinrich Rohrer received the 1986 Nobel Prize in physics, revealed the topography of surfaces, atom by atom. As the STM evolved, its capabilities and those of related instruments have greatly expanded the abilities of research scientists to study a wide variety of atomic-scale structures and properties, and even to manipulate individual atoms and molecules.

Scanning tunneling microscopes make it possible not just to view atoms but to push them and even to rearrange them.