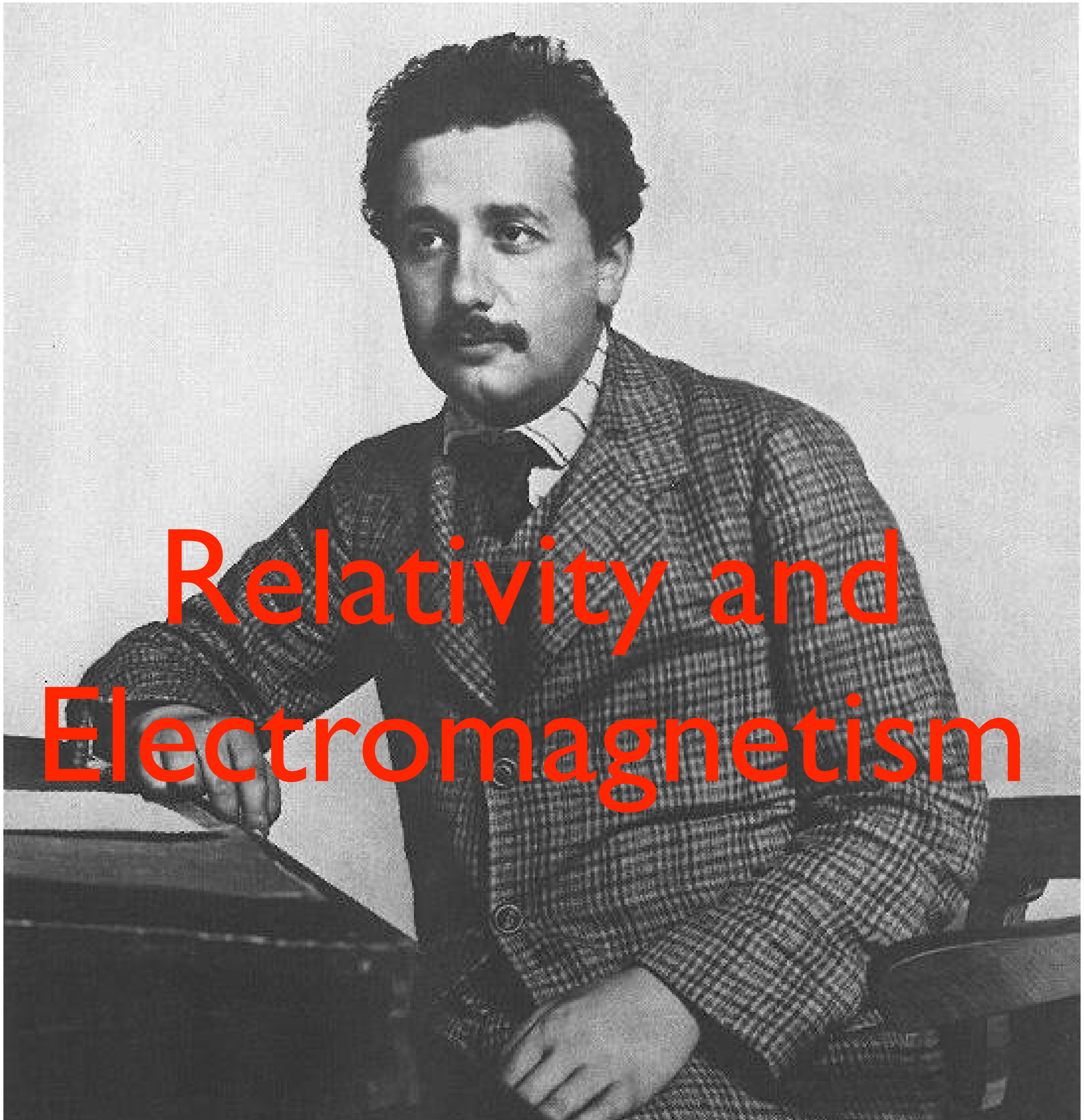
Physics 5K Lecture 9 Friday June 1, 2012



Reminder: Special Relativity is based on two postulates:

1. The Principle of Relativity: If a system of coordinates K is chosen so that, in relation to it, physical laws hold good in their simplest form [i.e., K is an inertial reference system], the same laws hold good in relation to any other system of coordinates K' moving in uniform translation relatively to K.

2. Invariance of the Speed of Light: Light in vacuum propagates with the speed c in terms of any system of inertial coordinates, regardless of the state of motion of the light source.

Relativity implies that the momentum p of a particle of rest mass m and velocity v is $p = m\gamma v$, where $\gamma = 1/(1- v^2/c^2)^{1/2}$. The energy of the particle, including its rest energy mc^2, is $E = m\gamma c^2$.

The laws of electrodynamics formulated by Maxwell are consistent with Special Relativity, and in fact Einstein was led to this theory through electrodynamics. His great 1905 paper on relativity is titled "On the Electrodynamics of Moving Bodies." In particular, the Lorentz force law

$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

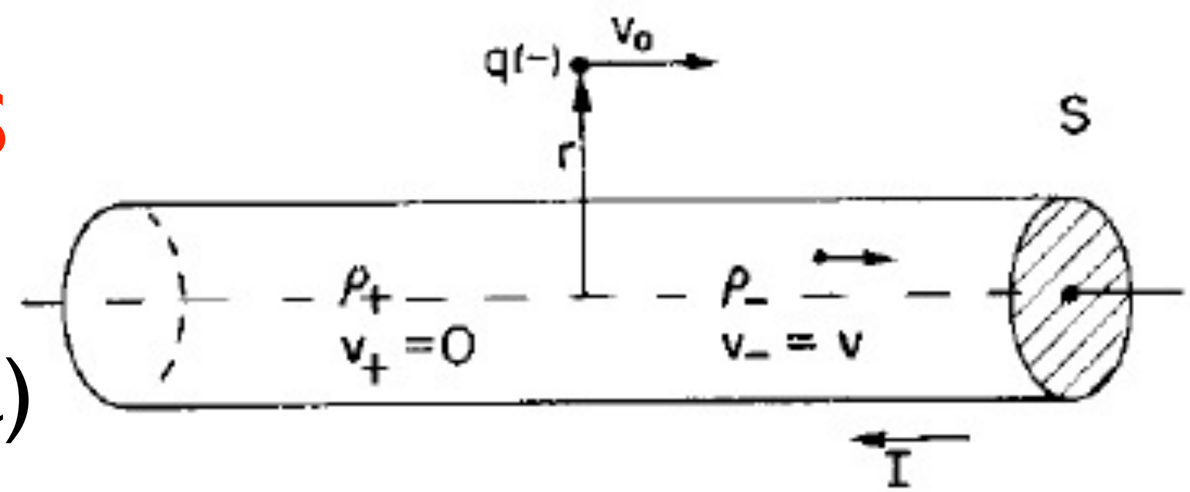
is true in any inertial reference frame.

We showed last week, using the Einstein's Rocket thought experiments, that lengths are contracted by a factor of $1/\gamma$ along the direction of motion ("Lorentz-Fitzgerald contraction," and that moving clocks run slow by the same factor ("time dilation").

Interaction of a Current with a Charge

To see why electromagnetism leads to relativity, let's look at this phenomenon from two different viewpoints. First, as in Fig. (a):

In Frame S

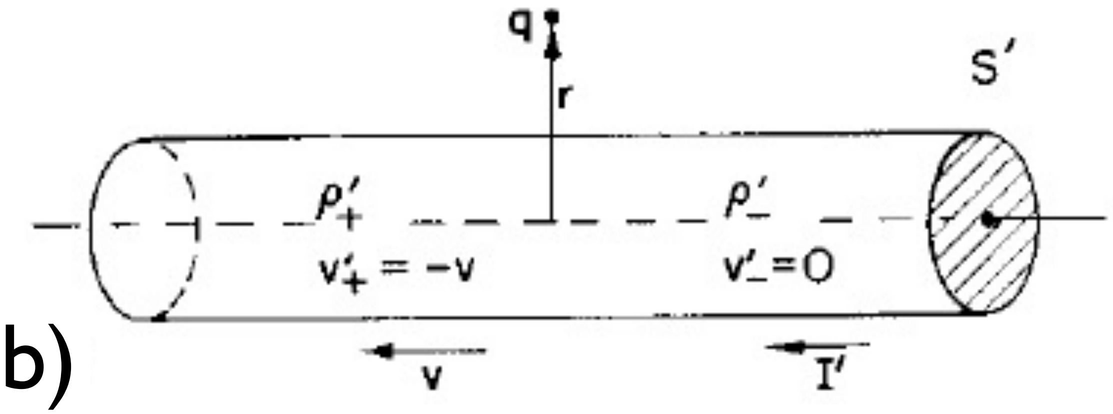


(a)

There is a current $I = \rho_- vA$ flowing in the wire, where ρ_- is the density of negative charges (i.e., the number of free electrons in the wire per unit volume), v is their average speed, and A is the area of the wire. This current produces a magnetic field **B** that circles around the wire. Its magnitude is $B = \mu_0 I/2\pi r$, where r is the radial distance from the wire and $\mu_0 = 4\pi \times 10^{-7}$ Tm/A is the "permeability of free space" (A=amp). Using the Lorentz force law, the force on the charge q is $F = qv_0 B = qv_0\, \mu_0 I/2\pi r = qv_0\, \mu_0 \rho_- vA/2\pi r = \mu_0 q\, v_0 v \rho_- A/2\pi r$. Since we know that parallel currents attract, if q is negative the force is toward the wire. Let's assume for simplicity that $v_0 = v$.

Now let's consider the situation from a reference frame S' moving with the speed v_0 of the charge q:

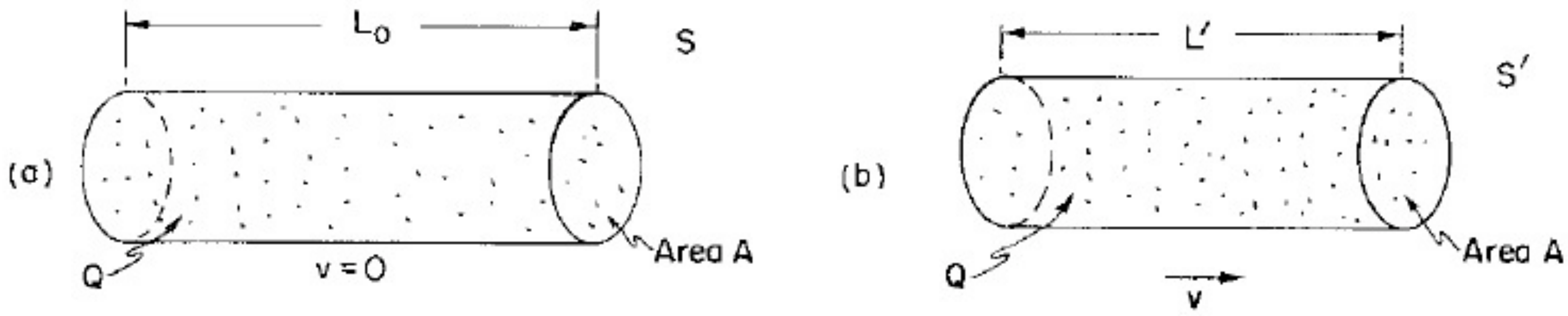
In Frame S'



(b)

There is a magnetic field in this reference frame since the positive charges (ions) are moving with speed -v. But since the charge q isn't moving in this reference frame, there can be no magnetic force on it. But there must be a force on q attracting it toward the wire, since we found such a force in frame S.

Because the ions are moving with speed v in frame S', the wire with the ions in it will be contracted along the direction of motion by a factor $1/\gamma$. This will cause the positive charge density to be greater than the negative charge density of the electrons by a factor γ, which will lead to an attractive electric force on charge q.



The positive charge density will be increased by the factor γ in frame S' since the length of the section of the wire shown has been decreased from L_0 to $L' = L_0/\gamma$. In frame S, the total charge density $\rho = \rho_- + \rho_+ = 0$ since the wire is neutral, so $\rho_- = - \rho_+$.
In frame S' we have just seen that $\rho'_+ = \rho_+ \gamma$. Since the electrons were moving with average speed v in frame S but are at rest in frame S', their charge density in S' will be $\rho'_- = \rho_-/\gamma$. The net charge density in S' will then be

$$\rho' = \rho'_- + \rho'_+ = \rho_- /\gamma + \rho_+\gamma = \rho_+(\gamma - 1/\gamma) = \rho_+ (v^2/c^2)\gamma$$

The E field from a line of charge density ρ' is $E = \rho' A/2\pi\varepsilon_0 r$, so the electric force on charge q due to the wire in frame S' is

$$F' = (q/2\pi\varepsilon_0 r)(\rho_+ A)(v^2/c^2)\gamma$$

It turns out that relativity implies that a force $F' = F\gamma$ in frame S' corresponds to a force F in frame S. (The drift velocity v is small compared to c, so this is a small effect in this situation.) Using this, we have calculated in frame S that the attractive magnetic force on q is

$$F = \mu_0 q\, v^2\, \rho_- A/2\pi r$$

while in frame S' we found an attractive electric force

$$F' = (q/2\pi\varepsilon_0 r)(\rho_+ A)(v^2/c^2)\ .$$

Taking into account that $\rho_- = \rho_+$, these will be equal if

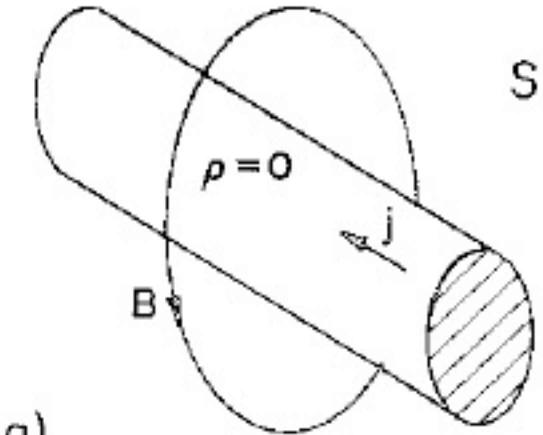
$$\mu_0 = 1/\varepsilon_0 c^2\ .$$

This is a valid equation — see Giancoli Eq. (31-12). Maxwell deduced that the speed of electromagnetic waves in vacuum is

$$c = 1/(\varepsilon_0\mu_0)^{1/2}\ .$$

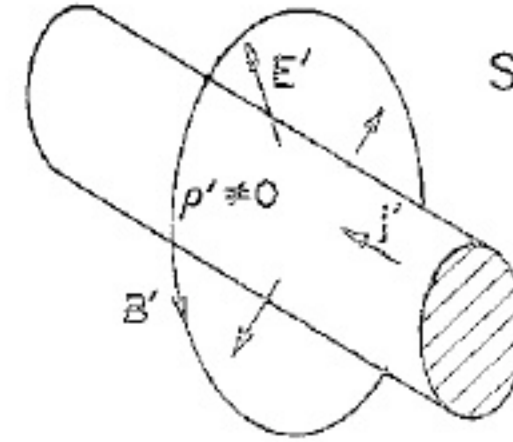
Thus a magnetic field in one reference frame turns into a combination of electric and magnetic fields in another frame!

A magnetic field in frame S



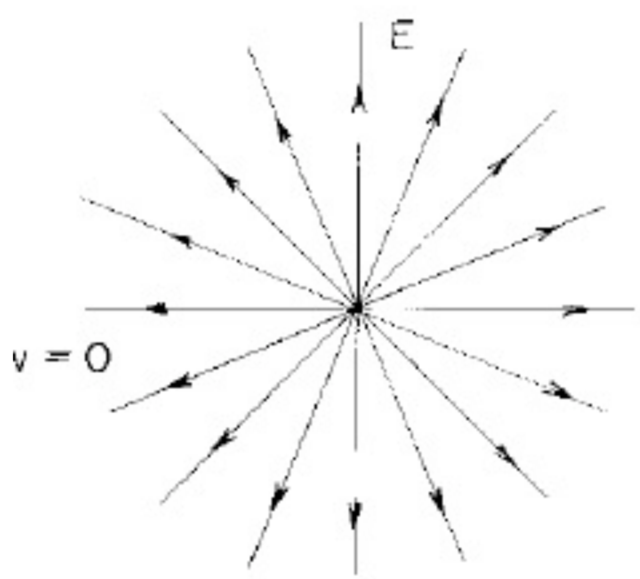
(a)

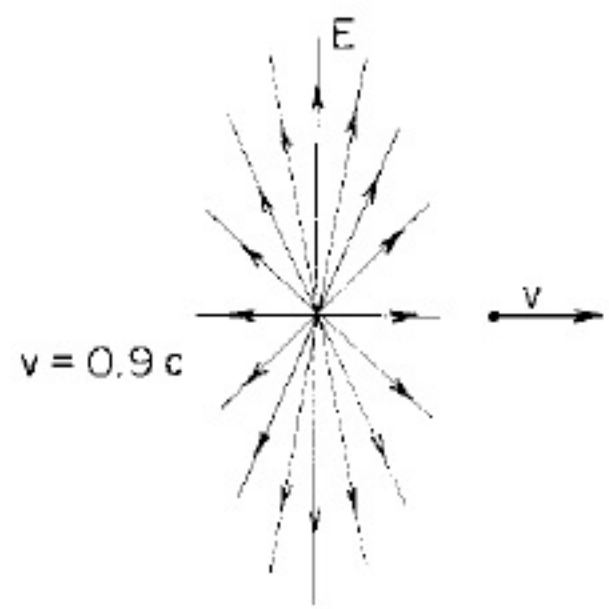
turns into a combination of electric and magnetic fields in frame S’



(b)

Since lengths along the direction of motion are contracted, you should not be surprised to find that the static electric field of a point charge in one reference frame is squashed in the direction of motion in a frame where the charge is moving rapidly





The Lorentz transformation of the electric and magnetic fields (Note: $c = 1$)

parallel to **v**

perpendicular to **v**

$$E'_x = E_x \qquad B'_x = B_x$$

$$E'_y = \frac{E_y - vB_z}{\sqrt{1 - v^2}} \qquad B'_y = \frac{B_y + vE_z}{\sqrt{1 - v^2}}$$

$$E'_z = \frac{E_z + vB_y}{\sqrt{1 - v^2}} \qquad B'_z = \frac{B_z - vE_y}{\sqrt{1 - v^2}}.$$

An alternative form for the field transformations (Note: $c = 1$)

parallel to **v**

perpendicular to **v**

$$E'_x = E_x \qquad B'_x = B_x$$

$$E'_y = \frac{(E + v \times B)_y}{\sqrt{1 - v^2}} \qquad B'_y = \frac{(B - v \times E)_y}{\sqrt{1 - v^2}}$$

$$E'_z = \frac{(E + v \times B)_z}{\sqrt{1 - v^2}} \qquad B'_z = \frac{(B - v \times E)_z}{\sqrt{1 - v^2}}$$

Still another form for the Lorentz transformation of E and B

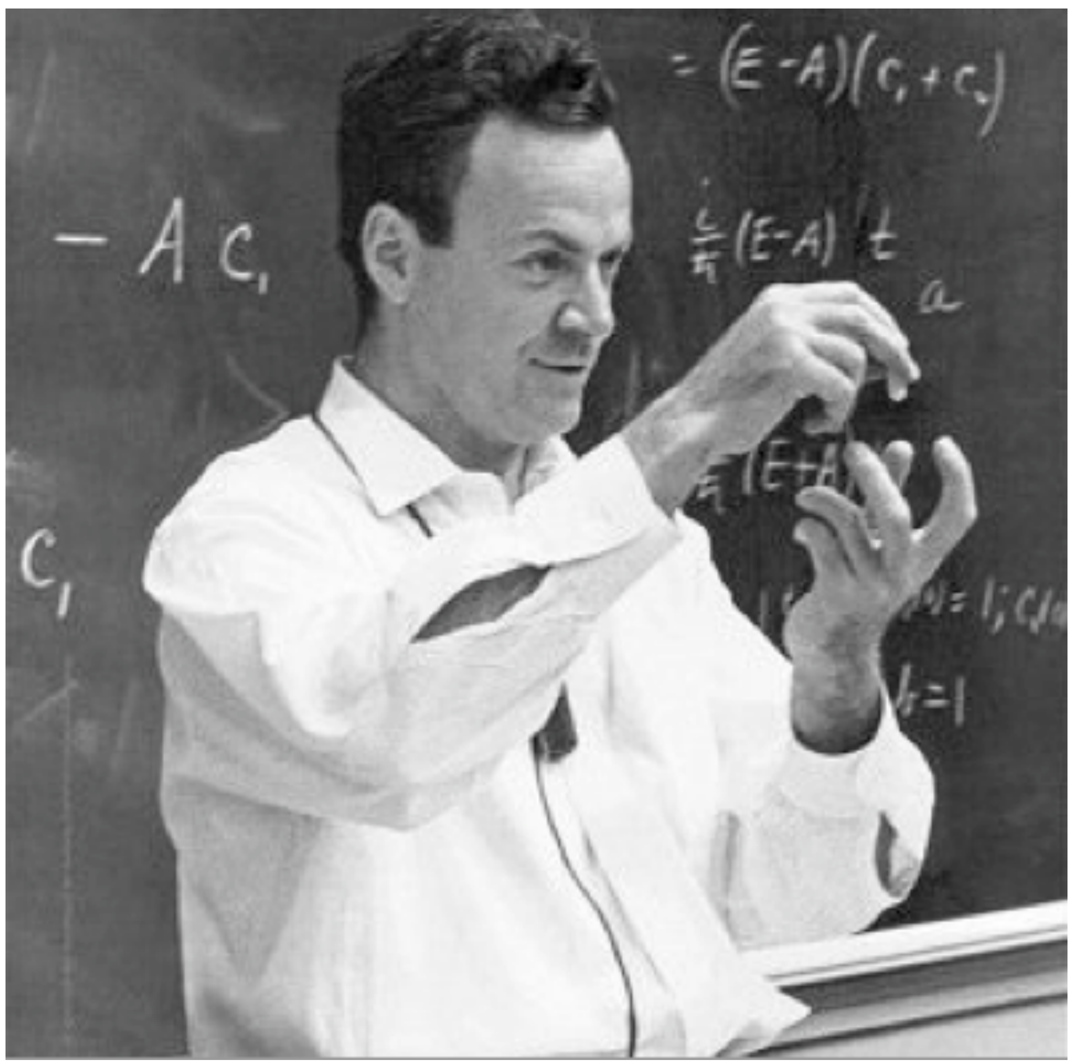
parallel to **v**

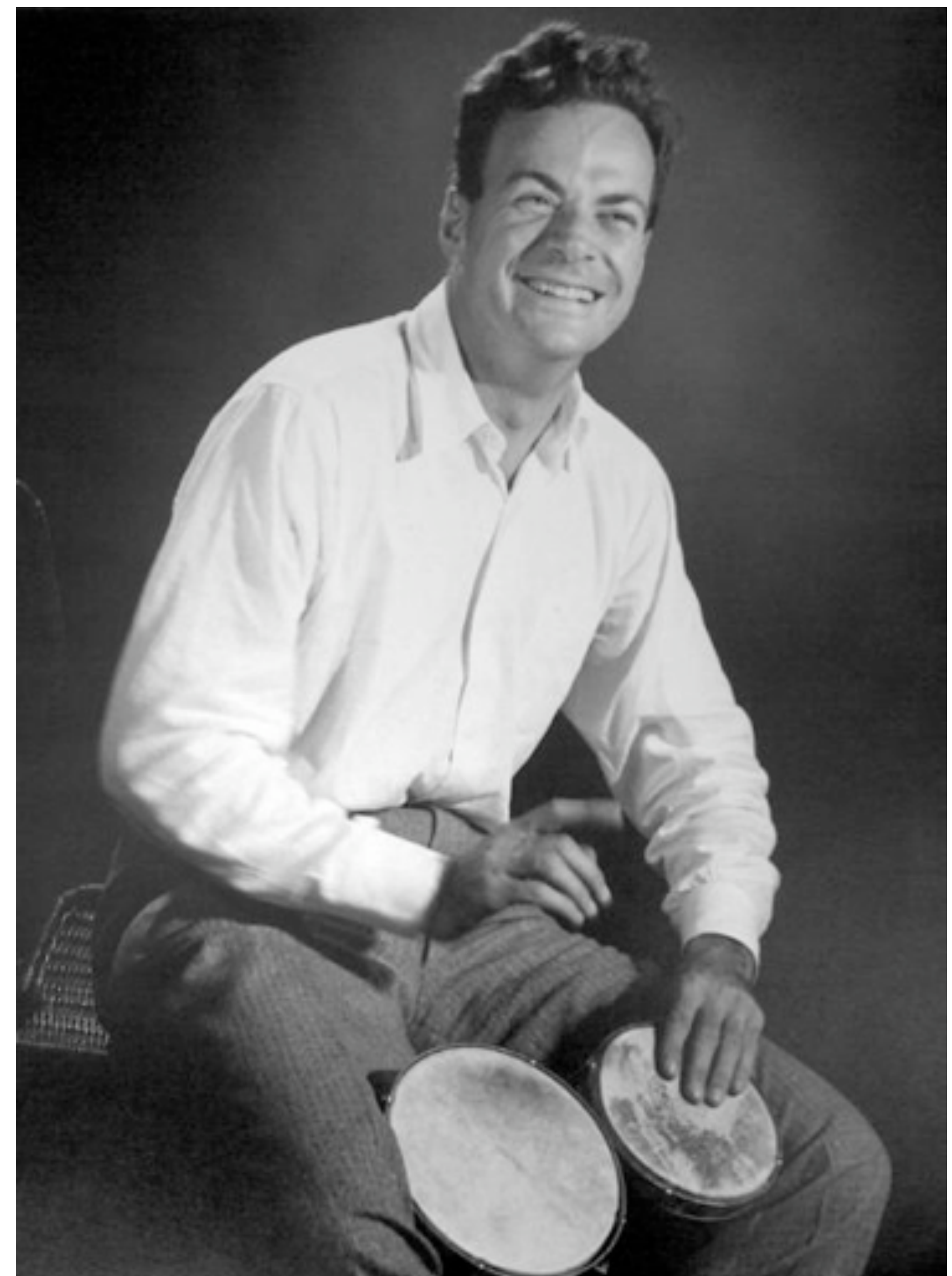
perpendicular to **v**

$$E'_\parallel = E \qquad B'_\parallel = B$$

$$E'_\perp = \frac{(E + v \times B)_\perp}{\sqrt{1 - v^2/c^2}} \qquad B'_\perp = \frac{\left(B - \dfrac{v \times E}{c^2}\right)_\perp}{\sqrt{1 - v^2/c^2}}$$

This material is taken from the Feynman Lectures on Physics, Volume 2, chapters 13 and 26.

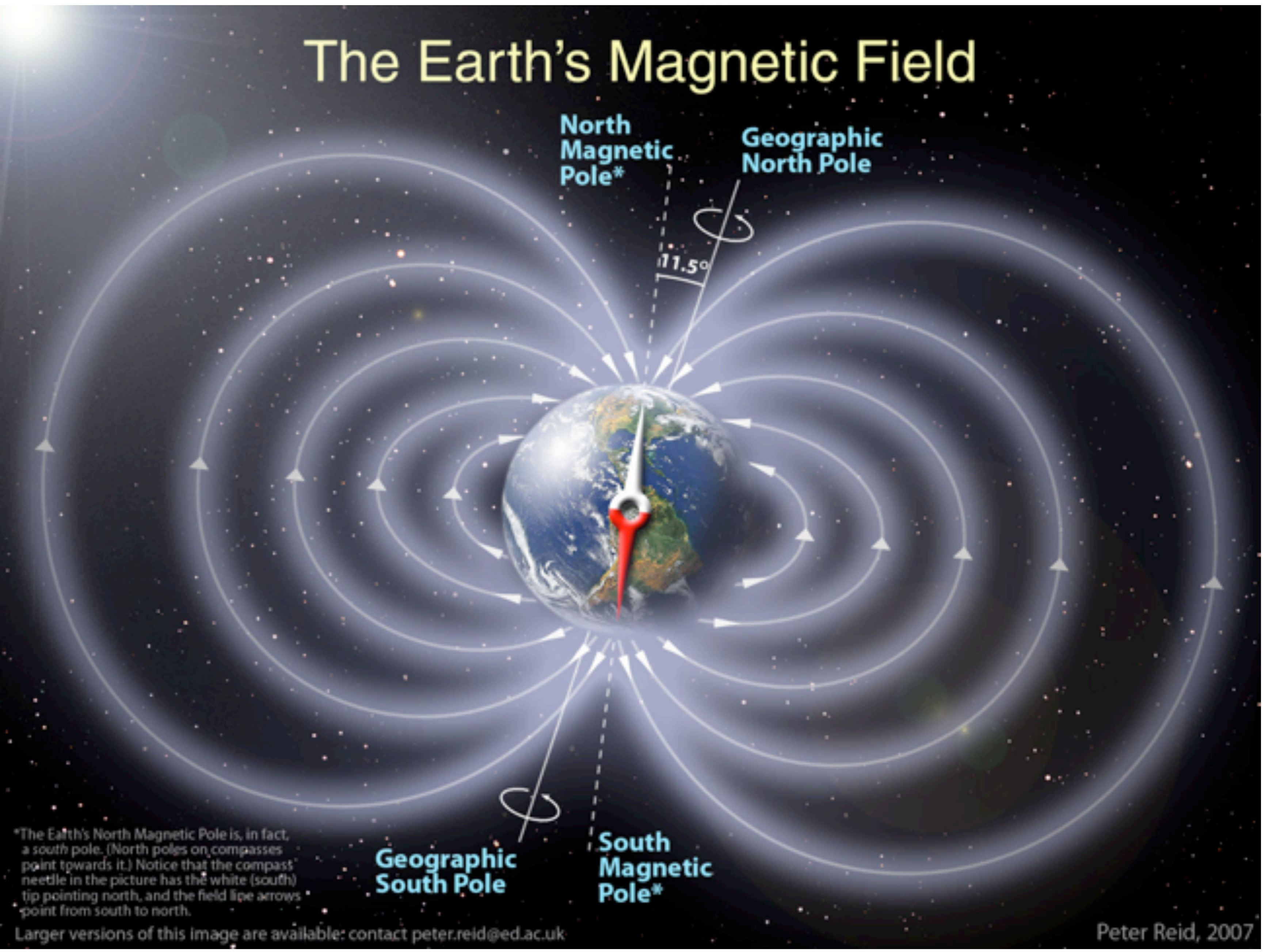




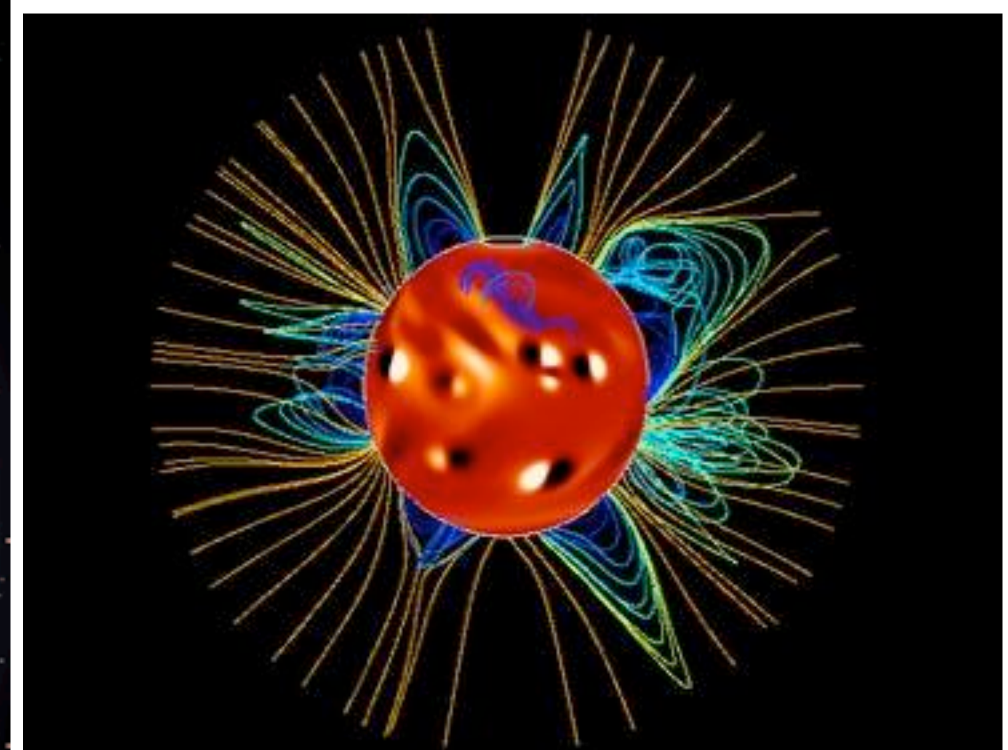
Einstein’s Special Theory of Relativity

http://physics.ucsc.edu/~snof/er.html

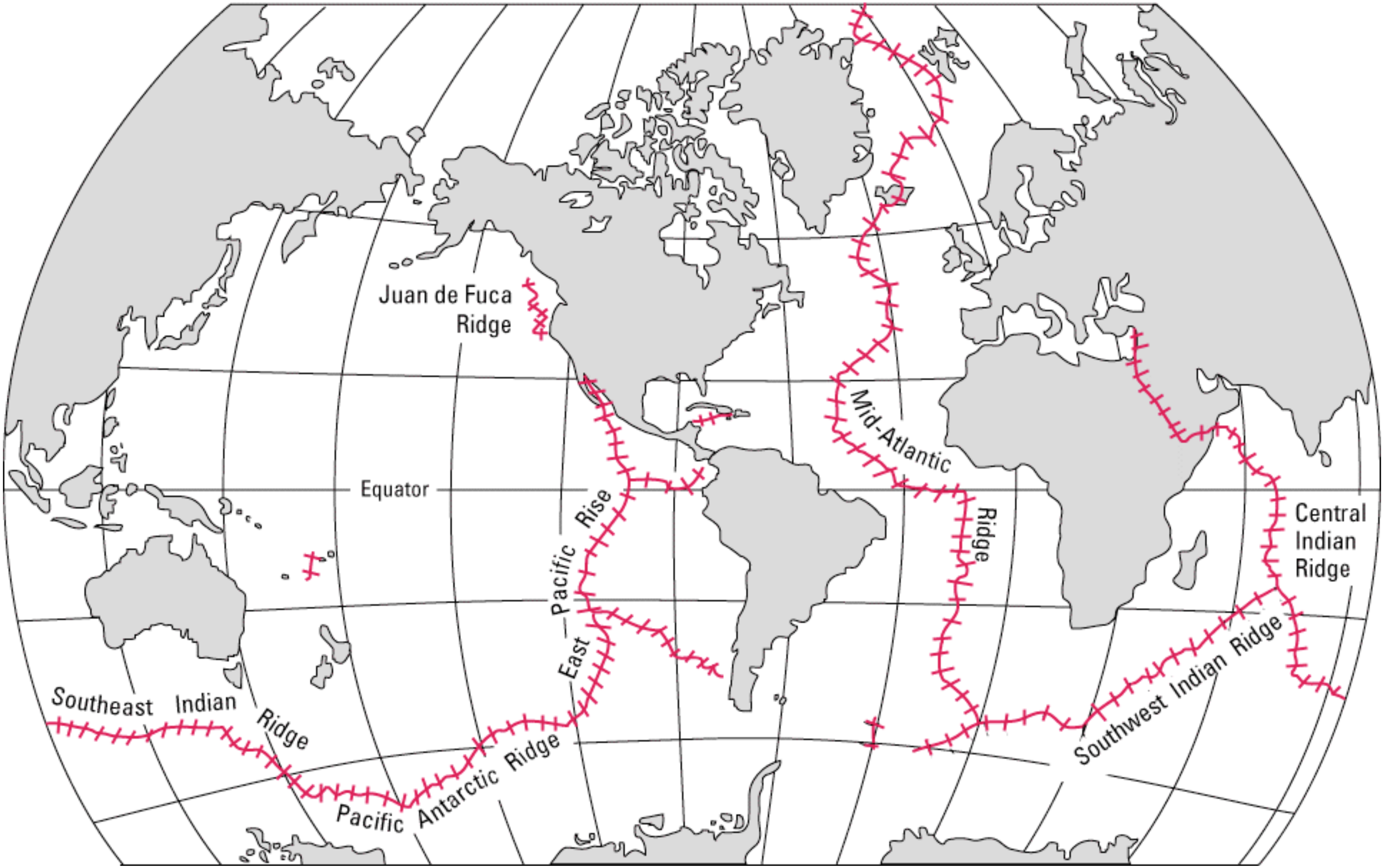
Magnetic Fields of the Earth and Sun and Their History



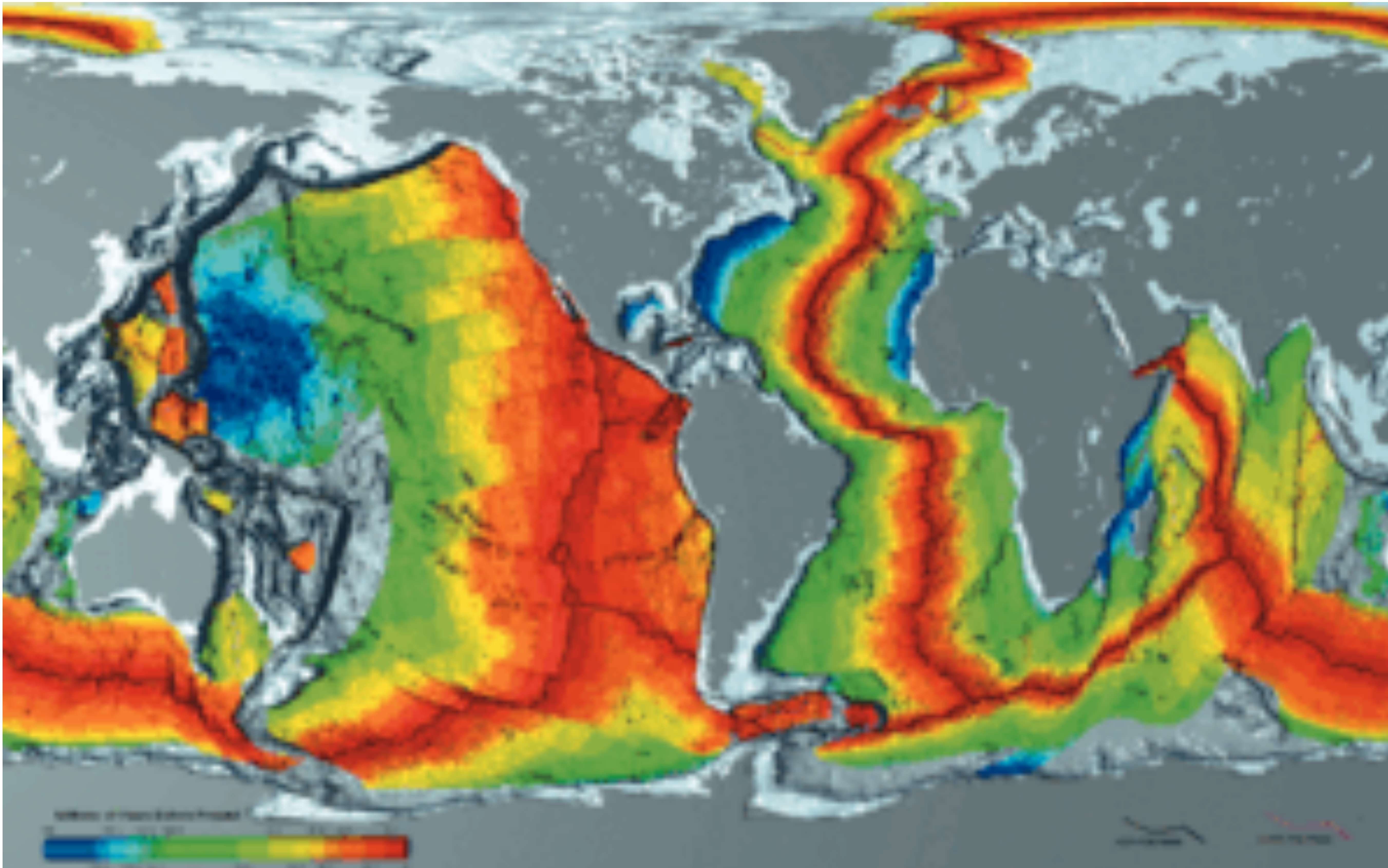
The Sun's Magnetic Field



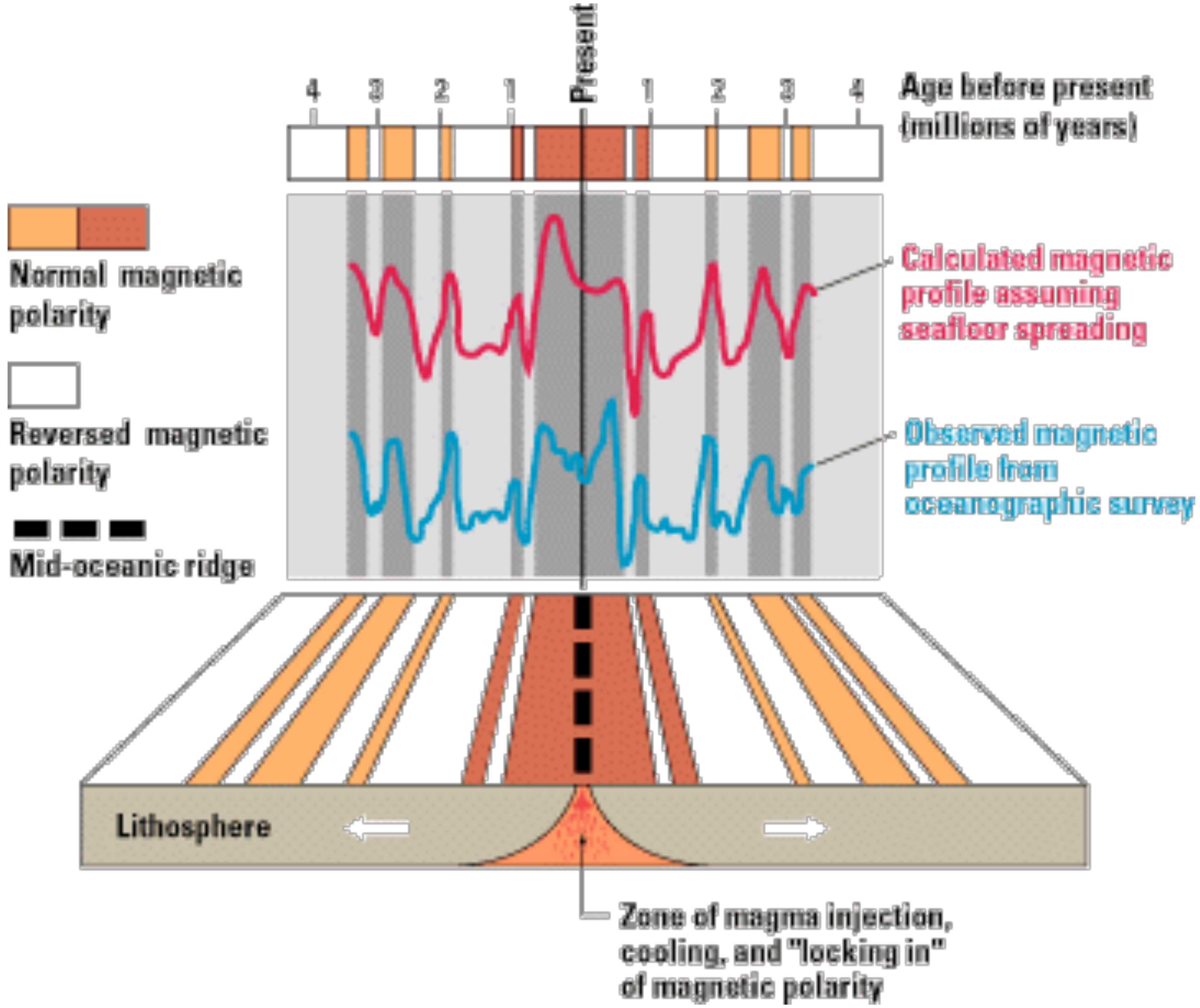
Mid-Ocean Ridges and Plate Tectonics



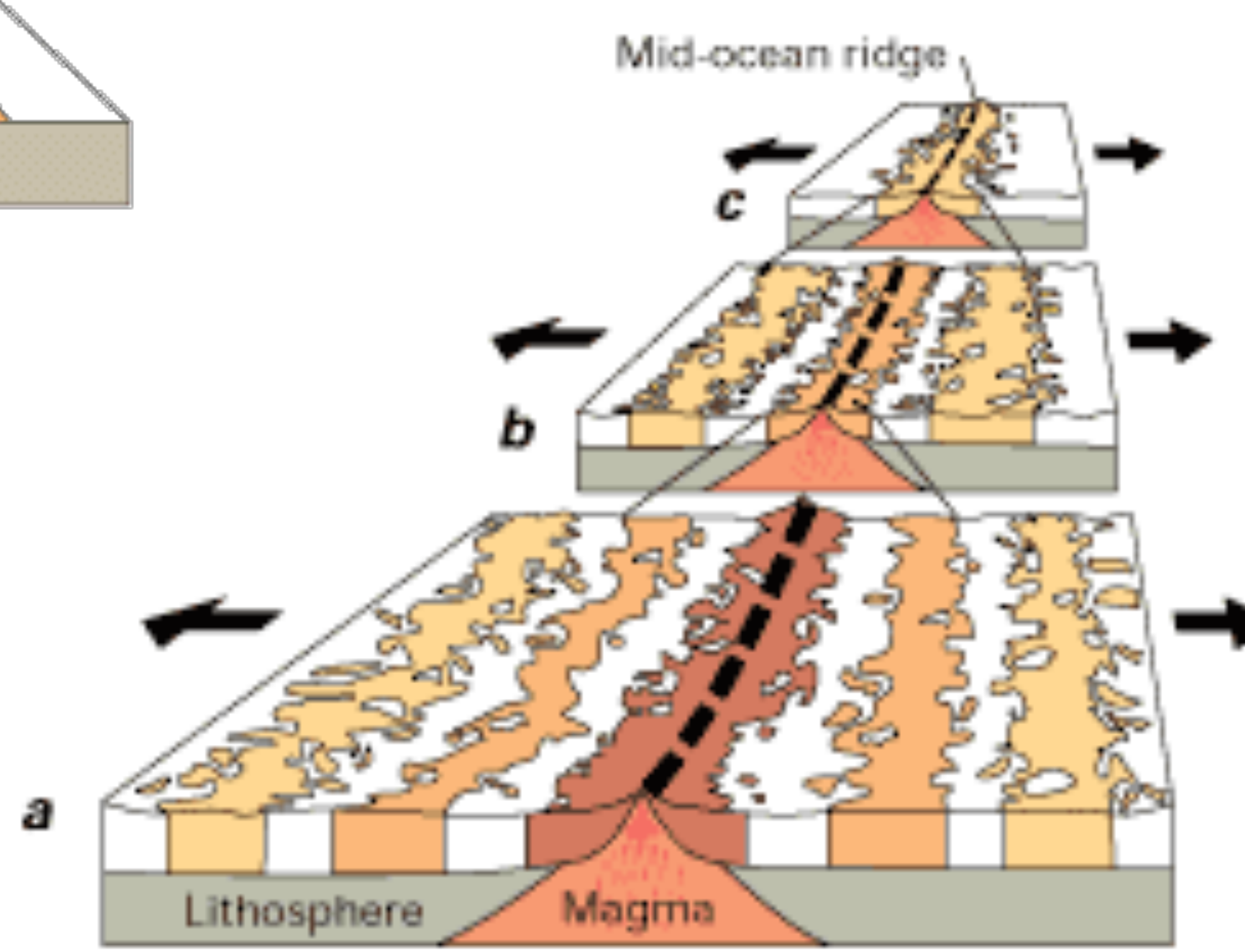
Seafloor Spreading Zones



Age of oceanic crust; youngest (red) is along spreading centers.

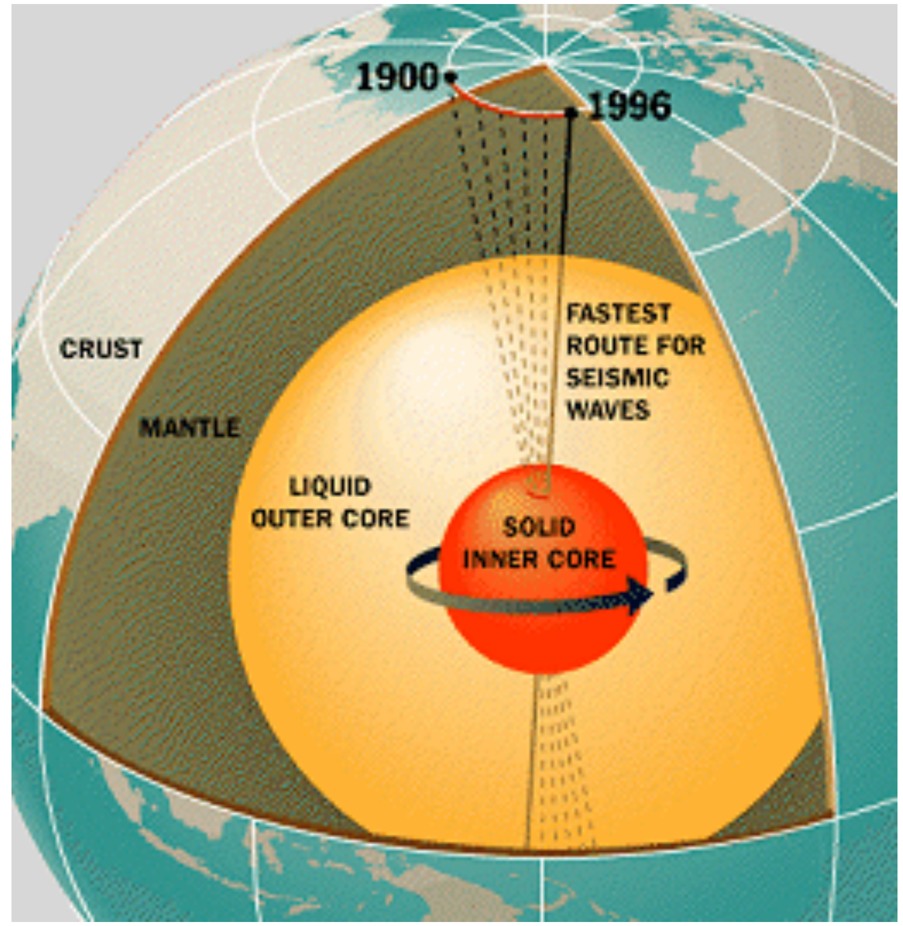


An observed magnetic profile (blue) for the ocean floor across the East Pacific Rise is matched quite well by a calculated profile (red) based on the Earth's magnetic reversals for the past 4 million years and an assumed constant rate of movement of ocean floor away from a hypothetical spreading center (bottom). The remarkable similarity of these two profiles provided one of the clinching arguments in support of the seafloor spreading hypothesis.

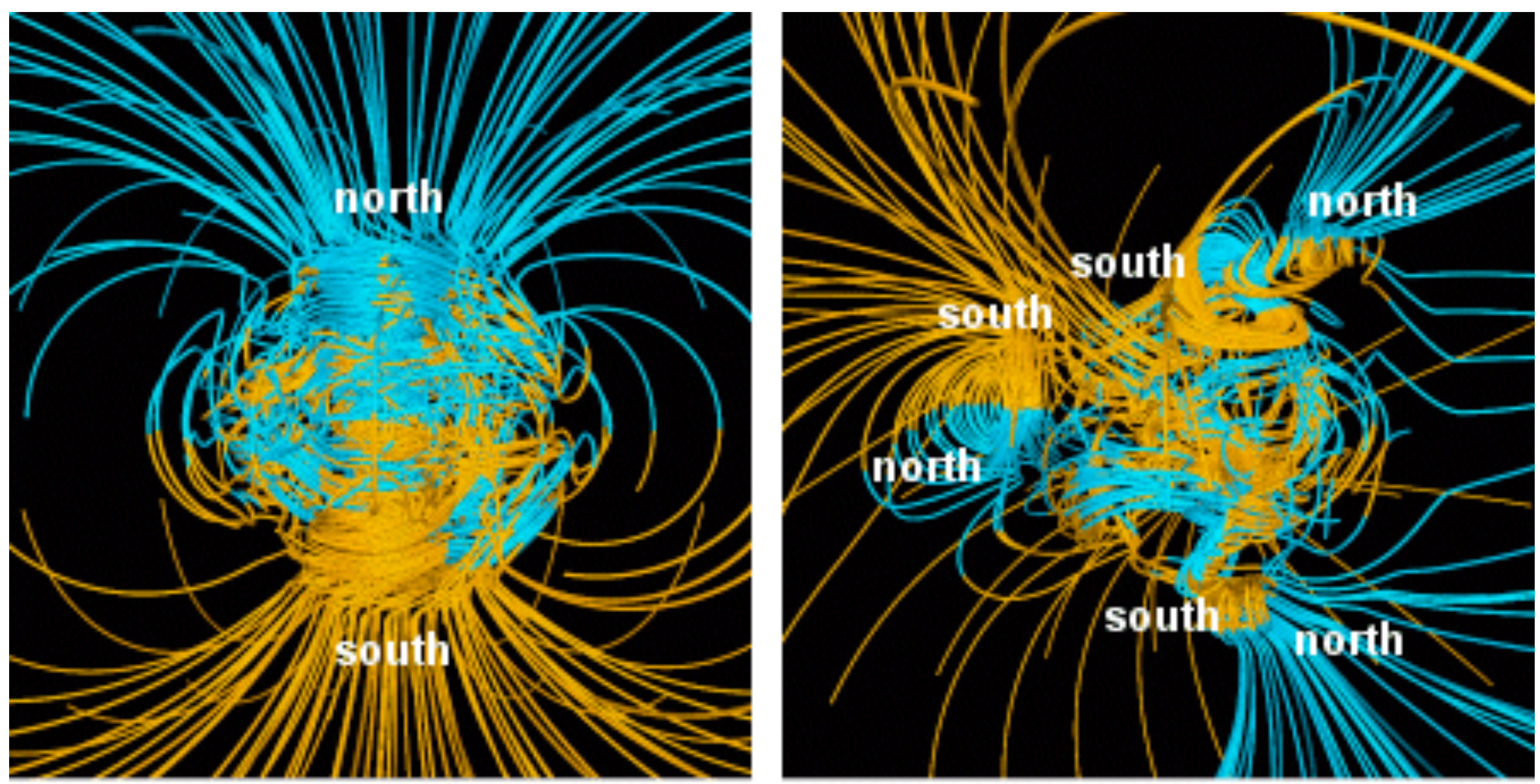


Geophysicists Allan Cox and Richard Doell and isotope geochemist Brent Dalrymple reconstructed the history of magnetic reversals for the past 4 million years using a dating technique based on the isotopes of the chemical elements potassium and argon. The potassium-argon technique -- like other "isotopic clocks" -- works because certain elements, such as potassium, contain unstable, *parent* radioactive isotopes that decay at a steady rate over geologic time to produce *daughter* isotopes. The decay of the radioactive potassium isotope (potassium-40) yields a stable daughter isotope (argon-40), which does not decay further. The age of a rock can be determined ("dated") by measuring the total amount of potassium in the rock, the amount of the remaining radioactive potassium-40 that has not decayed, and the amount of argon-40. Potassium-40 isotope has a half-life of 1,310 million years, it can be used in dating rocks millions of years old.

The Moving Magnetic Pole




Gary Glazmaier



between reversals during a reversal

Simulation by Gary Glazmaier, UCSC


SCIENTIFIC AMERICAN
WHAT FLIPS THE MAGNETIC POLES? THE ANSWER LURKS IN THE EARTH'S
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Contemporary Physics
Timescales of the Paleomagnetic Field
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Towards smaller memory devices
A giant carnivorous dinosaur
Chromatin dynamics in vivo
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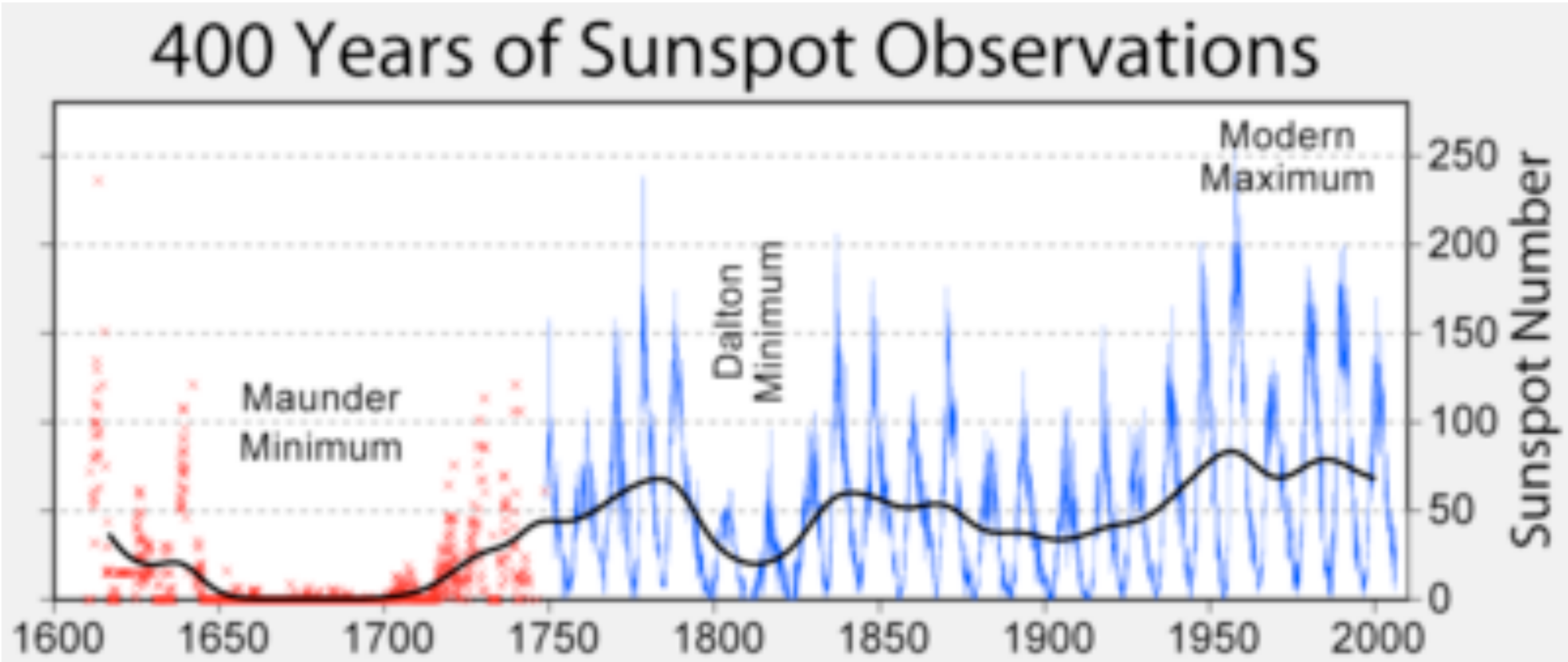
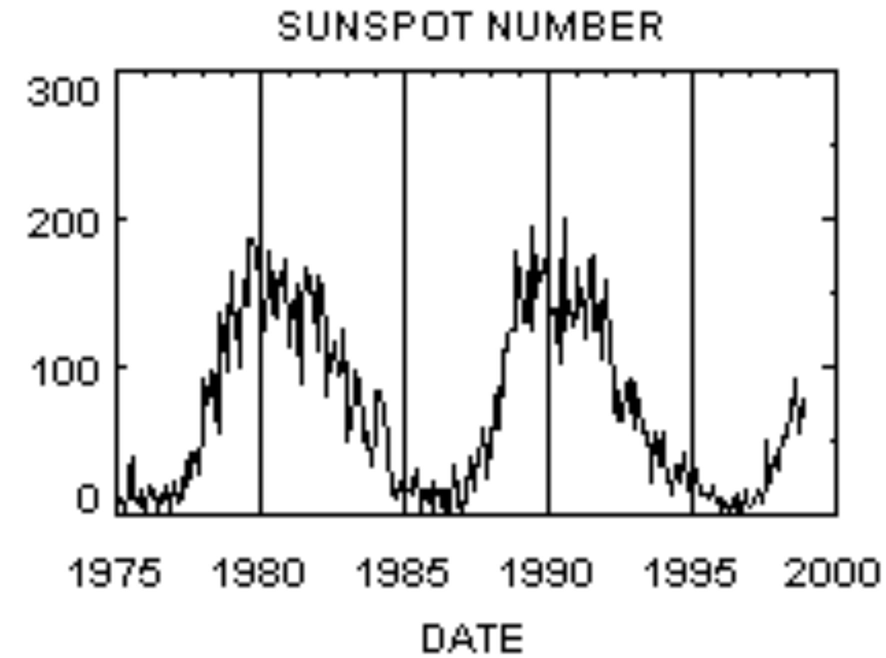
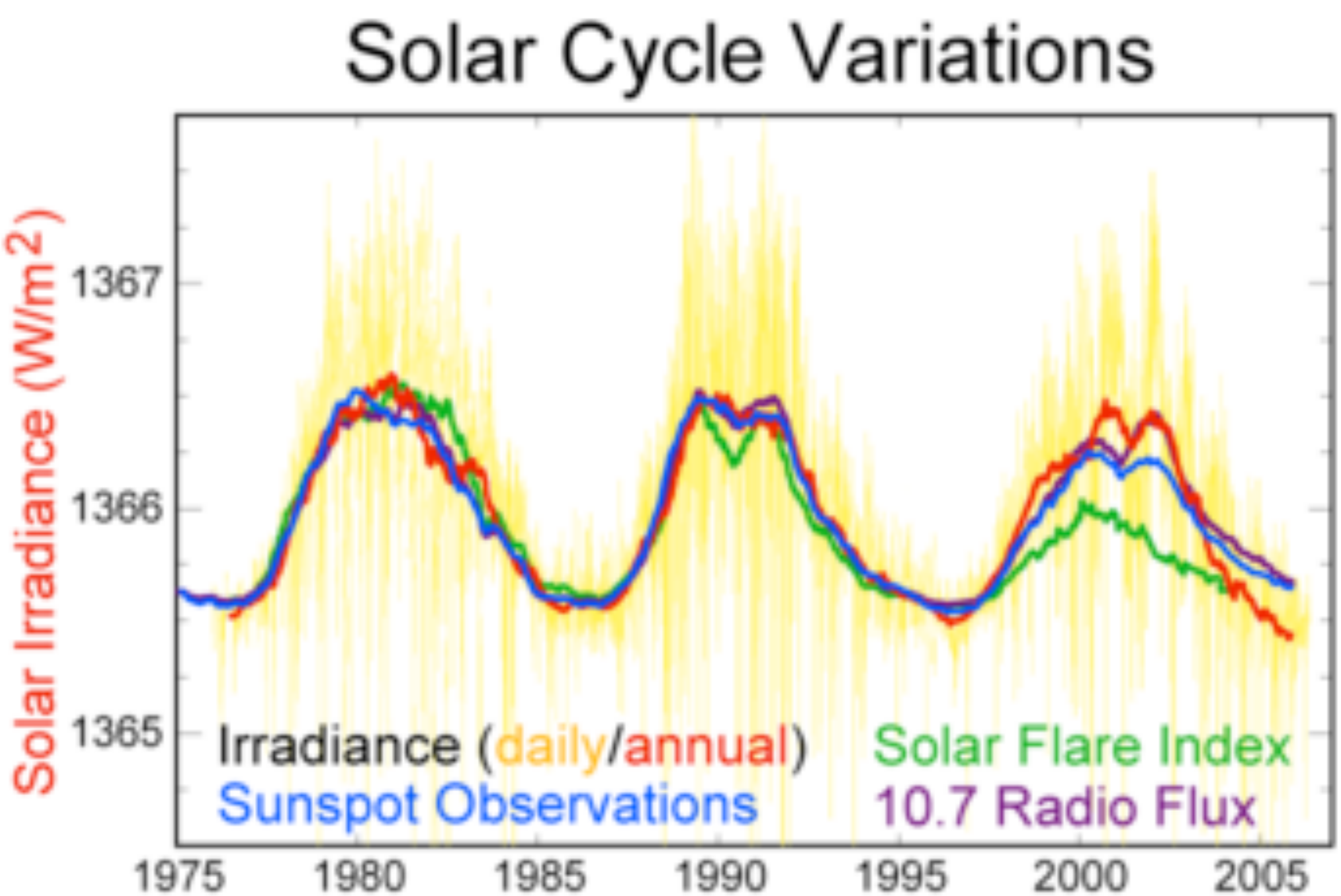
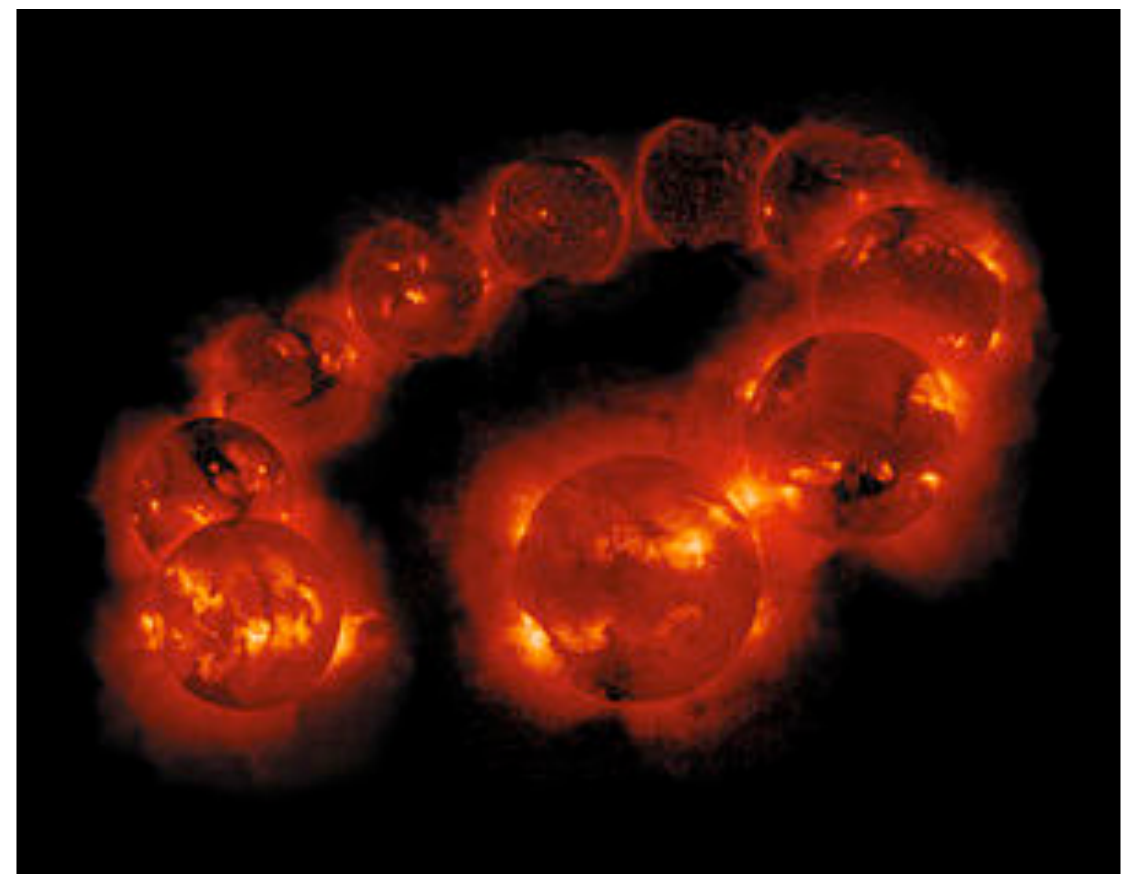
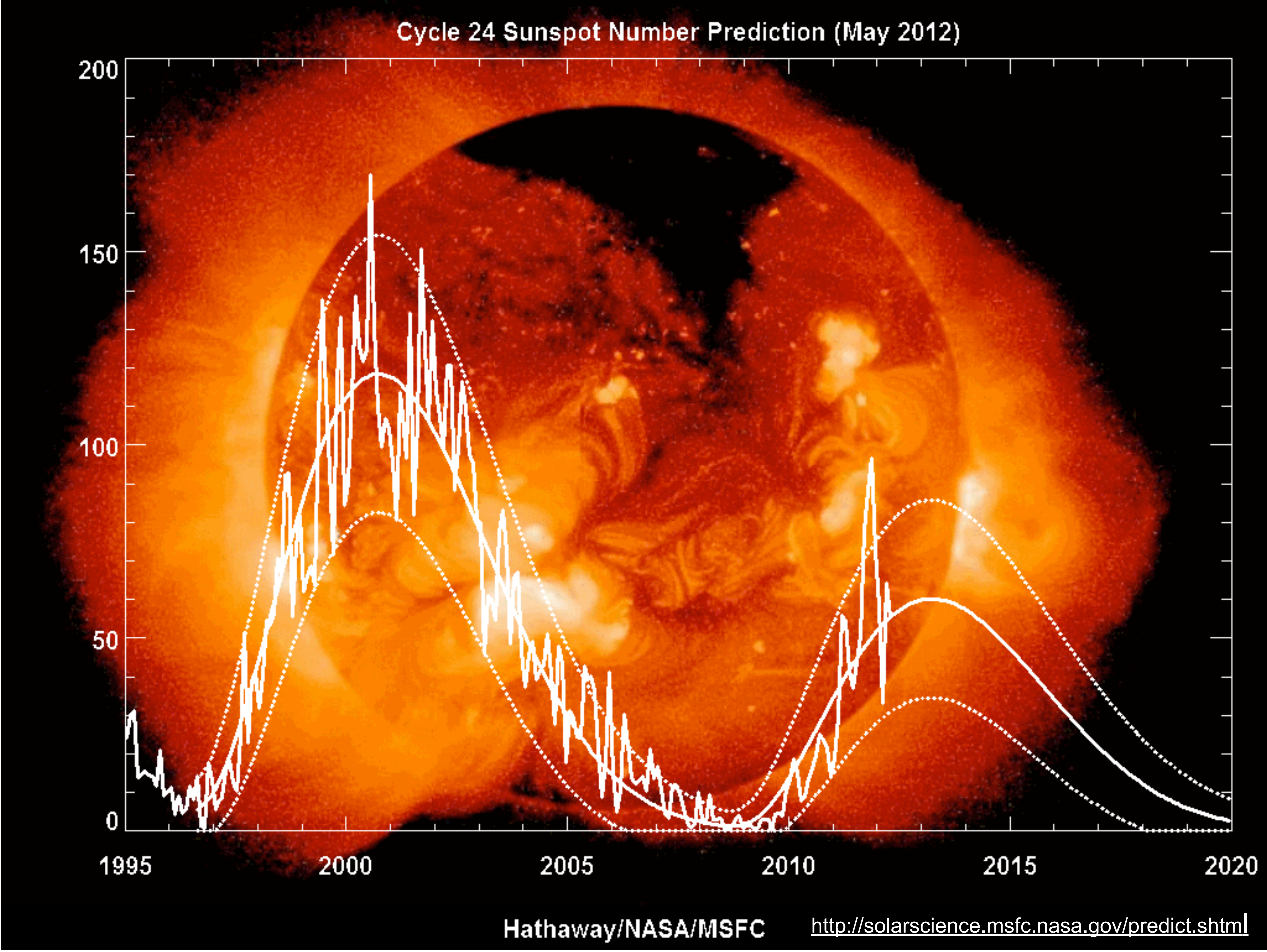








A solar cycle: a montage of ten years' worth of Yohkoh SXT images, demonstrating the variation in solar activity during a sunspot cycle, from after August 30, 1991, to September 6, 2001. Credit: the Yohkoh mission of ISAS (Japan) and NASA (US).


Cycle 24 Sunspot Number Prediction (May 2012)
200
150
100
50
0
1995
2000
2005
2010
2015
2020
Hathaway/NASA/MSFC
http://solarscience.msfc.nasa.gov/predict.shtml