

Modern Cosmology

A series of major discoveries has laid a lasting foundation for cosmology. Einstein's general relativity (1916) provided the conceptual foundation for the modern picture. Then Hubble discovered that "spiral nebulae" are large galaxies like our own Milky Way (1925), and that distant galaxies are receding from the Milky Way with a speed proportional to their distance (1929), which means that we live in an expanding universe. The discovery of the cosmic background radiation (1965) showed that the universe began in a very dense, hot, and homogeneous state: the Big Bang. This was confirmed by the discovery that the cosmic background radiation has exactly the same spectrum as heat radiation (1989), and the measured abundances of the light elements agree with the predictions of Big Bang theory if the abundance of ordinary matter is about 4% of critical density. Most of the matter in the universe is invisible particles which move very sluggishly in the early universe ("Cold Dark Matter Most of the energy density is mysterious dark energy.

Experimental and Historical Sciences both make predictions about new knowledge, whether from experiments or from the past

Historical Explanation Is Always Inferential

Our age cannot look back to earlier things Except where reasoning reveals their traces Lucretius

Patterns of Explanation Are the Same in the Historical Sciences as in the Experimental Sciences

Specific conditions + General laws ⇒ Particular phenomenon

In history as anywhere else in empirical science, the explanation of a phenomenon consists in subsuming it under general empirical laws; and the criterion of its soundness is ... exclusively whether it rests on empirically well confirmed assumptions concerning initial conditions and general laws.

C.G. Hempel, Aspects of Scientific Explanation (1965), p. 240.

Successful Predictions of the Big Bang

First Prediction

First Confirmation

Expansion of the Universe

Friedmann 1922, Lemaitre 1927 based on Einstein 1916 Hubble 1929

Cosmic Background Radiation

Existence of CBR

Gamow, Alpher, Hermann 1948

Penzias & Wilson 1965

CBR Thermal Spectrum

Peebles 1966

COBE 1989

CBR Fluctuation Amplitude

Cold Dark Matter theory 1984

COBE 1992

CBR Acoustic Peak

BOOMERANG 2000 MAXIMA 2000

Light Element Abundances

Peebles 1966, Wagoner 1967

D/H Tytler et al.1997

Three Pillars of the Big Bang Expanding Universe Universe The variation of the Earth's display a perfect fit with heat radiation with a term

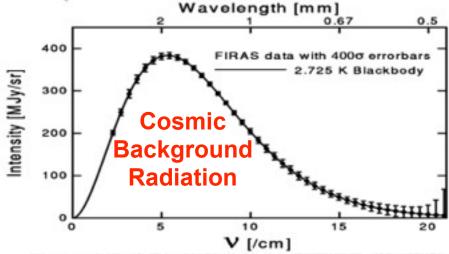
A modern illustration of Hubble's Law, displaying the increase of recession speed of galaxies growing in direct proportion to their distance.

Big Bang Nucleosynthesis

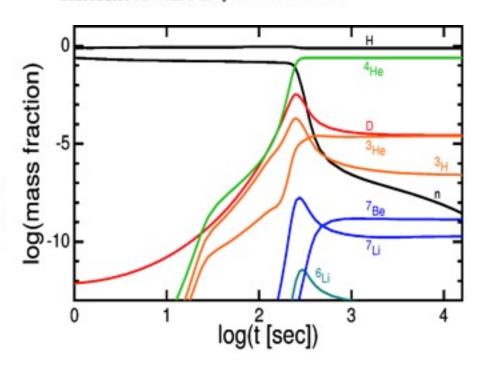
Distance [Mpc]

The detailed production of the lightest elements out of protons and neutrons during the first three minutes of the universe's history. The nuclear reactions occur rapidly when the temperature falls below a billion degrees Kelvin. Subsequently, the reactions are shut down, because of the rapidly falling temperature and density of matter in the expanding universe.

Caution: 7Li may now be discordant



The variation of the intensity of the microwave background radiation with its frequency, as observed by the COBE satellite from above the Earth's atmosphere. The observations (boxes) display a perfect fit with the (solid) curve expected from pure heat radiation with a temperature of 2.73°K.



General Relativity

GR follows from the principle of equivalence and Einstein's equation $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi GT_{\mu\nu}$.* Einstein had intuited the local equivalence of gravity and acceleration in 1907 (Pais, p. 179), but it was not until November 1915 that he developed the final form of the GR equation.

(Gravitation & Cosmology)

It can be derived from the following assumptions (Weinberg, p. 153):

- 1. The <u>l.h.s.</u> $G_{\mu\nu}$ is a tensor
- 2. $G_{\mu\nu}$ consists only of terms linear in second derivatives or quadratic in first derivatives of the metric tensor $g_{\mu\nu}$ ($\Leftrightarrow G_{\mu\nu}$ has dimension L⁻²)
- 3. Since $T_{\mu\nu}$ is symmetric in $\mu\nu$, so is $G_{\mu\nu}$
- 4. Since $T_{\mu\nu}$ is conserved (covariant derivative $T^{\mu}_{\nu;\mu}=0$) so also $G^{\mu}_{\nu;\mu}=0$
- 5. In the weak field limit where $g_{00} \approx -(1+2\phi)$, satisfying the Poisson equation $\nabla^2 \phi = 4\pi G \rho$ (i.e., $\nabla^2 g_{00} = -8\pi G T_{00}$), we must have $G_{00} = \nabla^2 g_{00}$

^{*}Note: we're here using the metric -1, 1,1,1 as in Dodelson, Weinberg.

Einstein's equation can also be derived from an action principle, varying the total action $I = I_M + I_G$, where I_M is the action of matter and I_G is that of gravity:

$$I_G = -\frac{1}{16\pi G} \int R(x) \sqrt{g(x)} d^4x$$

(see, e.g., Weinberg, p. 364). The curvature scalar $R \equiv R_{\mu\nu}$ g^{$\mu\nu$} is the obvious term to insert in I_G since a scalar connected with the metric is needed and it is the only one, unless higher powers R^2 , R^3 or higher derivatives $\Box R$ are used, which will lead to higher-order or higher-derivative terms in the gravity equation.

Einstein realized in 1916 that the 5^{th} postulate above isn't strictly necessary – merely that the equation reduce to the Newtonian Poisson equation within observational errors, which allows the inclusion of a small cosmological constant term. In the action derivation, such a term arises if we just add a constant to R.

General Relativity and Cosmology

John Wheeler:

GR: MATTER TELLS SPACE HOW TO CURVE

CURVED SPACE TELLS MATTER HOW TO MOVE

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = 8\pi G T^{\mu\nu} + \Lambda g^{\mu\nu}$$

$$\frac{du^{\mu}}{ds} + \Gamma^{\mu}_{\alpha\beta} u^{\alpha} u^{\beta} = 0$$

Cosmological Principle: on large scales, space is uniform and isotropic. COBE-Copernicus Theorem: If all observers observe a nearly-isotropic Cosmic Background Radiation (CBR), then the universe is locally nearly homogeneous and isotropic – i.e., is approximately described by the Friedmann-Robertson-Walker metric

$$ds^{2} = dt^{2} - a^{2}(t) \left[dr^{2} (1 - kr^{2})^{-1} + r^{2} d\Omega^{2} \right]$$

with curvature constant k = -1, 0, or +1. Substituting this metric into the Einstein equation at left above, we get the Friedmann eq.

Robertson- FRW E(ii)
$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p - \frac{k}{a^2} + \Lambda$$
 Walker

$$H_0 \equiv 100 h \,\mathrm{km \, s^{-1} Mpc^{-1}} \equiv 70 h_{70} \,\mathrm{km \, s^{-1} Mpc^{-1}}$$

Framework
$$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda \text{ with } H \equiv \frac{\dot{a}}{a}, \ a_0 \equiv 1, \ \Omega_0 \equiv \frac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}, \\ \rho_{c,0} \equiv \frac{3H_0^2}{8\pi G} = 1.36 \times 10^{11} h_{70}^2 M_{\odot} \mathrm{Mpc}^{-3}$$

(homogeneous, universe)

isotropic
$$E(ii) - E(00) \Rightarrow \frac{2\ddot{a}}{a} = -\frac{8\pi}{3}G\rho - 8\pi Gp + \frac{2}{3}\Lambda$$

Divide by
$$2E(00) \Rightarrow q_0 \equiv -\left(\frac{\ddot{a}}{a} \frac{a^2}{\dot{a}^2}\right)_0 = \frac{\Omega_0}{2} - \Omega_{\Lambda}$$

$$E(00) \Rightarrow t_0 = \int_0^1 \frac{da}{a} \left[\frac{8\pi}{3} G \rho - \frac{k}{a^2} + \frac{\Lambda}{3} \right]^{-\frac{1}{2}} = H_0^{-1} \int_0^1 \frac{da}{a} \left[\frac{\Omega_0}{a^3} - \frac{k}{H_0^2 a^2} + \Omega_{\Lambda} \right]^{-\frac{1}{2}}$$

$$t_0 = H_0^{-1} f(\Omega_0, \Omega_{\Lambda}) \qquad H_0^{-1} = 9.78 h^{-1} \text{Gyr} \qquad f(1, 0) = \frac{2}{3}$$

$$= 13.97 \text{ h}_{70}^{-1} \text{ Gyr} \qquad f(0, 0) = 1$$

$$f(0, 1) = \infty$$

$$f(0.3, 0.7) = 0.964$$

$$[E(00)a^3]'$$
 vs. $E(ii) \Rightarrow \frac{\partial}{\partial a}(\rho a^3) = -3pa^2$ ("continuity")

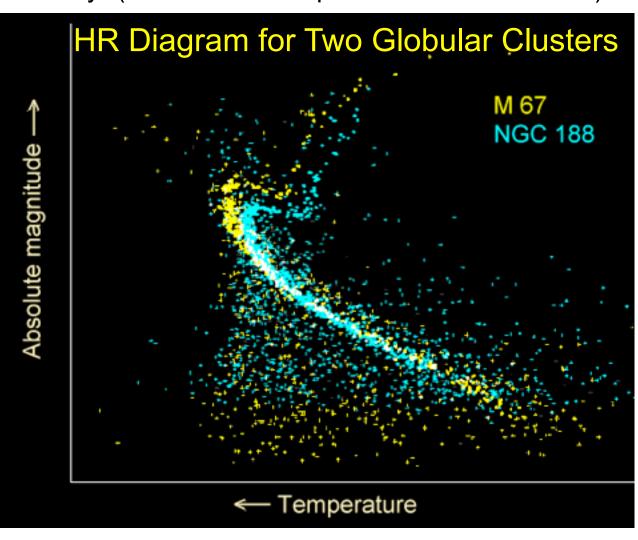
Given eq. of state $p = p(\rho)$, integrate to determine $\rho(a)$, integrate E(00) to determine a(t)

 $p = 0 \Rightarrow \rho = \rho_0 a^{-3}$ (assumed above in q_0 , t_0 eqs.) Radiation: $p = \frac{\rho}{3}, k = 0 \Rightarrow \rho \propto a^{-4}$

The Age of the Universe

In the mid-1990s there was a crisis in cosmology, because the age of the old Globular Cluster stars in the Milky Way, then estimated to be 16±3 Gyr, was higher than the expansion age of the universe, which for a critical density $(\Omega_m = 1)$ universe is 9±2 Gyr (with the Hubble parameter h=0.72±0.07).

But when the data from the Hipparcos astrometric satellite became available in 1997, it showed that the distance to the Globular Clusters had been underestimated, which implied that their ages are instead only 12±3 Gyr.



The Age of the Universe

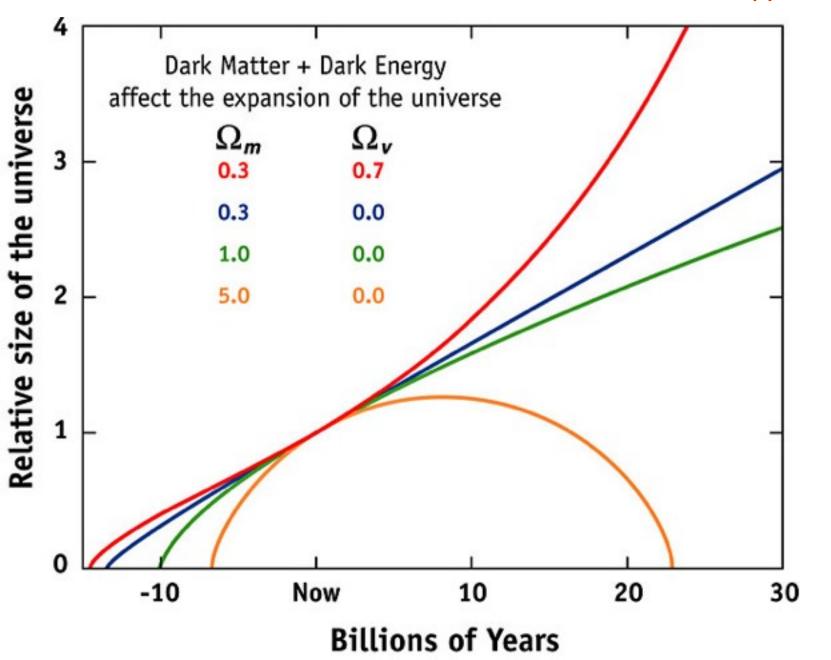
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Several lines of evidence now show that the universe does not have $\Omega_{\rm m}$ = 1 but rather $\Omega_{\rm tot}$ = $\Omega_{\rm m}$ + Ω_{Λ} = 1.0 with $\Omega_{\rm m}$ ≈ 0.3, which gives an expansion age of about 14 Gyr.

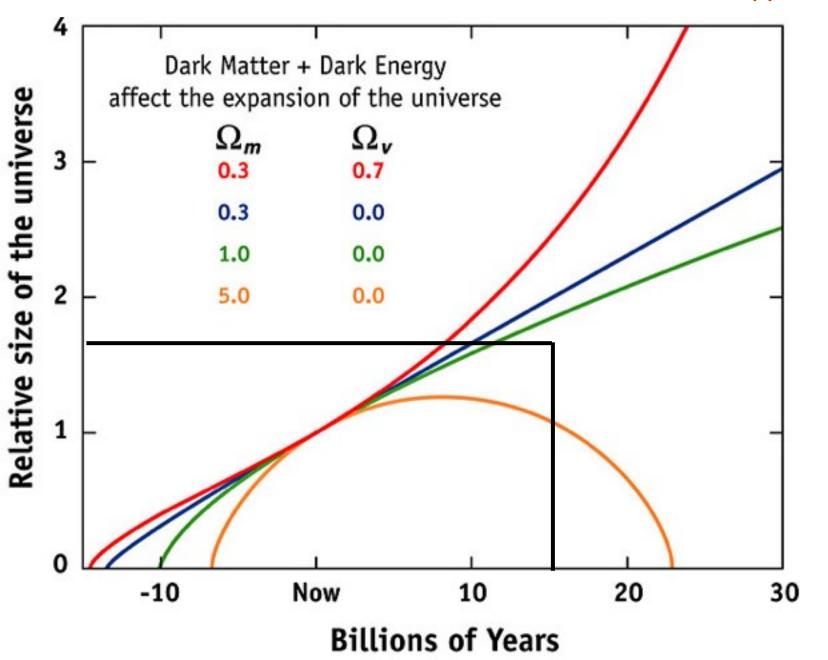
Moreover, age measurements based on radioactive decay of Thorium-232 (half-life 14.1 Gyr) measured in a number of stars gives a completely independent age of 14±3 Gyr. Similar measurements, based on detection in stars of Uranium-238 (half-life 4.47 Gyr), give ~13 Gyr.

All the recent measurements of the age of the universe are thus in excellent agreement. It is reassuring that three completely different clocks – stellar evolution, expansion of the universe, and radioactive decay – agree so well.

History of Cosmic Expansion for General $\Omega_{\rm M}$ & Ω_{Λ}



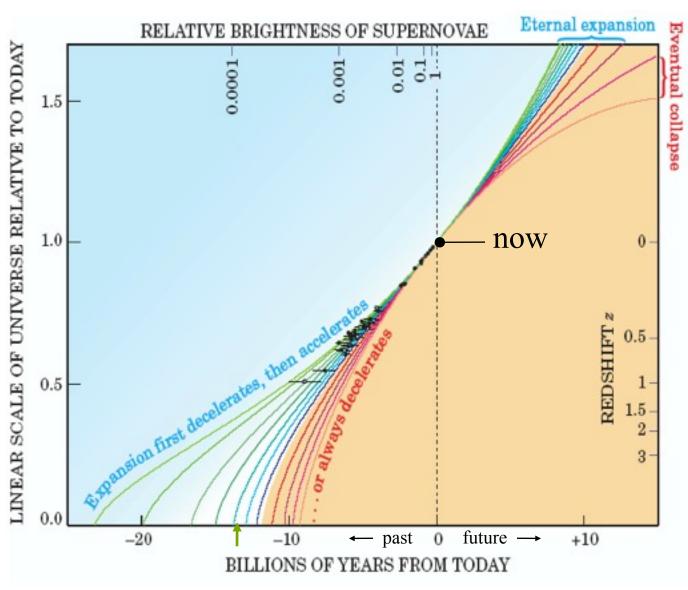
History of Cosmic Expansion for General $\Omega_{\rm M}$ & Ω_{Λ}



History of Cosmic Expansion for Ω_{Λ} = 1- Ω_{M}

With Ω_{Λ} = 0 the age of the decelerating universe would be only 9 Gyr, but Ω_{Λ} = 0.7, $\Omega_{\rm m}$ = 0.3 gives an age of 14 Gyr, consistent with stellar and radioactive decay ages

Figure 4. The history of cosmic expansion, as measured by the high-redshift supernovae (the black data points), assuming flat cosmic geometry. The scale factor R of the universe is taken to be 1 at present, so it equals 1/(1 + z). The curves in the blue shaded region represent cosmological models in which the accelerating effect of vacuum energy eventually overcomes the decelerating effect of the mass density. These curves assume vacuum energy densities ranging from 0.95 ρ_c (top curve) down to 0.4 $\rho_{\rm c}$. In the yellow shaded region, the curves represent models in which the cosmic expansion is always decelerating due to high mass density. They assume mass densities ranging (left to right) from $0.8 \rho_e$ up to $1.4 \rho_e$. In fact, for the last two curves, the expansion eventually halts and reverses into a cosmic collapse.



Saul Perlmutter, *Physics Today*, Apr 2003

LCDM Benchmark Cosmological Model: Ingredients & Epochs

	List of Ingredients
photons:	$\Omega_{\gamma,0} = 5.0 \times 10^{-5}$
neutrinos:	$\Omega_{\nu,0} = 3.4 \times 10^{-5}$
total radiation:	$\Omega_{r,0} = 8.4 \times 10^{-5}$
baryonic matter:	$\Omega_{\rm bary,0} = 0.04$
nonbaryonic dark matter:	$\Omega_{\rm dm,0} = 0.26$
total matter:	$\Omega_{m,0} = 0.30$
cosmological constant:	$\Omega_{\Lambda,0} \approx 0.70$

Important Epochs

important Epochs				
radiation-matter equality:	$a_{rm} = 2.8 \times 10^{-4}$	$t_{rm} = 4.7 \times 10^4 \mathrm{yr}$		
matter-lambda equality:	$a_{m\Lambda} = 0.75$	$t_{m\Lambda} = 9.8 \mathrm{Gyr}$		
Now:	$a_0 = 1$	$t_0 = 13.5 \text{Gyr}$		

Benchmark Model: Scale Factor vs. Time

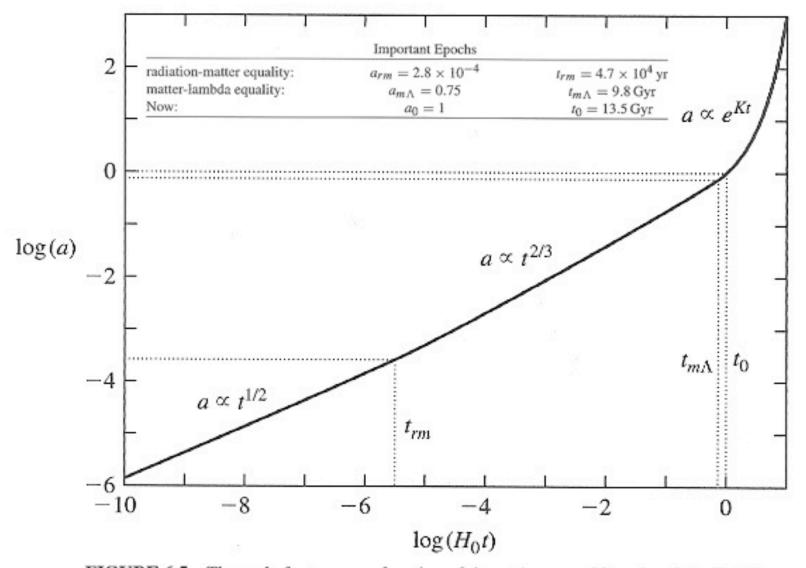
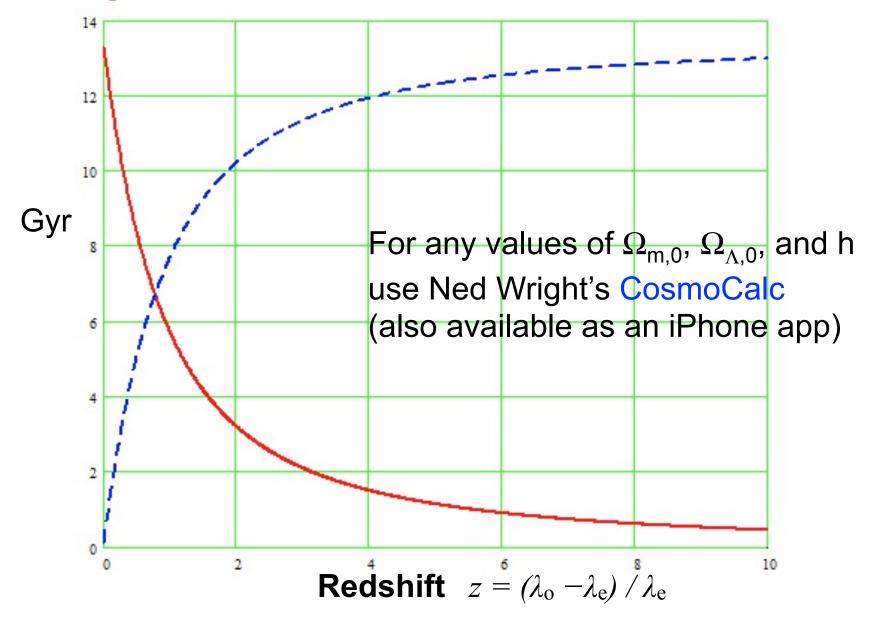


FIGURE 6.5 The scale factor a as a function of time t (measured in units of the Hubble time), computed for the Benchmark Model. The dotted lines indicate the time of radiation-matter equality, $a_{rm} = 2.8 \times 10^{-4}$, the time of matter-lambda equality, $a_{m\Lambda} = 0.75$, and the present moment, $a_0 = 1$. Barbara Ryden, Introduction to Cosmology (Addison-Wesley, 2003)

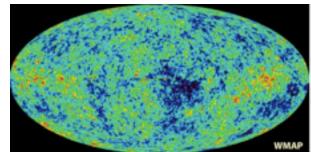
Age of the Universe and Lookback Time



These are for the Benchmark Model $\Omega_{\rm m,0}$ =0.3, $\Omega_{\Lambda,0}$ =0.7, h=0.7.

Brief History of the Universe

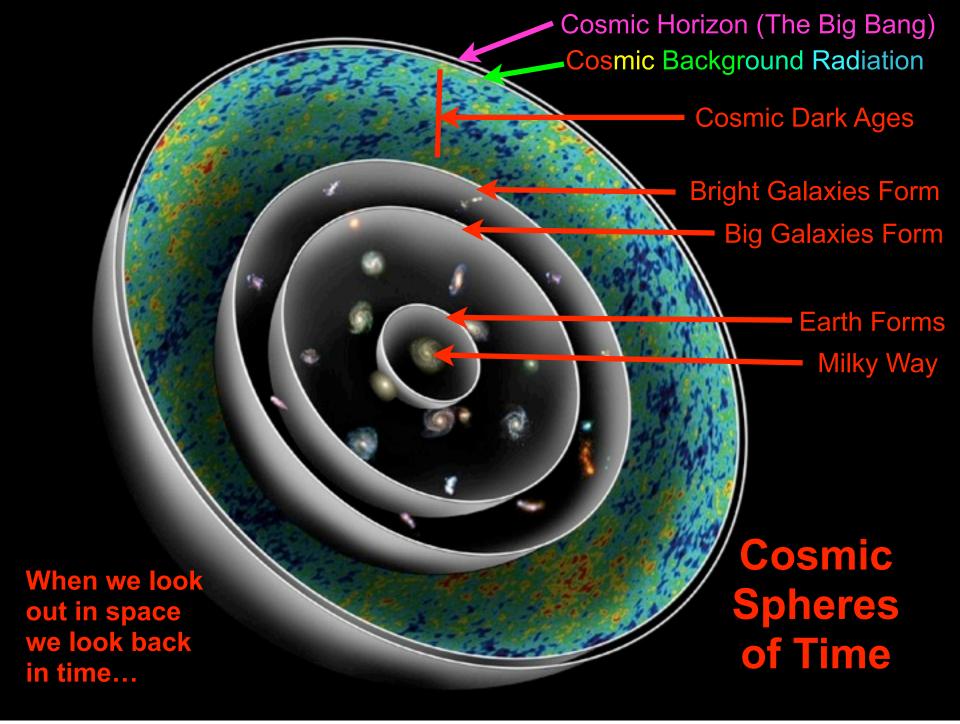
- Cosmic Inflation generates density fluctuations
- Symmetry breaking: more matter than antimatter
- All antimatter annihilates with almost all the matter (1s)
- Big Bang Nucleosynthesis makes light nuclei (10 min)
- Electrons and light nuclei combine to form atoms,
 - and the cosmic background radiation fills the newly transparent universe (380,000 yr)



- Galaxies and larger structures form (~0.5 Gyr)
- Carbon, oxygen, iron, ... are made in stars
- Earth-like planets form around 2nd generation stars
- Life somehow starts (~4 Gyr ago) and evolves on earth

Mapping the large scale structure of the universe ...





Evolution of Densities of Radiation, Matter, & Λ

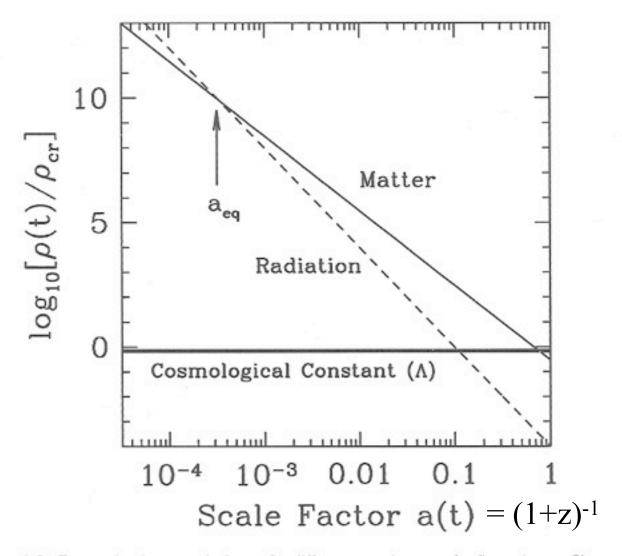


Figure 1.3. Energy density vs scale factor for different constituents of a flat universe. Shown are nonrelativistic matter, radiation, and a cosmological constant. All are in units of the critical density today. Even though matter and cosmological constant dominate today, at early times, the radiation density was largest. The epoch at which matter and radiation are equal is $a_{\rm eq}$.

z = redshift

Dodelson, Chapter 1

COSMIC BLACK-BODY RADIATION*

One of the basic problems of cosmology is the singularity characteristic of the familiar cosmological solutions of Einstein's field equations. Also puzzling is the presence of matter in excess over antimatter in the universe, for baryons and leptons are thought to be conserved. Thus, in the framework of conventional theory we cannot understand the origin of matter or of the universe. We can distinguish three main attempts to deal with these problems.

1. The assumption of continuous creation (Bondi and Gold 1948; Hoyle 1948), which avoids the singularity by postulating a universe expanding for all time and a continuous but slow creation of new matter in the universe.

2. The assumption (Wheeler 1964) that the creation of new matter is intimately related to the existence of the singularity, and that the resolution of both paradoxes may be found in a proper quantum mechanical treatment of Einstein's field equations.

3. The assumption that the singularity results from a mathematical over-idealization,

* This research was supported in part by the National Science Foundation and the Office of Naval Research of the U.S. Navy.

Fig. 1 —Possible thermal history of the Universe. The figure shows the previous thermal history of the Universe assuming a homogeneous isotropic general-relativity cosmological model (no scalar field) with present matter density 2 × 10⁻²⁹ gm/cm³ and present thermal radiation temperature 3.5° K. The bottom horizontal scale may be considered simply the proper distance between two chosen fiducial co-moving galaxies (points) The top horizontal scale is the proper world time. The line marked "temperature" refers to the temperature of the thermal radiation Matter remains in thermal equilibrium with the radiation until the plasma recombines, at the time indicated Thereafter further expansion cools matter not gravitationally bound faster than the radiation. The mass density in radiation is ρ_r . At present ρ_r is substantially below the mass density in matter, ρ_m , but, in the early Universe ρ_r exceeded ρ_m We have indicated the time when the Universe exhibited a transition from the characteristics of a radiation-filled model to those of a matter-filled model.

Looking back in time, as the temperature approaches 10¹⁰ ° K the electrons become relativistic, and thermal electron-pair creation sharply increases the matter density. At temperatures somewhat greater than 10¹⁰ °K these electrons should be so abundant as to assure a thermal neutrino abundance and a thermal neutron-proton abundance ratio. A temperature of this order would be required also to decompose the nuclei from the previous cycle in an oscillating Universe. Notice that the nucleons are nonrelativistic here.

The thermal neutrons decay at the right-hand limit of the indicated region of helium formation. There is a left-hand limit on this region because at higher temperatures photodissociation removes the deuterium necessary to form helium The difficulty with this model is that most of the matter would end up in helium.

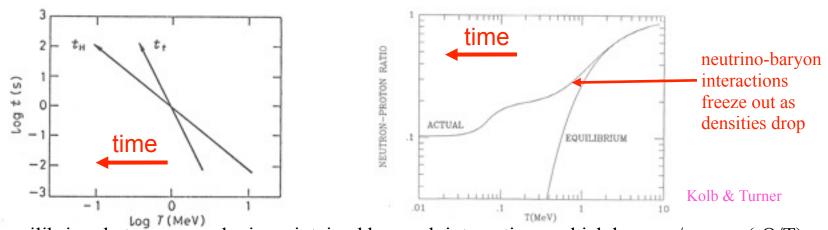
We deeply appreciate the helpfulness of Drs. Penzias and Wilson of the Bell Telephone Laboratories, Crawford Hill, Holmdel, New Jersey, in discussing with us the result of their measurements and in showing us their receiving system. We are also grateful for several helpful suggestions of Professor J. A. Wheeler.

> R. H. DICKE P. J. E. PEEBLES P. G. ROLL D. T. WILKINSON

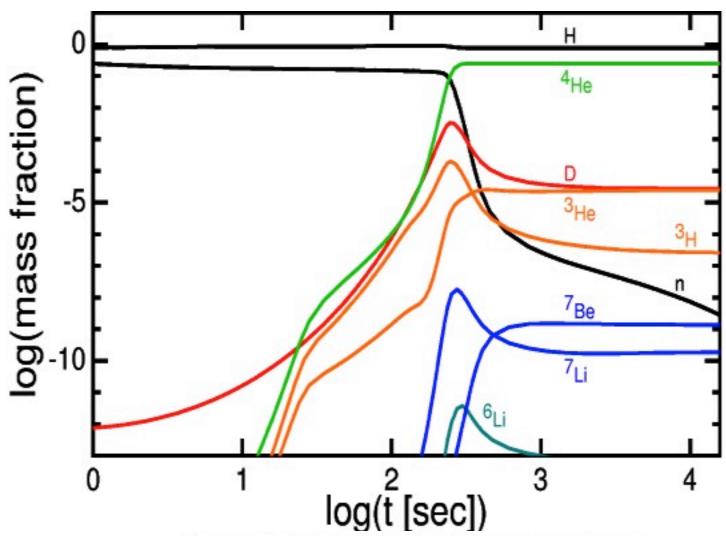
May 7, 1965 PALMER PHYSICAL LABORATORY PRINCETON, NEW JERSEY

Big Bang Nucleosynthesis

BBN was conceived by Gamow in 1946 as an explanation for the formation of all the elements, but the absence of any stable nuclei with A=5,8 makes it impossible for BBN to proceed past Li. The formation of carbon and heavier elements occurs instead through the triple- α process in the centers of red giants (Burbidge², Fowler, & Hoyle 57). At the BBN baryon density of 2×10^{-29} Ω_b h² (T/T₀)³ g cm⁻³ $\approx 2\times10^{-5}$ g cm⁻³, the probability of the triple- α process is negligible even though T $\approx 10^9$ K.

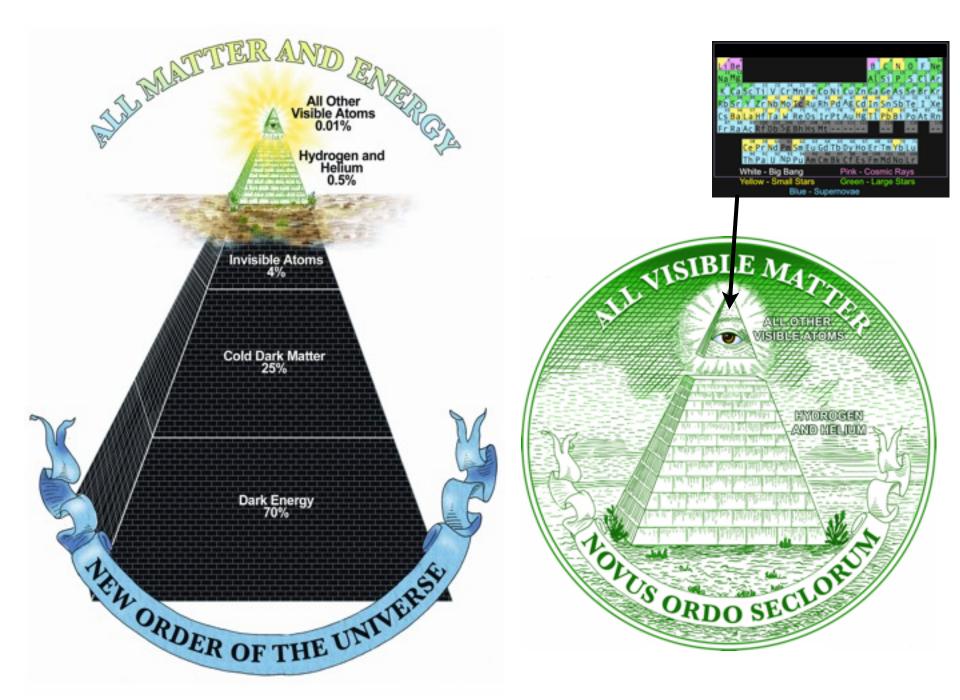


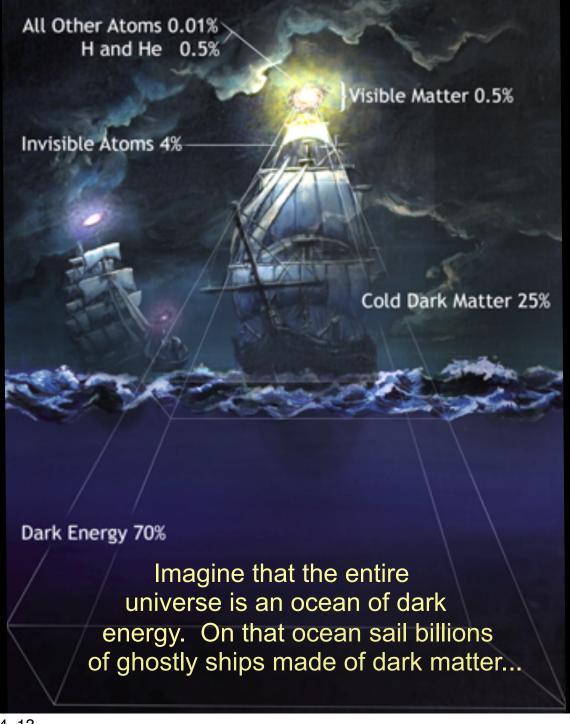
Thermal equilibrium between n and p is maintained by weak interactions, which keeps n/p = exp(-Q/T) (where Q = $m_n - m_p = 1.293$ MeV) until about t ≈ 1 s. But because the neutrino mean free time $t_v^{-1} \approx \sigma_v \, n_{e\pm} \approx (G_F T)^2 (T^3)$ is increasing as $t_v \approx T^{-5}$ (here the Fermi constant $G_F \approx 10^{-5}$ GeV⁻²), while the horizon size is increasing only as $t_H \approx (G\rho)^{-1/2} \approx M_{Pl} \, T^{-2}$, these interactions freeze out when T drops below about 0.8 MeV. This leaves n/(p+n) ≈ 0.14. The neutrons then decay with a mean lifetime 887 ± 2 s until they are mostly fused into D and then ⁴He. The higher the baryon density, the higher the final abundance of ⁴He and the lower the abundance of D that survives this fusion process. Since D/H is so sensitive to baryon density, David Schramm called deuterium the "baryometer." He and his colleagues also pointed out that since the horizon size increases more slowly with T⁻² the larger the number of light neutrino species N_v contributing to the energy density ρ , BBN predicted that $N_v \approx 3$ before N_v was measured at accelerators by measuring the width of the Z^0 .



The detailed production of the lightest elements out of protons and neutrons during the first three minutes of the universe's history. The nuclear reactions occur rapidly when the temperature falls below a billion degrees Kelvin. Subsequently, the reactions are shut down, because of the rapidly falling temperature and density of matter in the expanding universe.

Ken Kawano's (1992) BBN code is available at http://www-thphys.physics.ox.ac.uk/users/SubirSarkar/bbn.html



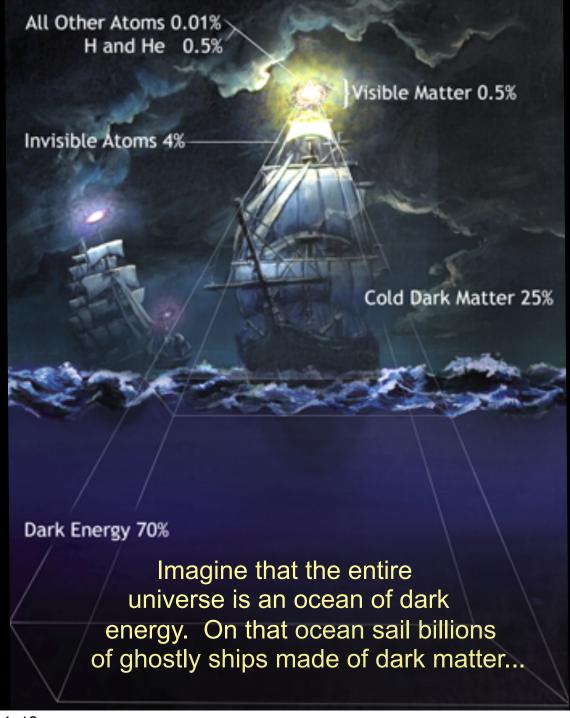


Matter and Energy Content of the Universe

Dark Matter Ships

on a

Dark Energy Ocean



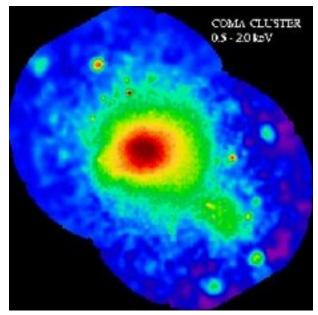
Matter and Energy Content of the Universe

NCDM

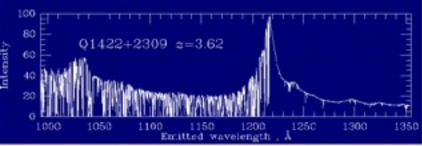
Double Dark Theory

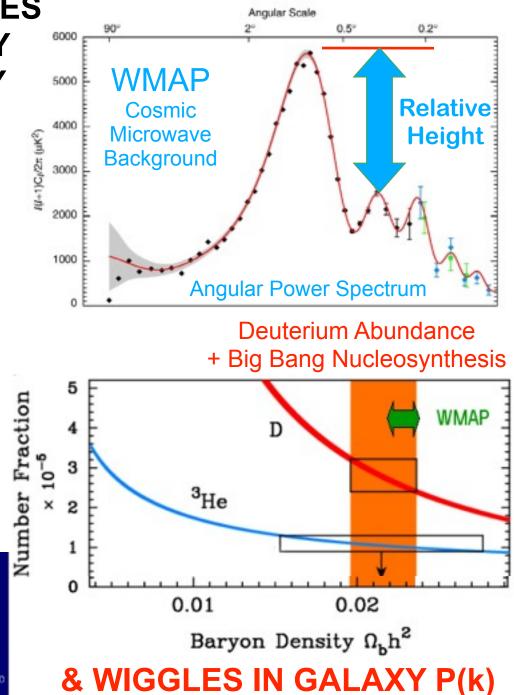
5 INDEPENDENT MEASURES AGREE: ATOMS ARE ONLY 4½% OF COSMIC DENSITY

Galaxy Cluster in X-rays



Absorption of Quasar Light



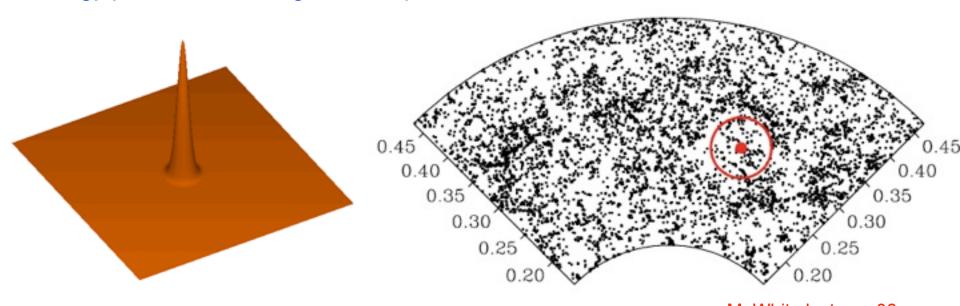


Wednesday, September 4, 13

BAO WIGGLES IN GALAXY P(k)

Sound waves that propagate in the opaque early universe imprint a characteristic scale in the clustering of matter, providing a "standard ruler" whose length can be computed using straightforward physics and parameters that are tightly constrained by CMB observations. Measuring the angle subtended by this scale determines a distance to that redshift and constrains the expansion rate.

The detection of the acoustic oscillation scale is one of the key accomplishments of the SDSS, and even this moderate signal-to-noise measurement substantially tightens constraints on cosmological parameters. Observing the evolution of the BAO standard ruler provides one of the best ways to measure whether the dark energy parameters changed in the past.



M. White lectures 08

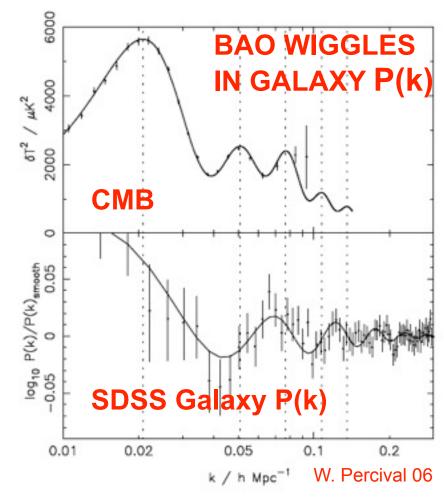
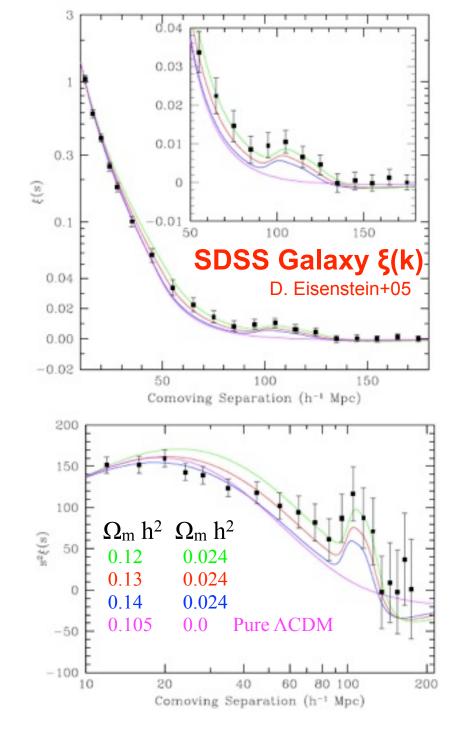
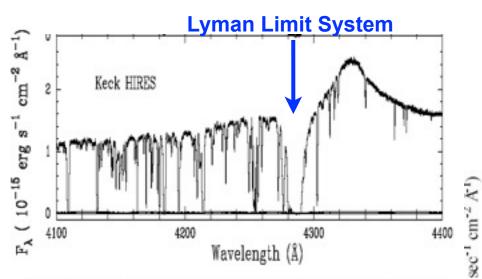


Fig. 3. Upper panel: The TT power spectrum recovered from the 3-year WMAP data (Hinshaw et al. 2006), projected into comoving space assuming a cosmological model with $\Omega_m =$ 0.25 and $\Omega_V =$ 0.75. For comparison, in the lower panel we plot the baryon oscillations calculated by dividing the SDSS power spectrum with a smooth cubic spline fit (Percival et al. 2007a). Vertical dotted lines show the positions of the peaks in the CMB power spectrum. As can be seen, there is still a long way to go before low redshift observations can rival the CMB in terms of the significance of the acoustic oscillation signal.



Deuterium absorption at redshift 2.525659 towards Quasar Q1243+3047

 $^{7}_{\lambda} \times 10^{-16}$ (ergs



The Ly α absorption near 4285 Å is from the system in which we measure D/H.

The detection of Deuterium and the modeling of this system seem convincing. This is just a portion of the evidence that the Tytler group presented in this paper. They have similarly convincing evidence for several other Lyman limit systems in quasar spectra.

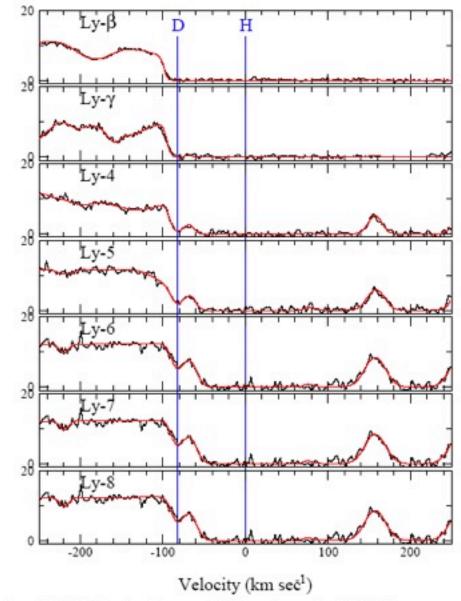
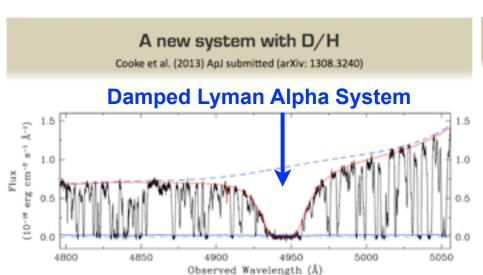


Fig. 7.— The HIRES spectrum of Ly-2 to 8, together with our model of the system, as given in Table 3.

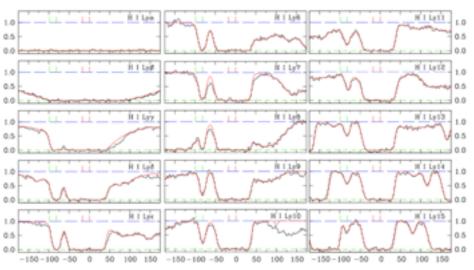
Kirkman, Tytler, Suzuki, O'Meara, & Lubin 2004



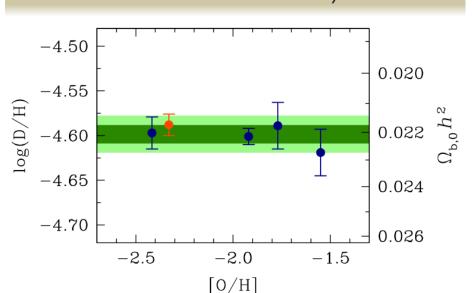
 $\log N(H I)/cm^{-2} = 20.49 \pm 0.01$

A new system with D/H

Cooke et al. (2013) ApJ submitted (arXiv: 1308.3240)



Precision Measures of D/H



The baryon density

ASSUMING STANDAND BIG BANG NUCLEOSYNTHESIS

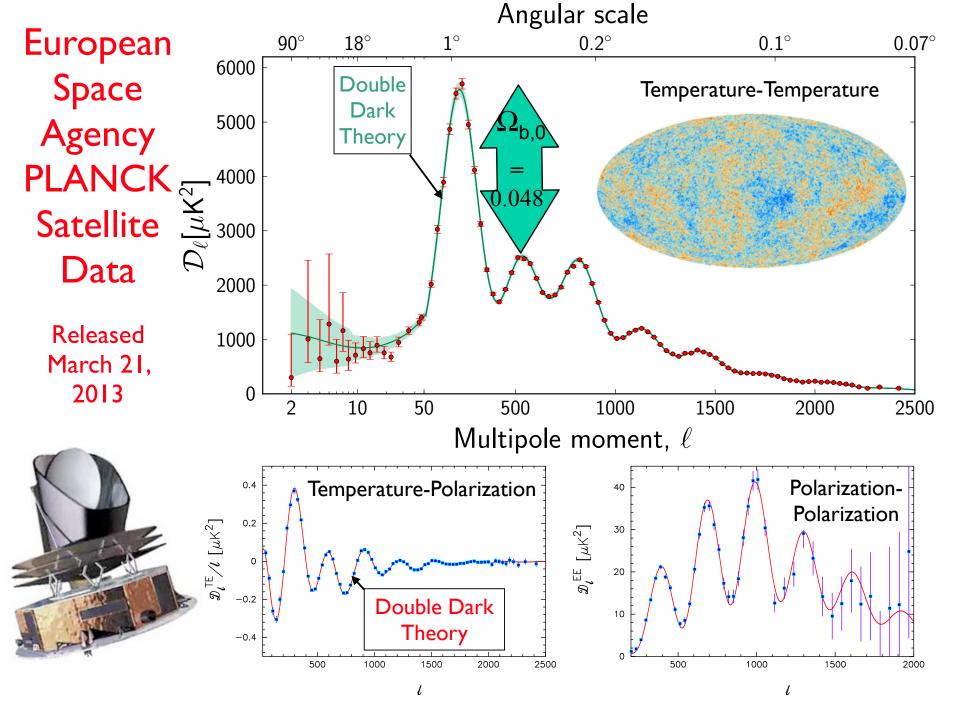
$$100 \ \Omega_{b,0} \ h^2(BBN) = 2.202 \pm 0.045$$



$$100 \Omega_{b.0} h^2 (CMB) = 2.205 \pm 0.028$$

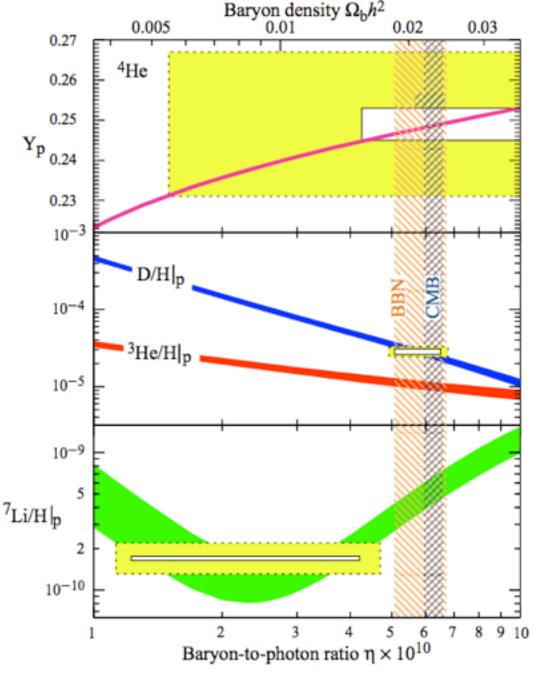


For
$$h = 0.7$$
, $\Omega_{b,0} = 0.045$



BBN
Predicted
vs.
Measured
Abundance
s of D, ³He,
⁴He, and ⁷Li

⁷Li IS NOW DISCORDANT unless stellar diffusion destroys ⁷Li



BBN is a Prototype for Hydrogen Recombination and DM Annihilation All three are examples of the universe dropping out of equilibrium!

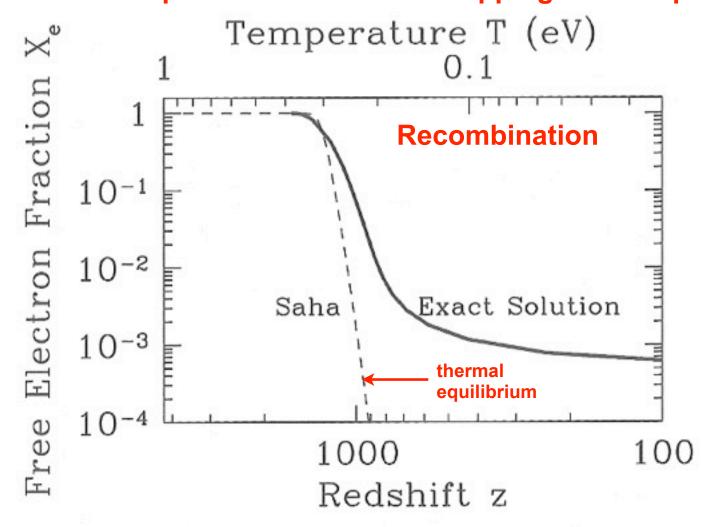


Figure 3.4. Free electron fraction as a function of redshift. Recombination takes place suddenly at $z\sim 1000$ corresponding to $T\sim 1/4$ eV. The Saha approximation, Eq. (3.37), holds in equilibrium and correctly identifies the redshift of recombination, but not the detailed evolution of X_e . Here $\Omega_b=0.06, \Omega_m=1, h=0.5$.

Dodelson, Modern Cosmology, p. 72

Dark Matter Annihilation

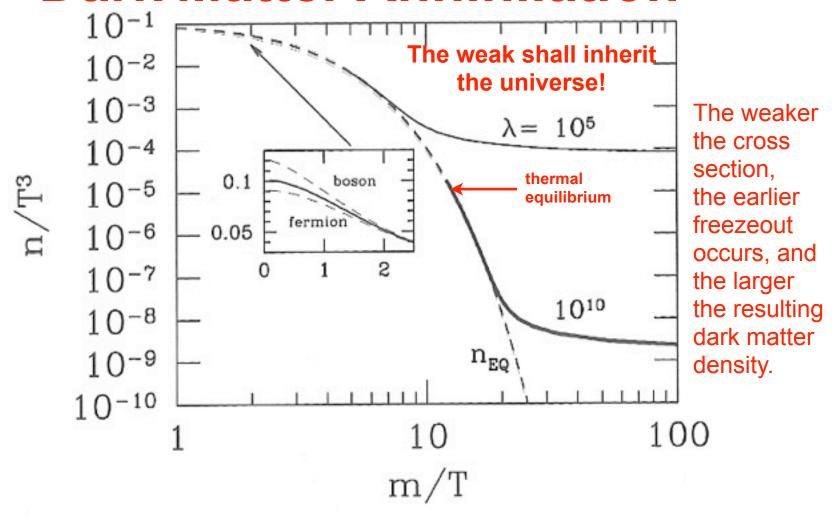


Figure 3.5. Abundance of heavy stable particle as the temperature drops beneath its mass. Dashed line is equilibrium abundance. Two different solid curves show heavy particle abundance for two different values of λ , the ratio of the annihilation rate to the Hubble rate. Inset shows that the difference between quantum statistics and Boltzmann statistics is important only at temperatures larger than the mass.

Dodelson, Modern Cosmology, p. 76

Dark Matter Annihilation

The abundance today of dark matter particles X of the WIMP variety is determined by their survival of annihilation in the early universe. Supersymmetric neutralinos can annihilate with each other (and sometimes with other particles: "co-annihilation").

Dark matter annihilation follows the same pattern as the previous discussions: initially the abundance of dark matter particles X is given by the equilibrium Boltzmann exponential $\exp(-m_X/T)$, but as they start to disappear they have trouble finding each other and eventually their number density freezes out. The freezeout process can be followed using the Boltzmann equation, as discussed in Kolb and Turner, Dodelson, Mukhanov, and other textbooks. For a detailed discussion of Susy WIMPs, see the review article by Jungman, Kamionkowski, and Griest (1996). The result is that the abundance today of WIMPs X is given in most cases by (Dodelson's Eqs. 3.59-60)

$$\Omega_X = \left[\frac{4\pi^3 G g_*(m)}{45} \right]^{1/2} \frac{x_f T_0^3}{30 \langle \sigma v \rangle \rho_{\rm cr}} = 0.3 h^{-2} \left(\frac{x_f}{10} \right) \left(\frac{g_*(m)}{100} \right)^{1/2} \frac{10^{-39} {\rm cm}^2}{\langle \sigma v \rangle}.$$

Here $x_f \approx 10$ is the ratio of m_X to the freezeout temperature T_f , and $g_*(m_X) \approx 100$ is the density of states factor in the expression for the energy density of the universe when the temperature equals m_X

$$\rho = \frac{\pi^2}{30} T^4 \left[\sum_{i=\text{bosons}} g_i + \frac{7}{8} \sum_{i=\text{fermions}} g_i \right] \equiv g_* \frac{\pi^2}{30} T^4.$$

The sum is over relativistic species i (see the graph of g(T) on the next slide). Note that more X's survive, the weaker the cross section σ . For Susy WIMPs the natural values are $\sigma \sim 10^{-39}$ cm², so $\Omega_{\rm X} \approx 1$ naturally. This is known as the "WIMP miracle."

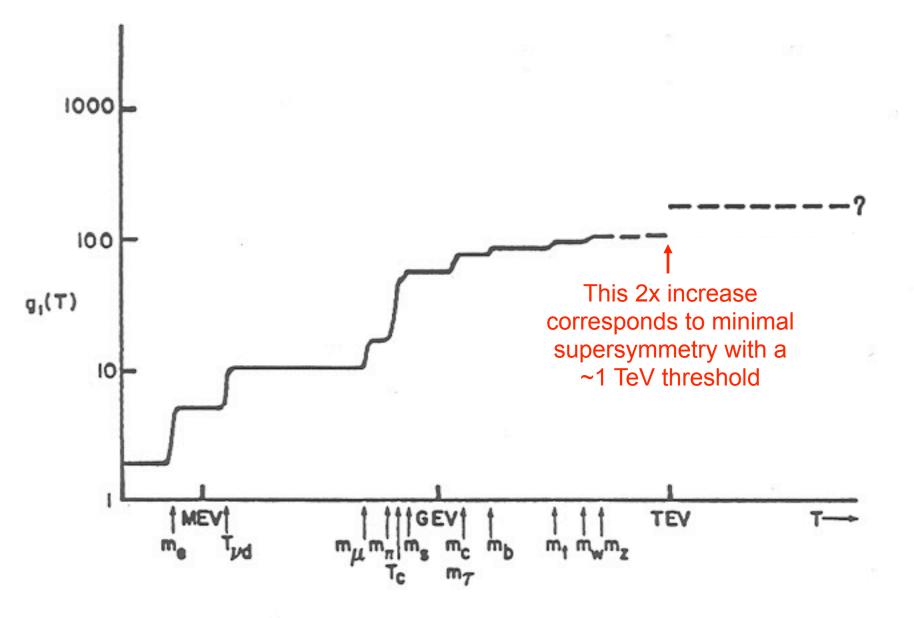


Fig. 1 The effective number of degrees of freedom of thermally interacting relativistic particles as a function of temperature.

Supersymmetry is the basis of most attempts, such as superstring theory, to go beyond the current "Standard Model" of particle physics. Heinz Pagels and Joel Primack pointed out in a 1982 paper that the lightest supersymmetric partner particle is stable because of R-parity, and is thus a good candidate for the dark matter particles – weakly interacting massive particles (**WIMP**s).

Michael Dine and others pointed out that the **axion**, a particle needed to save the strong interactions from violating CP symmetry, could also be the dark matter particle. Searches for both are underway.

Supersymmetric WIMPs

When the British physicist Paul Dirac first combined Special Relativity with quantum mechanics, he found that this predicted that for every ordinary particle like the electron, there must be another particle with the opposite electric charge – the anti-electron (positron). Similarly, corresponding to the proton there must be an anti-proton. Supersymmetry appears to be required to combine General Relativity (our modern theory of space, time, and gravity) with the other forces of nature (the electromagnetic, weak, and strong interactions). The consequence is another doubling of the number of particles, since supersymmetry predicts that for every particle that we now know, including the antiparticles, there must be another, thus far undiscovered particle with the same electric charge but with *spin* differing by half a unit.

Spin	Matter (fermions)	Forces (bosons)	
2		graviton	
1		photon, W^{\pm}, Z^{0} gluons	
1/2	quarks $u,d,$ leptons $e, \nu_e,$		
0		Higgs bosons	
		axion	

Supersymmetric WIMPs

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after doubling

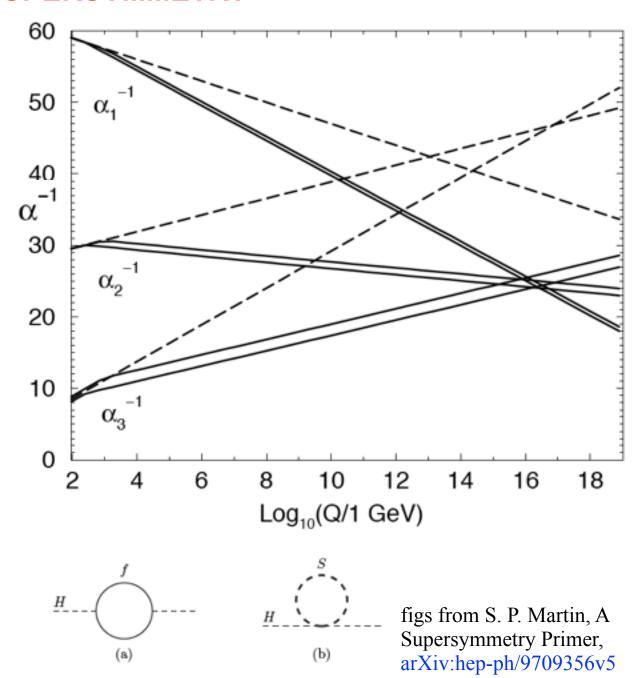
Spin	Matter (fermions)	Forces (bosons)	Hypothetical Superpartners	Spin
2		graviton	gravitino	3/2
1		photon, W^{\pm}, Z^0 gluons	photino, winos, zino, gluinos	1/2
1/2	quarks $u,d,$ leptons $e, \nu_e,$		squarks $\tilde{u}, \tilde{d}, \dots$ sleptons $\tilde{e}, \tilde{\nu}_e, \dots$	0
0		Higgs bosons axion	Higgsinos axinos	1/2

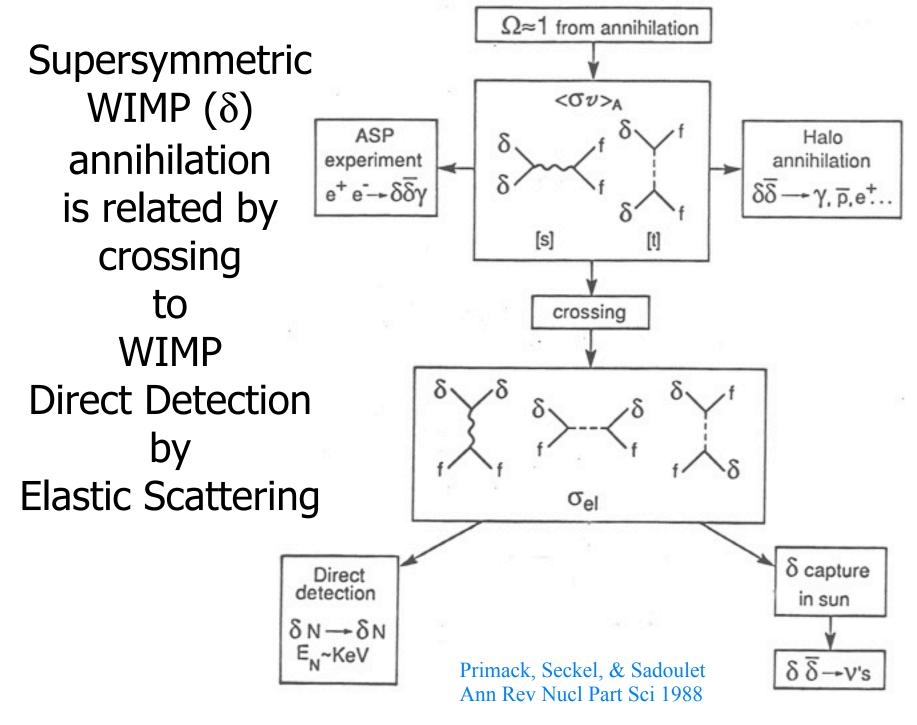
Note: Supersymmetric cold dark matter candidate particles are underlined.

SUPERSYMMETRY

The only experimental evidence for supersymmetry is that running of coupling constants in the Standard Model (dashed lines in firgure) does not lead to Grand Unification of the weak, electromagnetic, and strong interactions, while with supersymmetry the three couplings all do come together at a scale just above 10¹⁶ GeV. The figure assumes the Minimal Supersymmetric Standard Model (MSSM) with sparticle masses between 250 GeV and 1 TeV.

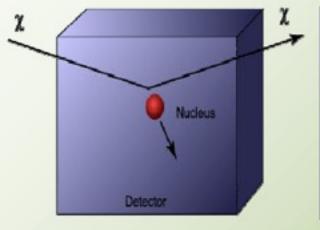
Other arguments for SUSY include: helps unification of gravity since it controls the vacuum energy and moderates loop divergences (fermion and boson loop divergences cancel), solves the hierarchy problem, and naturally leads to DM with $\Omega \sim 1$.



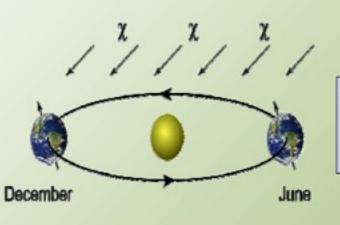


Experiments are Underway for Detection of WIMPs

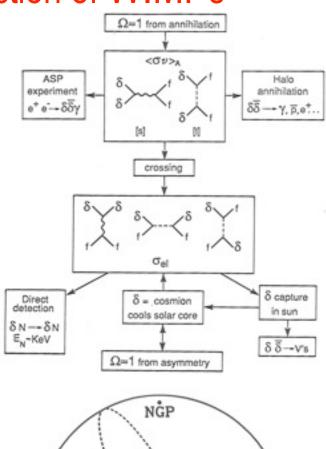
Direct detection - general principles

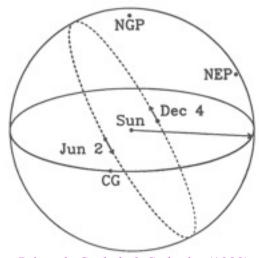


- WIMP + nucleus →
 WIMP + nucleus
- Measure the nuclear recoil energy
- Suppress backgrounds enough to be sensitive to a signal, or...



 Search for an annual modulation due to the Earth's motion around the Sun





Primack, Seckel, & Sadoulet (1988)

and also AXIONs

The diagram at right shows the layout of the axion search experiment now underway at the University of Washington. Axions would be detected as extra photons in the Microwave Cavity.

