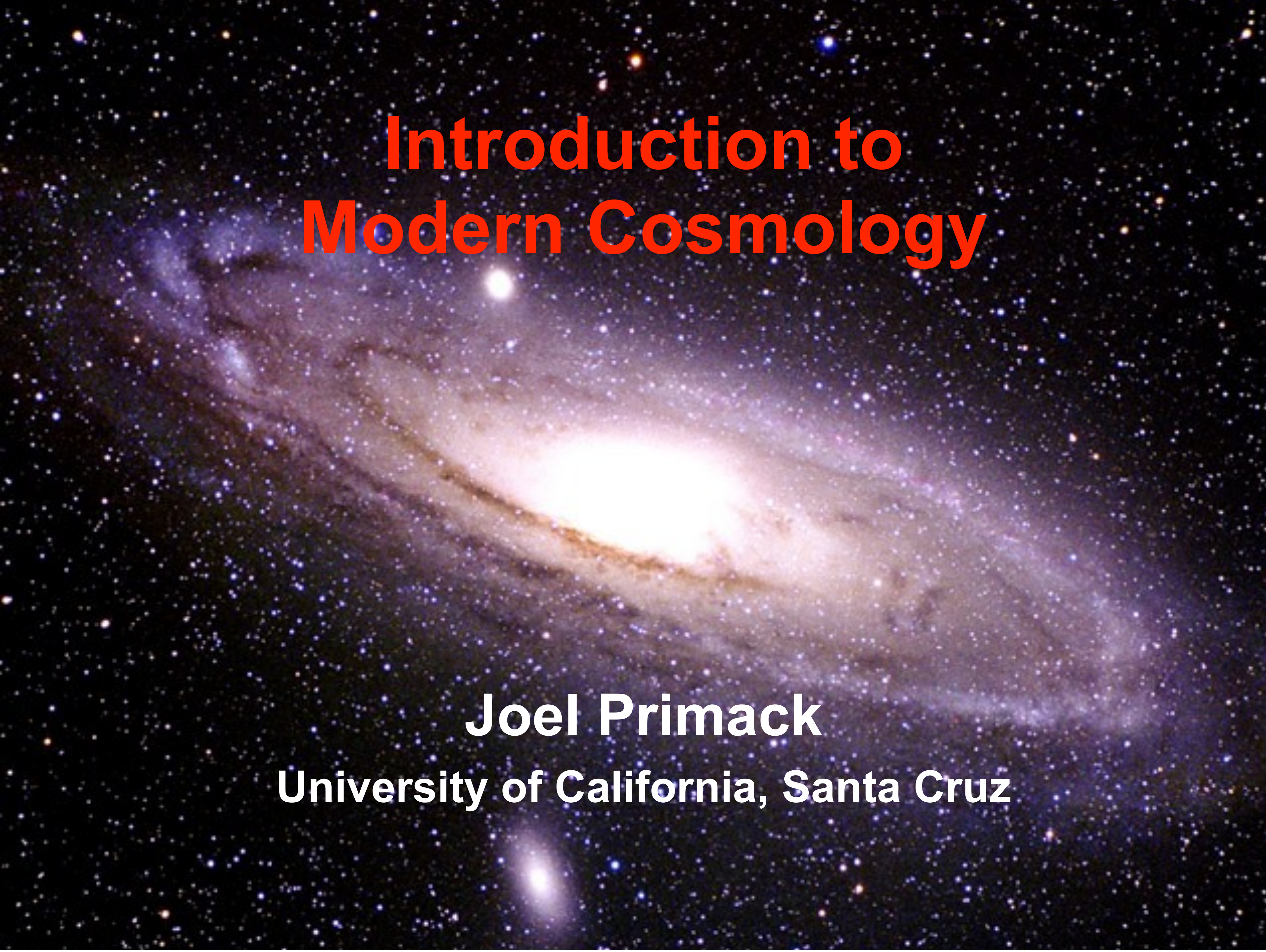
Introduction to Modern Cosmology



Joel Primack

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Modern Cosmology

A series of major discoveries has laid a lasting foundation for cosmology. Einstein's general relativity (1916) provided the conceptual foundation for the modern picture. Then Hubble discovered that "spiral nebulae" are large galaxies like our own Milky Way (1925), and that distant galaxies are receding from the Milky Way with a speed proportional to their distance (1929), which means that we live in an expanding universe. The discovery of the cosmic background radiation (1965) showed that the universe began in a very dense, hot, and homogeneous state: the Big Bang. This was confirmed by the discovery that the cosmic background radiation has exactly the same spectrum as heat radiation (1989), and the measured abundances of the light elements agree with the predictions of Big Bang theory if the abundance of ordinary matter is about 4% of critical density. Most of the matter in the universe is invisible particles which move very sluggishly in the early universe ("Cold Dark Matter"). Most of the energy density is mysterious dark energy.

Experimental and Historical Sciences

both make predictions about new knowledge, whether from experiments or from the past

Historical Explanation Is Always Inferential

Our age cannot look back to earlier things
Except where reasoning reveals their traces Lucretius

Patterns of Explanation Are the Same in the Historical Sciences as in the Experimental Sciences

Specific conditions + General laws ⇒ Particular event

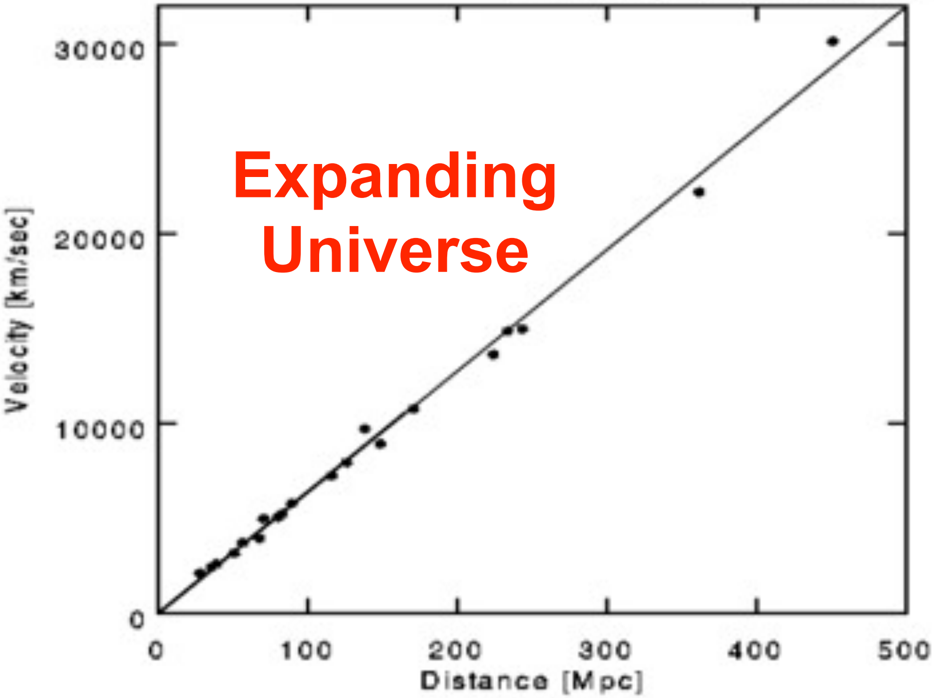
In history as anywhere else in empirical science, the explanation of a phenomenon consists in subsuming it under general empirical laws; and the criterion of its soundness is … exclusively whether it rests on empirically well confirmed assumptions concerning initial conditions and general laws.

C.G. Hempel, *Aspects of Scientific Explanation* (1965), p. 240.

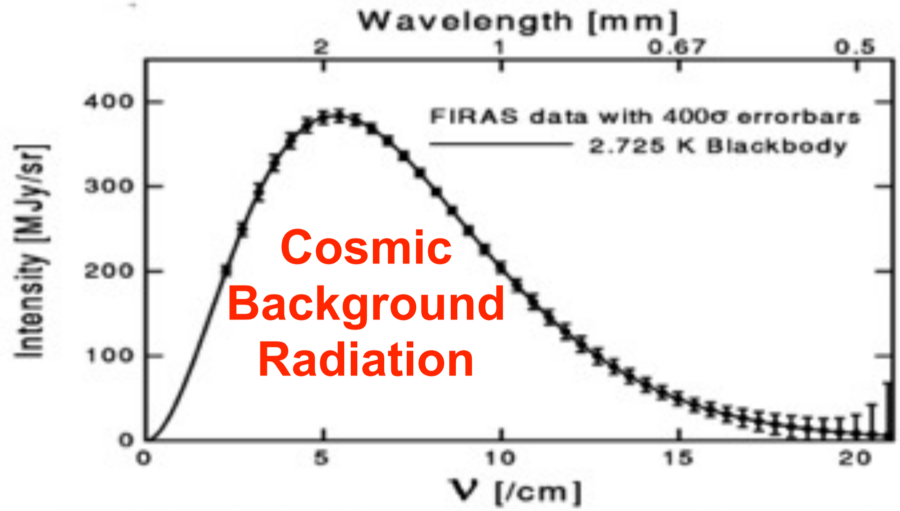
Successful Predictions of the Big Bang

First Prediction	First Confirmation
Expansion of the Universe	
Friedmann 1922, Lemaitre 1927 based on Einstein 1916	Hubble 1929
Cosmic Background Radiation	
Existence of CBR	
Gamow, Alpher, Hermann 1948	Penzias & Wilson 1965
CBR Thermal Spectrum	
Peebles 1966	COBE 1989
CBR Fluctuation Amplitude	
Cold Dark Matter theory 1984	COBE 1992
CBR Acoustic Peak	BOOMERANG 2000 MAXIMA 2000
Light Element Abundances	
Peebles 1966, Wagoner 1967	D/H Tytler et al.1997

Three Pillars of the Big Bang



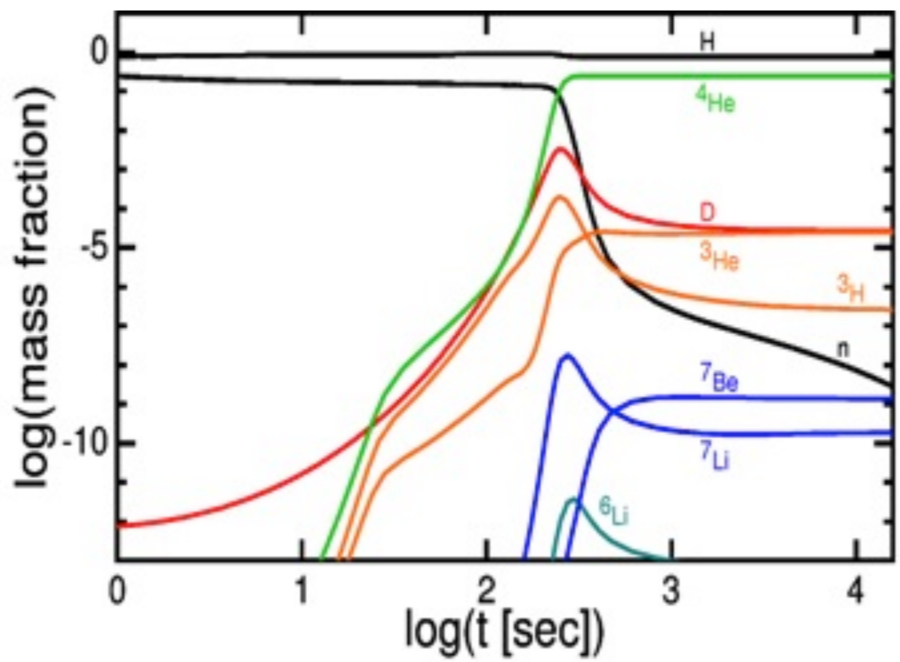
A modern illustration of Hubble's Law, displaying the increase of recession speed of galaxies growing in direct proportion to their distance.



The variation of the intensity of the microwave background radiation with its frequency, as observed by the COBE satellite from above the Earth's atmosphere. The observations (boxes) display a perfect fit with the (solid) curve expected from pure heat radiation with a temperature of 2.73°K.

Big Bang Nucleosynthesis

The detailed production of the lightest elements out of protons and neutrons during the first three minutes of the universe's history. The nuclear reactions occur rapidly when the temperature falls below a billion degrees Kelvin. Subsequently, the reactions are shut down, because of the rapidly falling temperature and density of matter in the expanding universe.



Caution: ^{7}Li may now be discordant

General Relativity and Cosmology

GR: MATTER TELLS SPACE HOW TO CURVE

$$R^{\mu\nu} - \tfrac{1}{2}Rg^{\mu\nu} = -\,8\pi G T^{\mu\nu} + \Lambda g^{\mu\nu}$$

CURVED SPACE TELLS MATTER HOW TO MOVE

$$\frac{du^\mu}{ds} + \Gamma^\mu{}_{\alpha\beta}\, u^\alpha u^\beta = 0$$

Cosmological Principle: on large scales, space is uniform and isotropic. COBE-Copernicus Theorem: If all observers observe a nearly-isotropic Cosmic Background Radiation (CBR), then the universe is locally nearly homogeneous and isotropic – i.e., is approximately described by the Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)\,[dr^2\,(1 - kr^2)^{-1} + r^2\, d\Omega^2]$$

with curvature constant $k = -1$, 0, or +1. Substituting this metric into the Einstein equation at left above, we get the Friedmann eq.

General Relativity

GR follows from the principle of equivalence and Einstein's equation $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi G T_{\mu\nu}$.* Einstein had intuited the local equivalence of gravity and acceleration in 1907 (Pais, p. 179), but it was not until November 1915 that he developed the final form of the GR equation.

(*Gravitation & Cosmology*)

It can be derived from the following assumptions (Weinberg, p. 153):

1. The l.h.s. $G_{\mu\nu}$ is a tensor
2. $G_{\mu\nu}$ consists only of terms linear in second derivatives or quadratic in first derivatives of the metric tensor $g_{\mu\nu}$ ($\Leftrightarrow G_{\mu\nu}$ has dimension L^{-2})
3. Since $T_{\mu\nu}$ is symmetric in $\mu\nu$, so is $G_{\mu\nu}$
4. Since $T_{\mu\nu}$ is conserved (covariant derivative $T^{\mu}{}_{\nu;\mu}=0$) so also $G^{\mu}{}_{\nu;\mu}=0$
5. In the weak field limit where $g_{00} \approx -(1+2\phi)$, satisfying the Poisson equation $\nabla^2\phi=4\pi G\rho$ (i.e., $\nabla^2 g_{00}=-8\pi G T_{00}$), we must have $G_{00}=\nabla^2 g_{00}$

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*Note: we're here using the metric –1, 1,1,1 as in Dodelson, Weinberg.

Einstein's equation can also be derived from an action principle, varying the total action $I = I_M + I_G$, where I_M is the action of matter and I_G is that of gravity:

$$I_G = -\frac{1}{16\pi G}\int R(x)\,\sqrt{g(x)}\,d^4x$$

(see, e.g., Weinberg, p. 364). The curvature scalar $R \equiv R_{\mu\nu}\, g^{\mu\nu}$ is the obvious term to insert in I_G since a scalar connected with the metric is needed and it is the only one, unless higher powers R^2, R^3 or higher derivatives $\Box R$ are used, which will lead to higher-order or higher-derivative terms in the gravity equation.

Einstein realized in 1916 that the 5th postulate above isn't strictly necessary – merely that the equation reduce to the Newtonian Poisson equation within observational errors, which allows the inclusion of a small cosmological constant term. In the action derivation, such a term arises if we just add a constant to R.

Friedmann-Robertson-Walker Framework (homogeneous, isotropic universe)

FRW E(00) $\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$ ← Friedmann equation

FRW E(ii) $\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi G p - \frac{k}{a^2} + \Lambda$

$$H_0 \equiv 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$$
$$\equiv 70h_{70} \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$\frac{E(00)}{H_0^2} \Rightarrow 1 = \Omega_0 - \frac{k}{H_0^2} + \Omega_\Lambda \text{ with } H \equiv \frac{\dot{a}}{a},\ a_0 \equiv 1,\ \Omega_0 \equiv \frac{\rho_0}{\rho_c}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2},$$

$$\rho_{c,0} \equiv \frac{3H_0^2}{8\pi G} = 1.36 \times 10^{11} h_{70}^2 M_\odot \text{Mpc}^{-3}$$

$$E(ii) - E(00) \Rightarrow \frac{2\ddot{a}}{a} = -\frac{8\pi}{3}G\rho - 8\pi G p + \frac{2}{3}\Lambda$$

$$\text{Divide by } 2E(00) \Rightarrow \quad q_0 \equiv -\left(\frac{\ddot{a}}{a}\frac{a^2}{\dot{a}^2}\right)_0 = \frac{\Omega_0}{2} - \Omega_\Lambda$$

$$E(00) \Rightarrow t_0 = \int_0^1 \frac{da}{a}\left[\frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}\right]^{-\frac{1}{2}} = H_0^{-1}\int_0^1 \frac{da}{a}\left[\frac{\Omega_0}{a^3} - \frac{k}{H_0^2 a^2} + \Omega_\Lambda\right]^{-\frac{1}{2}}$$

$$t_0 = H_0^{-1} f(\Omega_0, \Omega_\Lambda) \qquad H_0^{-1} = 9.78h^{-1}\text{Gyr}$$
$$= 13.97\ h_{70}^{-1} \text{ Gyr}$$

$$f(1,0) = \tfrac{2}{3}$$
$$f(0,0) = 1$$
$$f(0,1) = \infty$$
$$f(0.3, 0.7) = 0.964$$

$$[E(00)a^3]' \text{ vs. } E(ii) \Rightarrow \quad \frac{\partial}{\partial a}(\rho a^3) = -3pa^2 \ (\text{“continuity”})$$

Given eq. of state $p = p(\rho)$, integrate to determine $\rho(a)$,
integrate $E(00)$ to determine $a(t)$

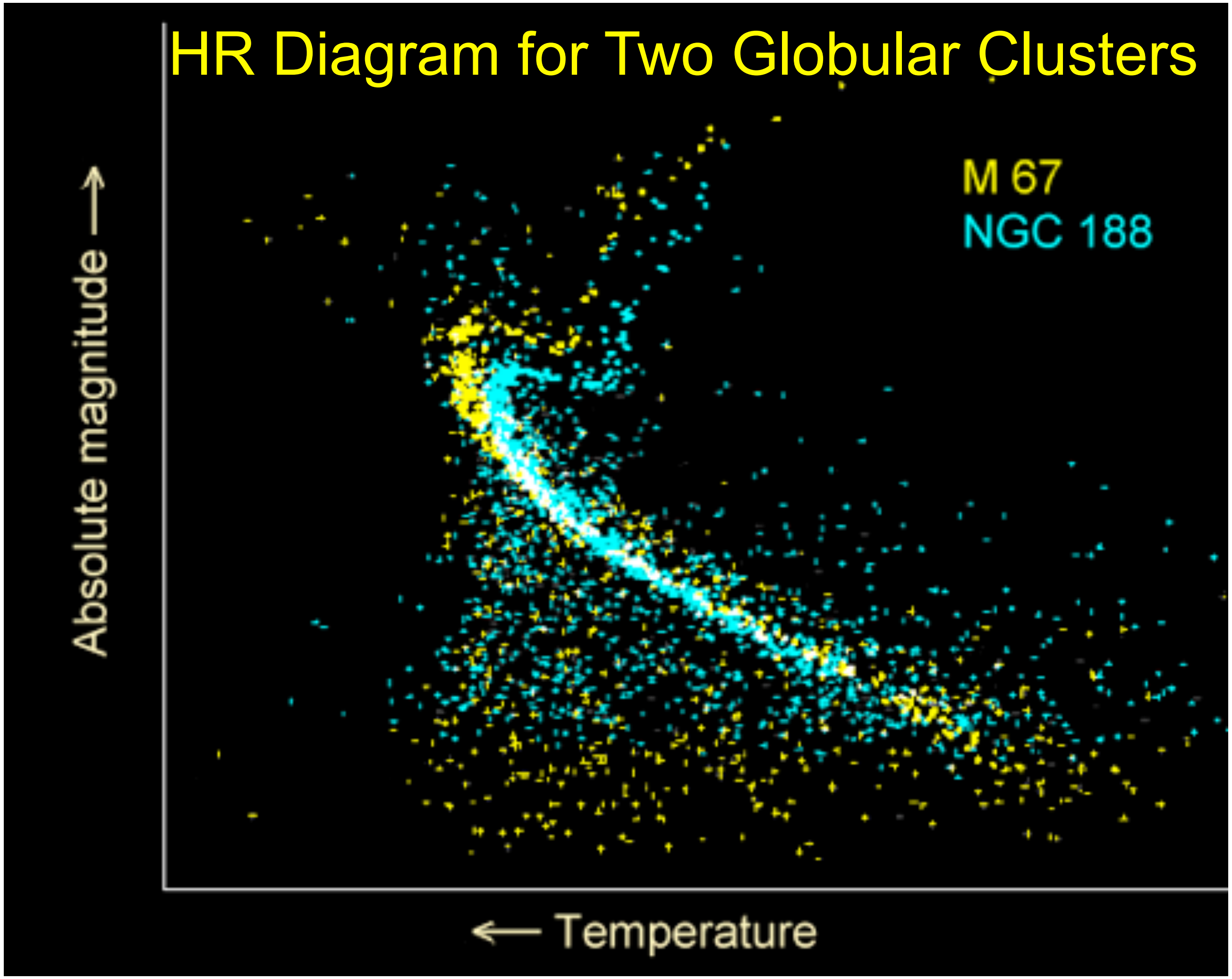
Matter: $p = 0 \Rightarrow \rho = \rho_0 a^{-3}$ (assumed above in q_0, t_0 eqs.)

Radiation: $p = \frac{\rho}{3},\ k = 0 \Rightarrow \rho \propto a^{-4}$

The Age of the Universe

In the mid-1990s there was a crisis in cosmology, because the age of the old Globular Cluster stars in the Milky Way, then estimated to be 16±3 Gyr, was higher than the expansion age of the universe, which for a critical density ($\Omega_m = 1$) universe is 9±2 Gyr (with the Hubble parameter h=0.72±0.07).

But when the data from the Hipparcos astrometric satellite became available in 1997, it showed that the distance to the Globular Clusters had been under-estimated, which implied that their ages are 12±3 Gyr.



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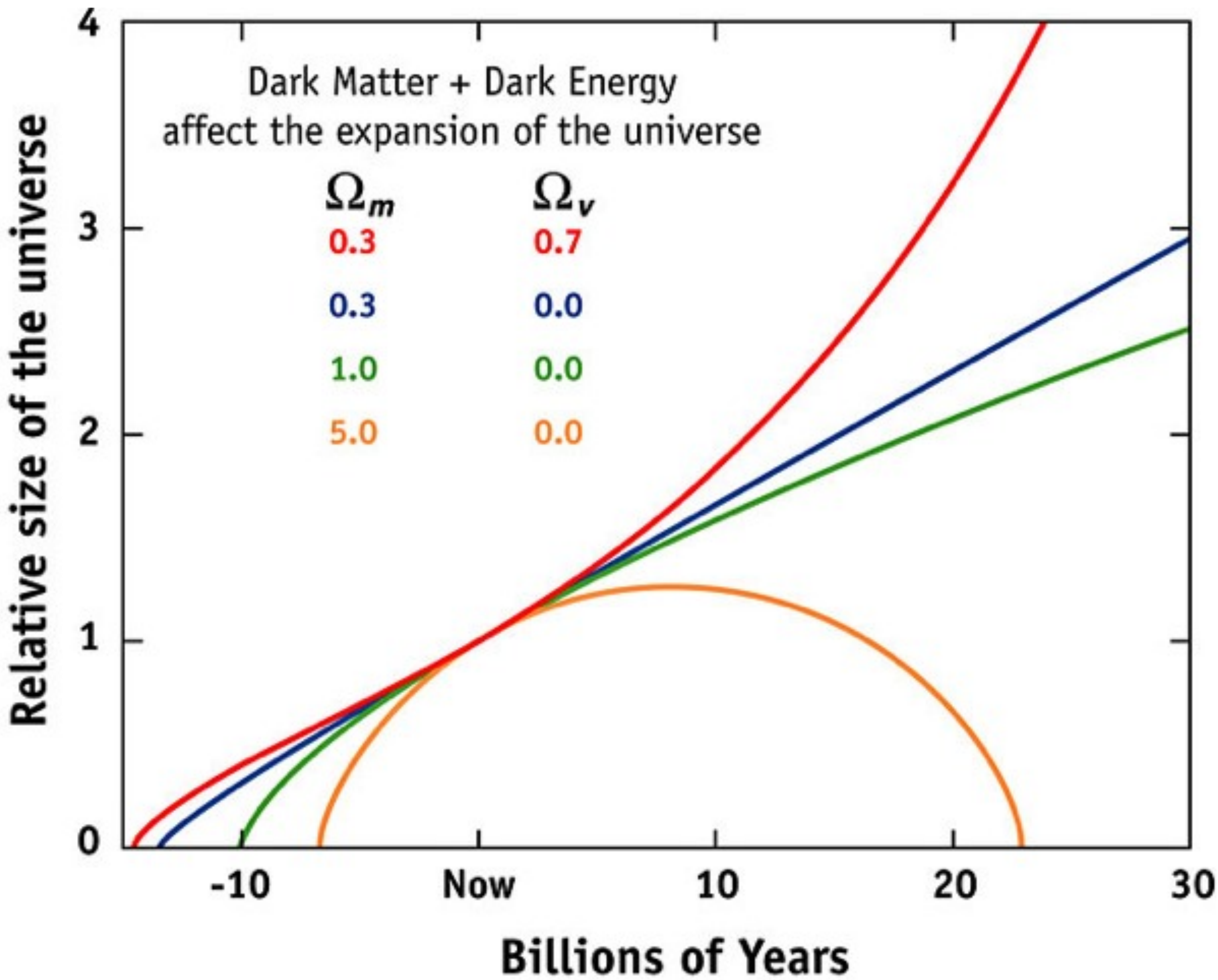
Many lines of evidence now show that the universe does not have $\Omega_m = 1$ but rather $\Omega_{tot} = \Omega_m + \Omega_\Lambda = 1.0$ with $\Omega_m \approx 0.3$, which gives an expansion age of about 14 Gyr.

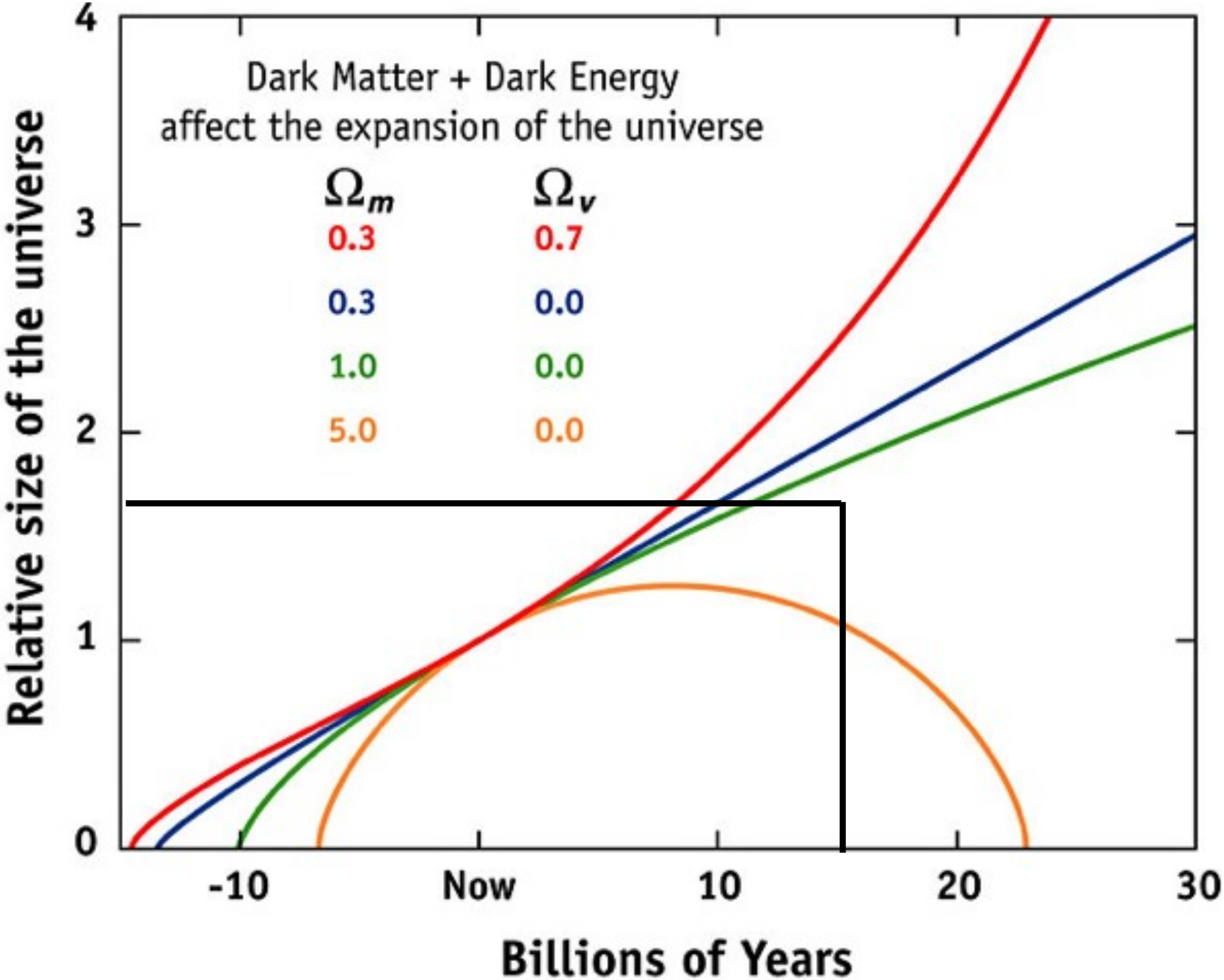
Moreover, age measurement based on radioactive decay of Thorium-232 (half-life 14.1 Gyr) measured in a number of stars gives a completely independent age of 14±3 Gyr. A similar measurement, based on Uranium-238 (half-life 4.47 Gyr), gives 12.5±3 Gyr.

All the recent measurements of the age of the universe are thus in excellent agreement. It is reassuring that three completely different clocks – stellar evolution, expansion of the universe, and radioactive decay – agree so well.

History of Cosmic Expansion for General Ω_M & Ω_Λ



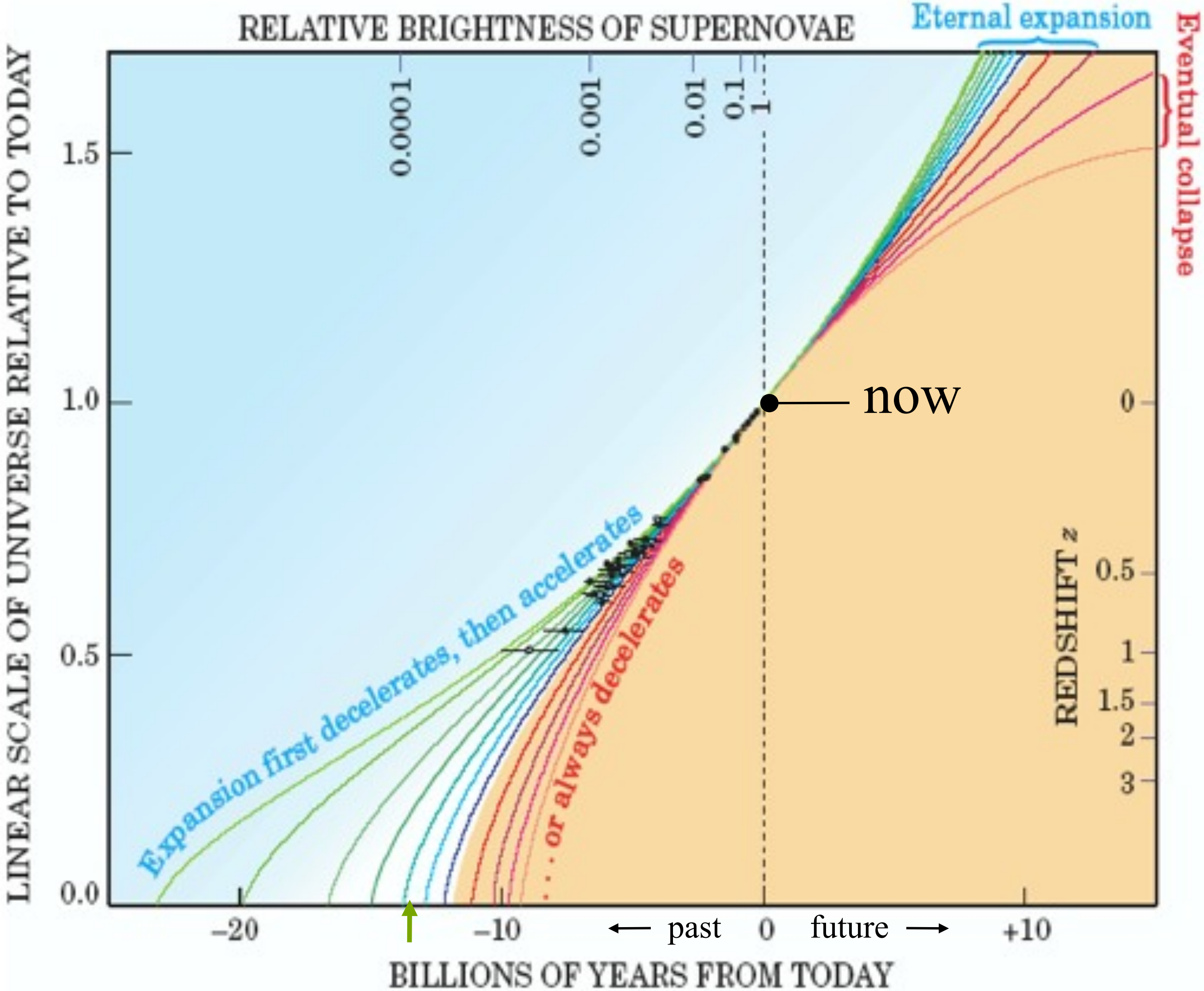
History of Cosmic Expansion for General Ω_M & Ω_Λ



History of Cosmic Expansion for $\Omega_\Lambda = 1 - \Omega_M$

With $\Omega_\Lambda = 0$ the age of the decelerating universe would be only 9 Gyr, but $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$ gives an age of 14 Gyr, consistent with stellar and radioactive decay ages

Figure 4. The history of cosmic expansion, as measured by the high-redshift supernovae (the black data points), assuming flat cosmic geometry. The scale factor R of the universe is taken to be 1 at present, so it equals $1/(1 + z)$. The curves in the blue shaded region represent cosmological models in which the accelerating effect of vacuum energy eventually overcomes the decelerating effect of the mass density. These curves assume vacuum energy densities ranging from 0.95 ρ_c (top curve) down to 0.4 ρ_c. In the yellow shaded region, the curves represent models in which the cosmic expansion is always decelerating due to high mass density. They assume mass densities ranging (left to right) from 0.8 ρ_c up to 1.4 ρ_c. In fact, for the last two curves, the expansion eventually halts and reverses into a cosmic collapse.



Saul Perlmutter, ***Physics Today***, Apr 2003

LCDM Benchmark Cosmological Model: Ingredients & Epochs

	List of Ingredients
photons:	$\Omega_{\gamma,0} = 5.0 \times 10^{-5}$
neutrinos:	$\Omega_{\nu,0} = 3.4 \times 10^{-5}$
total radiation:	$\Omega_{r,0} = 8.4 \times 10^{-5}$
baryonic matter:	$\Omega_{\text{bary},0} = 0.04$
nonbaryonic dark matter:	$\Omega_{\text{dm},0} = 0.26$
total matter:	$\Omega_{m,0} = 0.30$
cosmological constant:	$\Omega_{\Lambda,0} \approx 0.70$

Important Epochs

radiation-matter equality:	$a_{rm} = 2.8 \times 10^{-4}$	$t_{rm} = 4.7 \times 10^{4}$ yr
matter-lambda equality:	$a_{m\Lambda} = 0.75$	$t_{m\Lambda} = 9.8$ Gyr
Now:	$a_0 = 1$	$t_0 = 13.5$ Gyr

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

Benchmark Model: Scale Factor vs. Time

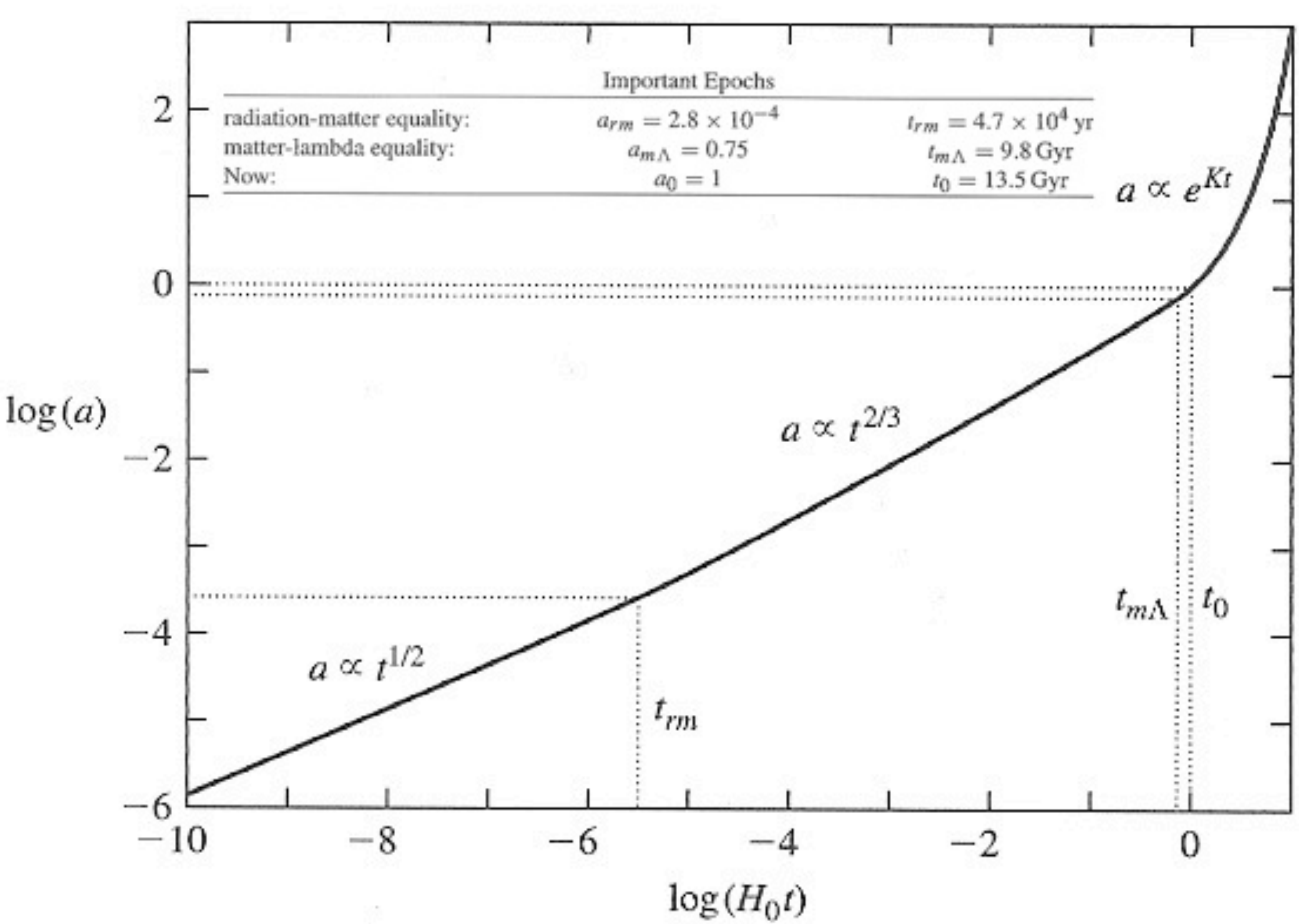
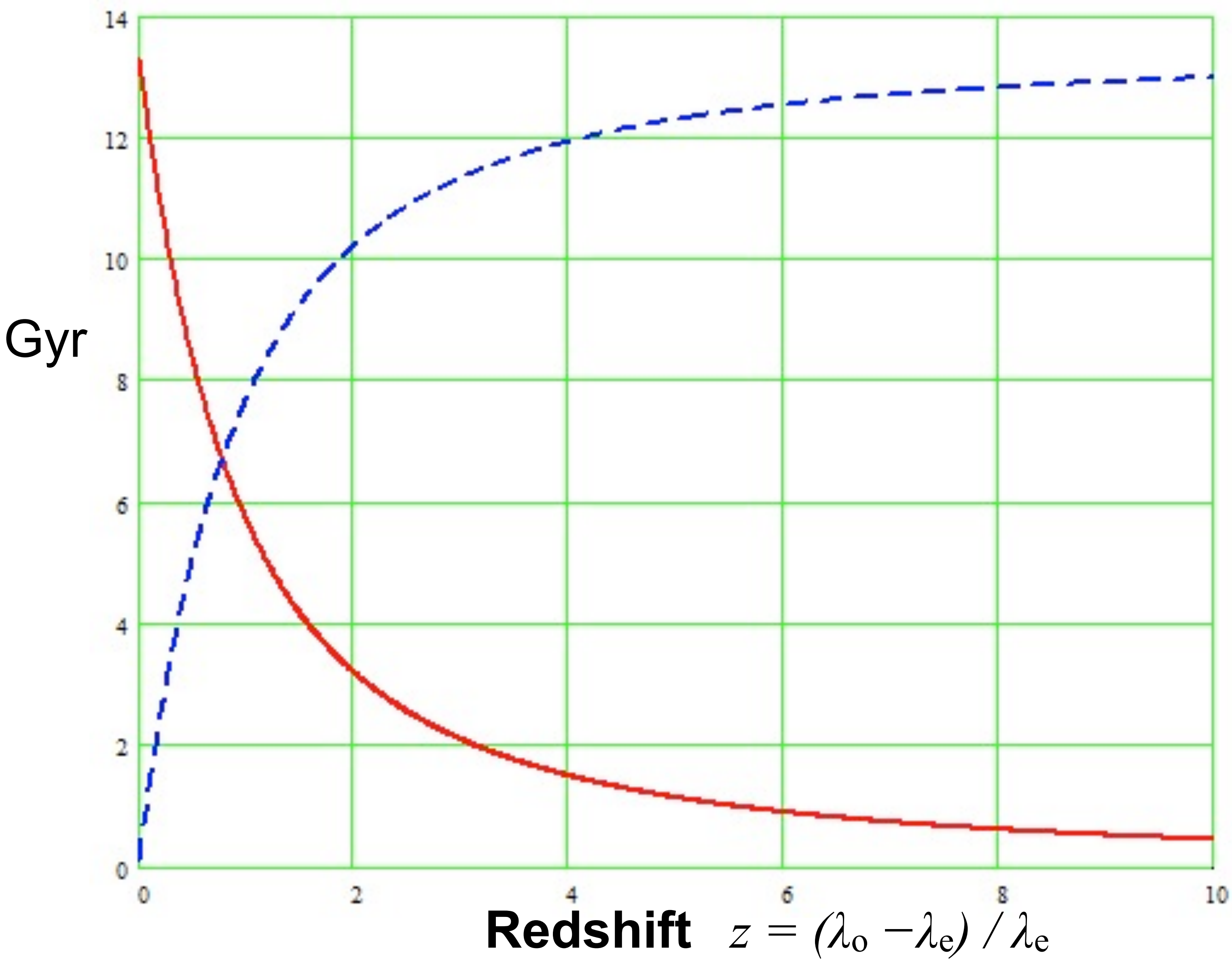


FIGURE 6.5 The scale factor a as a function of time t (measured in units of the Hubble time), computed for the Benchmark Model. The dotted lines indicate the time of radiation-matter equality, $a_{rm} = 2.8 \times 10^{-4}$, the time of matter-lambda equality, $a_{m\Lambda} = 0.75$, and the present moment, $a_0 = 1$.

Barbara Ryden, *Introduction to Cosmology* (Addison-Wesley, 2003)

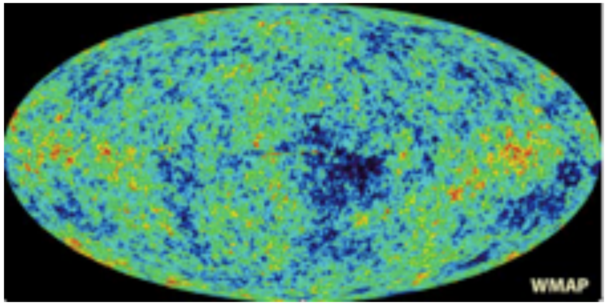
Age of the Universe and Lookback Time



These are for the Benchmark Model $\Omega_{m,0}$=0.3, $\Omega_{\Lambda,0}$=0.7, h=0.7.

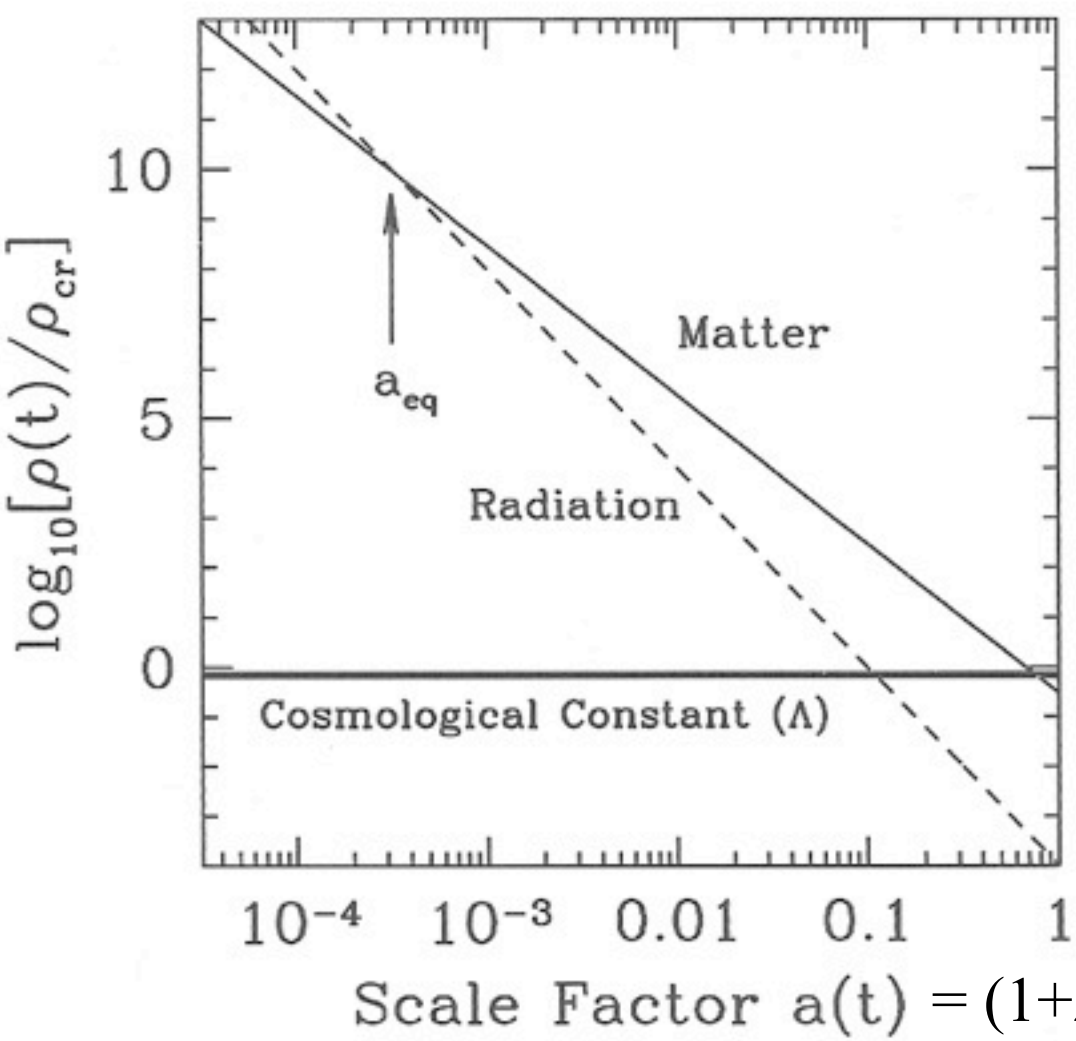
Brief History of the Universe

- Cosmic Inflation generates density fluctuations
- Symmetry breaking: more matter than antimatter
- All antimatter annihilates with almost all the matter (1s)
- Big Bang Nucleosynthesis makes light nuclei (10 min)
- Electrons and light nuclei combine to form atoms, and the cosmic background radiation fills the newly transparent universe (380,000 yr)



- Galaxies and larger structures form (~0.5 Gyr)
- Carbon, oxygen, iron, ... are made in stars
- Earth-like planets form around 2nd generation stars
- Life somehow starts (~4 Gyr ago) and evolves on earth

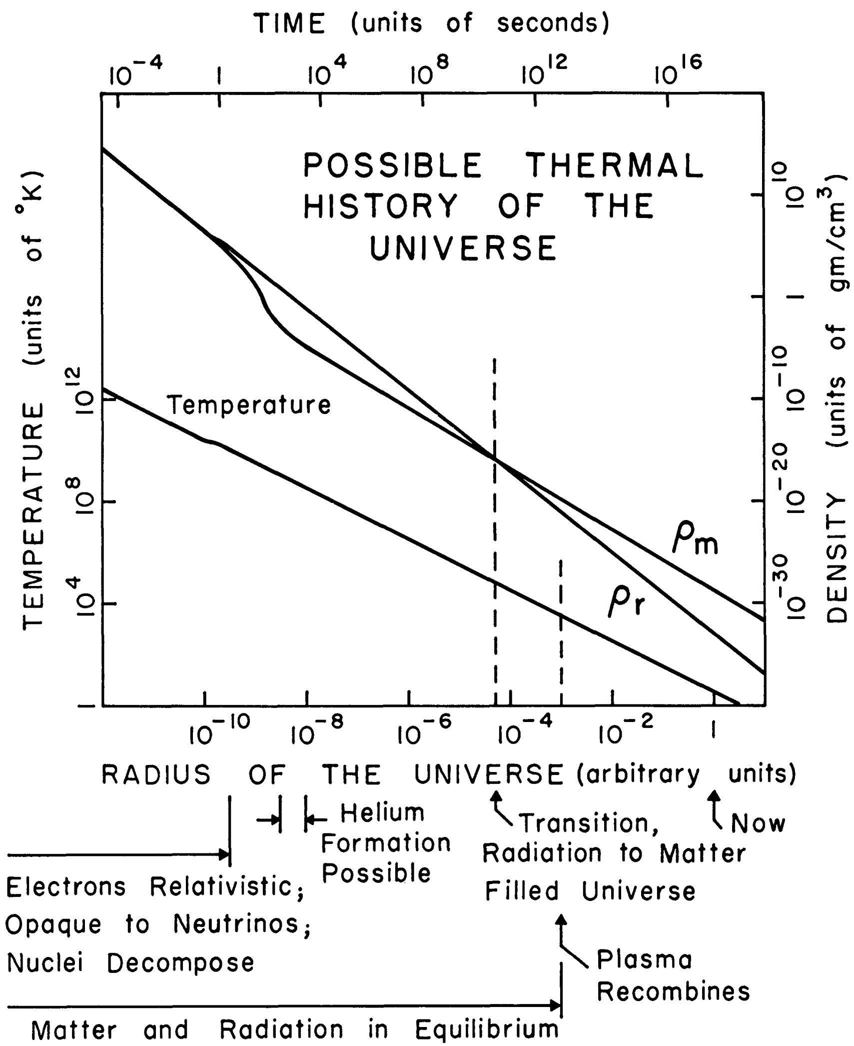
Evolution of Densities of Radiation, Matter, & Λ



$= (1+z)^{-1}$ z = redshift

Figure 1.3. Energy density vs scale factor for different constituents of a flat universe. Shown are nonrelativistic matter, radiation, and a cosmological constant. All are in units of the critical density today. Even though matter and cosmological constant dominate today, at early times, the radiation density was largest. The epoch at which matter and radiation are equal is a_{eq}.

Dodelson, Chapter 1



COSMIC BLACK-BODY RADIATION*

One of the basic problems of cosmology is the singularity characteristic of the familiar cosmological solutions of Einstein's field equations. Also puzzling is the presence of matter in excess over antimatter in the universe, for baryons and leptons are thought to be conserved. Thus, in the framework of conventional theory we cannot understand the origin of matter or of the universe. We can distinguish three main attempts to deal with these problems.

1. The assumption of continuous creation (Bondi and Gold 1948; Hoyle 1948), which avoids the singularity by postulating a universe expanding for all time and a continuous but slow creation of new matter in the universe.

2. The assumption (Wheeler 1964) that the creation of new matter is intimately related to the existence of the singularity, and that the resolution of both paradoxes may be found in a proper quantum mechanical treatment of Einstein's field equations.

3. The assumption that the singularity results from a mathematical over-idealization,

* This research was supported in part by the National Science Foundation and the Office of Naval Research of the U.S. Navy.

FIG 1 —Possible thermal history of the Universe. The figure shows the previous thermal history of the Universe assuming a homogeneous isotropic general-relativity cosmological model (no scalar field) with present matter density 2×10^{-29} gm/cm³ and present thermal radiation temperature 3.5° K The bottom horizontal scale may be considered simply the proper distance between two chosen fiducial co-moving galaxies (*points*) The top horizontal scale is the proper world time. The line marked "temperature" refers to the temperature of the thermal radiation Matter remains in thermal equilibrium with the radiation until the plasma recombines, at the time indicated Thereafter further expansion cools matter not gravitationally bound faster than the radiation. The mass density in radiation is ρ_r. At present ρ_r is substantially below the mass density in matter, ρ_m, but, in the early Universe ρ_r exceeded ρ_m We have indicated the time when the Universe exhibited a transition from the characteristics of a radiation-filled model to those of a matter-filled model.

Looking back in time, as the temperature approaches 10^{10} ° K the electrons become relativistic, and thermal electron-pair creation sharply increases the matter density At temperatures somewhat greater than 10^{10} ° K these electrons should be so abundant as to assure a thermal neutrino abundance and a thermal neutron-proton abundance ratio. A temperature of this order would be required also to decompose the nuclei from the previous cycle in an oscillating Universe. Notice that the nucleons are non-relativistic here.

The thermal neutrons decay at the right-hand limit of the indicated region of helium formation. There is a left-hand limit on this region because at higher temperatures photodissociation removes the deuterium necessary to form helium The difficulty with this model is that most of the matter would end up in helium.

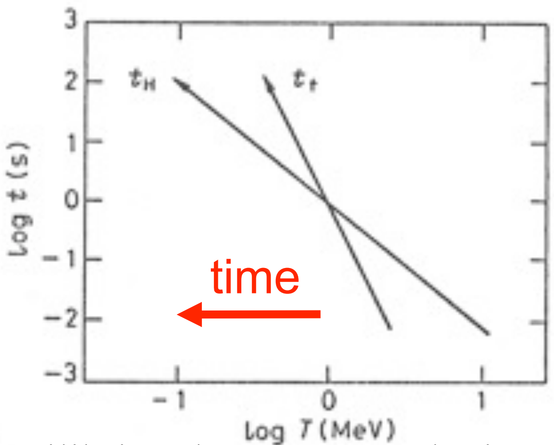
We deeply appreciate the helpfulness of Drs. Penzias and Wilson of the Bell Telephone Laboratories, Crawford Hill, Holmdel, New Jersey, in discussing with us the result of their measurements and in showing us their receiving system. We are also grateful for several helpful suggestions of Professor J. A. Wheeler.

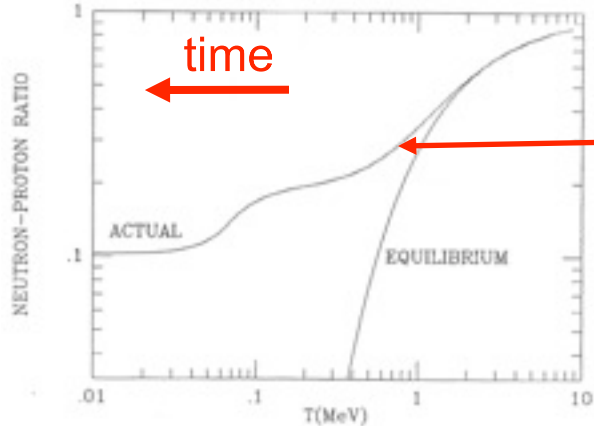
R. H. DICKE
P. J. E. PEEBLES
P. G. ROLL
D. T. WILKINSON

May 7, 1965
PALMER PHYSICAL LABORATORY
PRINCETON, NEW JERSEY

Big Bang Nucleosynthesis

BBN was conceived by Gamow in 1946 as an explanation for the formation of all the elements, but the absence of any stable nuclei with A=5,8 makes it impossible for BBN to proceed past Li. The formation of carbon and heavier elements occurs instead through the triple-α process in the centers of red giants (Burbidge[2], Fowler, & Hoyle 57). At the BBN baryon density of $2\times10^{-29}\ \Omega_b\ h^2\ (T/T_0)^3$ g cm^{-3} $\approx 2\times10^{-5}$ g cm^{-3}, the probability of the triple-α process is negligible even though $T \approx 10^9$K.

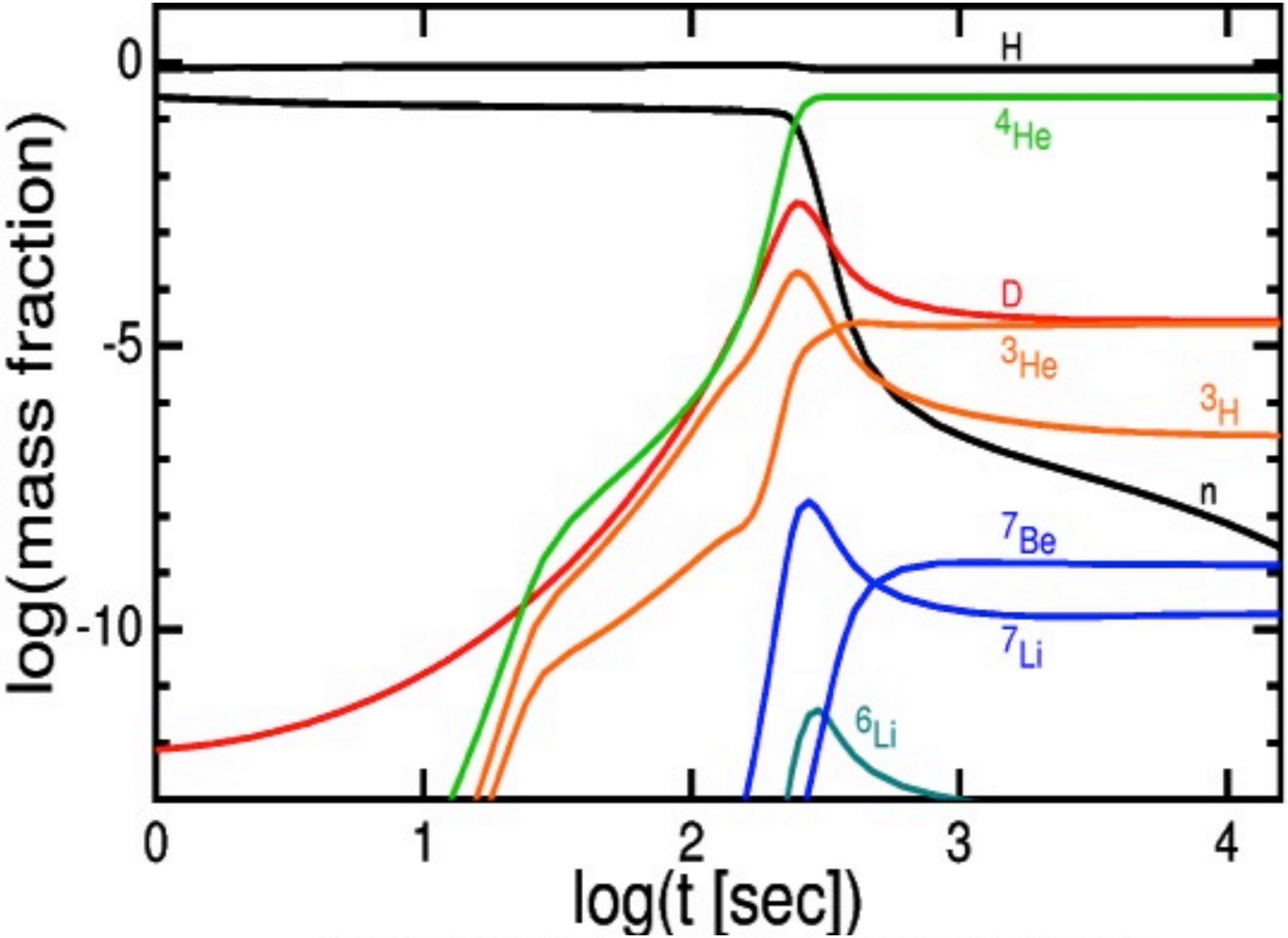




neutrino-baryon interactions freeze out as densities drop

Kolb & Turner

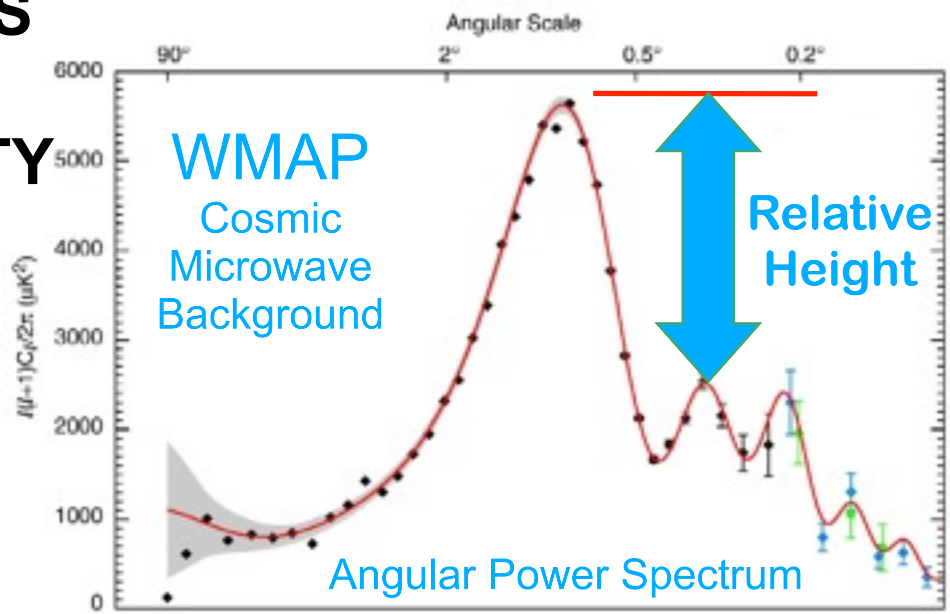
Thermal equilibrium between n and p is maintained by weak interactions, which keeps $n/p = \exp(-Q/T)$ (where $Q = m_n - m_p = 1.293$ MeV) until about $t \approx 1$ s. But because the neutrino mean free time $t_\nu^{-1} \approx \sigma_\nu n_{e\pm} \approx (G_F T)^2 (T^3)$ is increasing as $t_\nu \propto T^{-5}$ (here the Fermi constant $G_F \approx 10^{-5}$ GeV^{-2}), while the horizon size is increasing only as $t_H \approx (G\rho)^{-1/2} \approx M_{Pl}\, T^{-2}$, these interactions freeze out when T drops below about 0.8 MeV. This leaves $n/(p+n) \approx 0.14$. The neutrons then decay with a mean lifetime 887 ± 2 s until they are mostly fused into D and then ^{4}He. The higher the baryon density, the higher the final abundance of ^{4}He and the lower the abundance of D that survives this fusion process. Since D/H is so sensitive to baryon density, David Schramm called deuterium the "baryometer." He and his colleagues also pointed out that since the horizon size increases more slowly with T^{-2} the larger the number of light neutrino species N_ν contributing to the energy density ρ, BBN predicted that $N_\nu \approx 3$ before N_ν was measured at accelerators by measuring the width of the Z^0 .



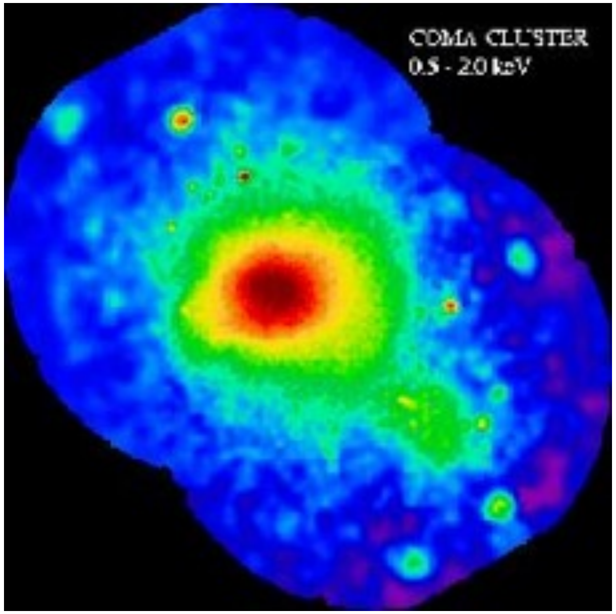
The detailed production of the lightest elements out of protons and neutrons during the first three minutes of the universe's history. The nuclear reactions occur rapidly when the temperature falls below a billion degrees Kelvin. Subsequently, the reactions are shut down, because of the rapidly falling temperature and density of matter in the expanding universe.

Ken Kawano's (1992) BBN code is available at
http://www-thphys.physics.ox.ac.uk/users/SubirSarkar/bbn.html

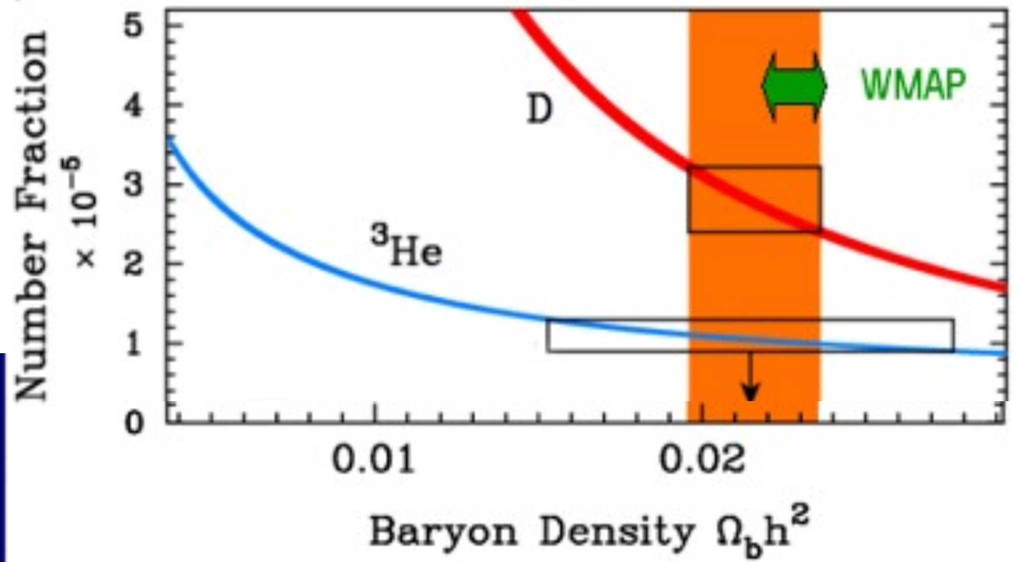
5 INDEPENDENT MEASURES AGREE: ATOMS ARE ONLY 4% OF THE COSMIC DENSITY



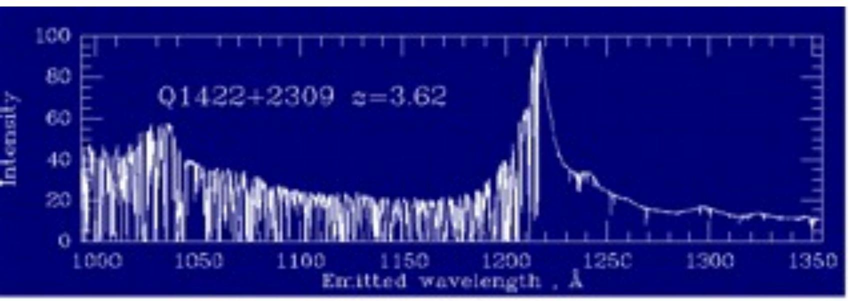
Galaxy Cluster in X-rays



Deuterium Abundance
+ Big Bang Nucleosynthesis



Absorption of Quasar Light

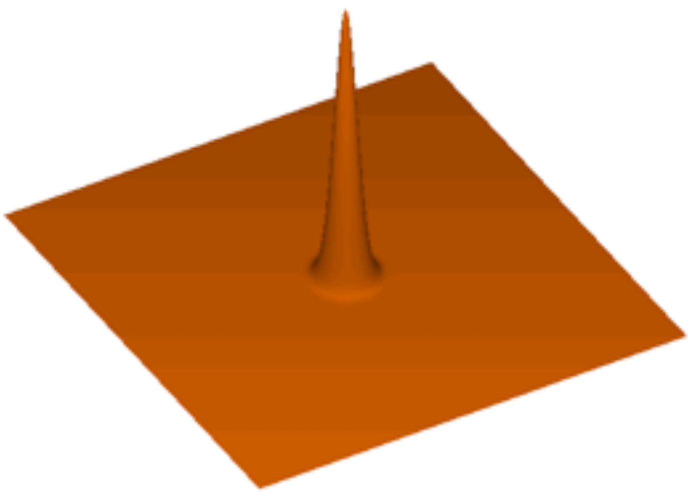


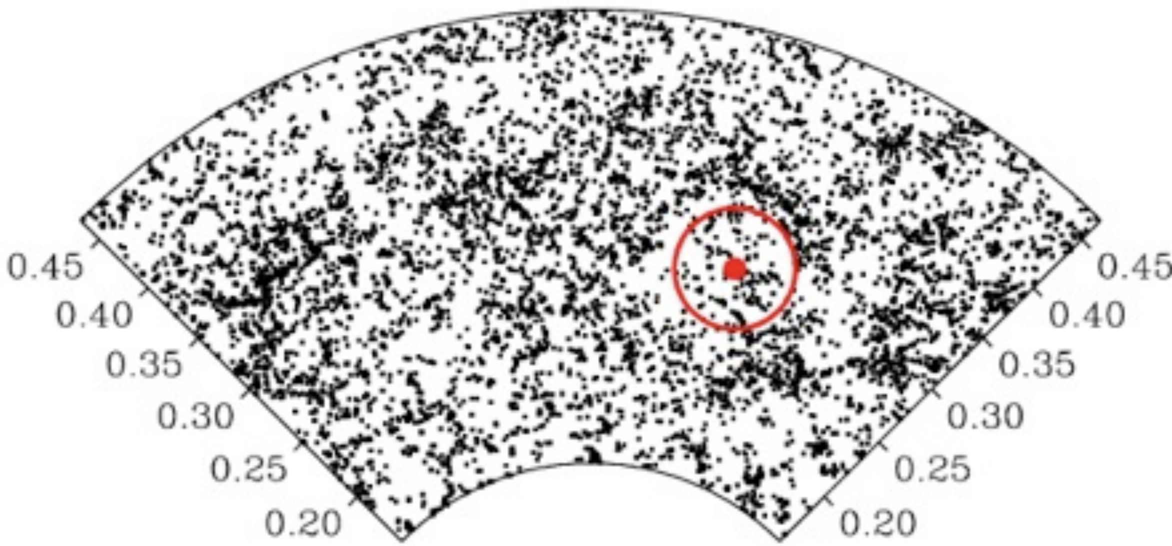
& WIGGLES IN GALAXY P(k)

BAO WIGGLES IN GALAXY P(k)

Sound waves that propagate in the opaque early universe imprint a characteristic scale in the clustering of matter, providing a “standard ruler” whose length can be computed using straightforward physics and parameters that are tightly constrained by CMB observations. Measuring the angle subtended by this scale determines a distance to that redshift and constrains the expansion rate.

The detection of the acoustic oscillation scale is one of the key accomplishments of the SDSS, and even this moderate signal-to-noise measurement substantially tightens constraints on cosmological parameters. Observing the evolution of the BAO standard ruler provides one of the best ways to measure whether the dark energy parameters changed in the past.





M. White lectures 08

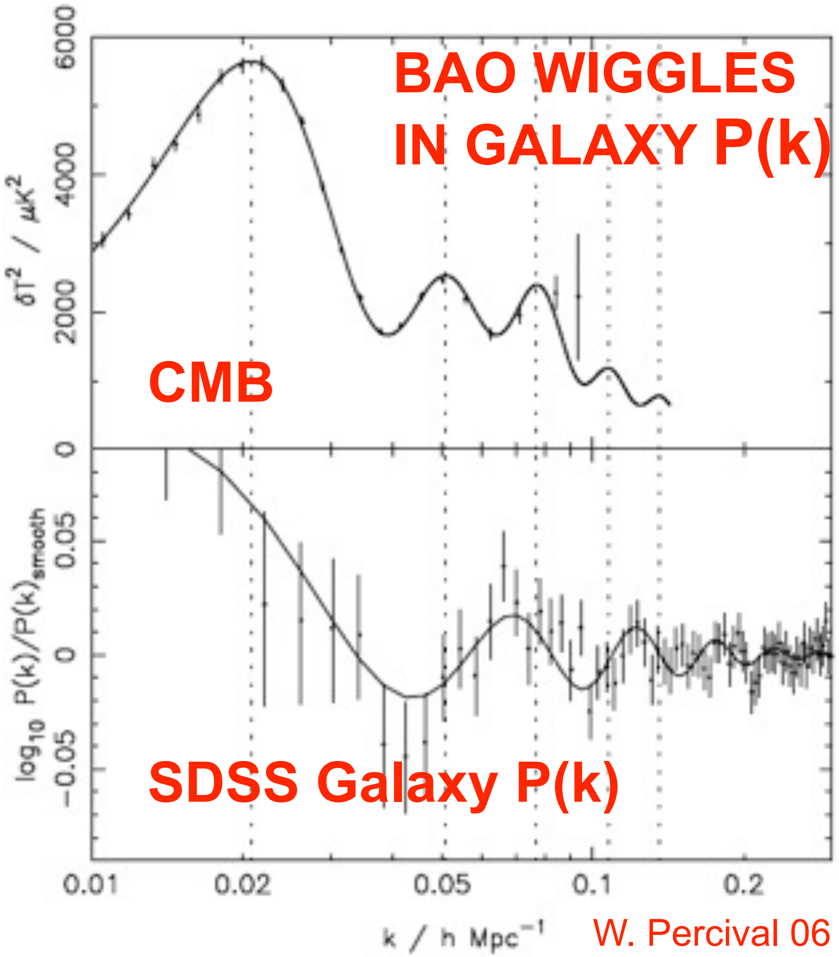
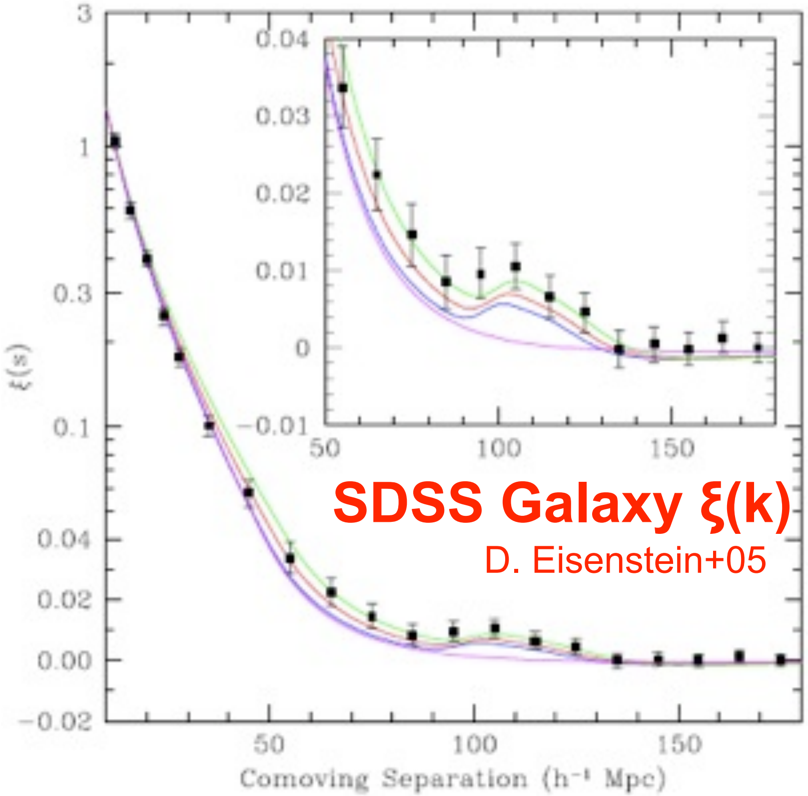
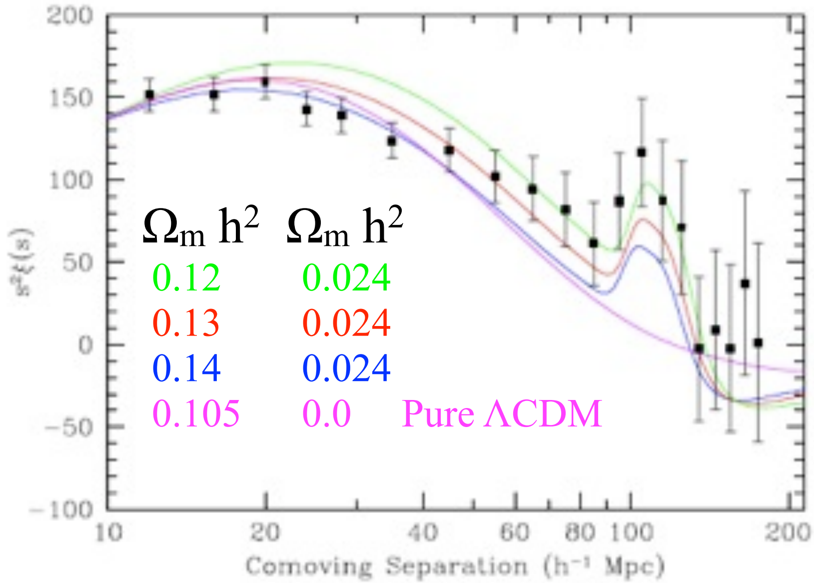
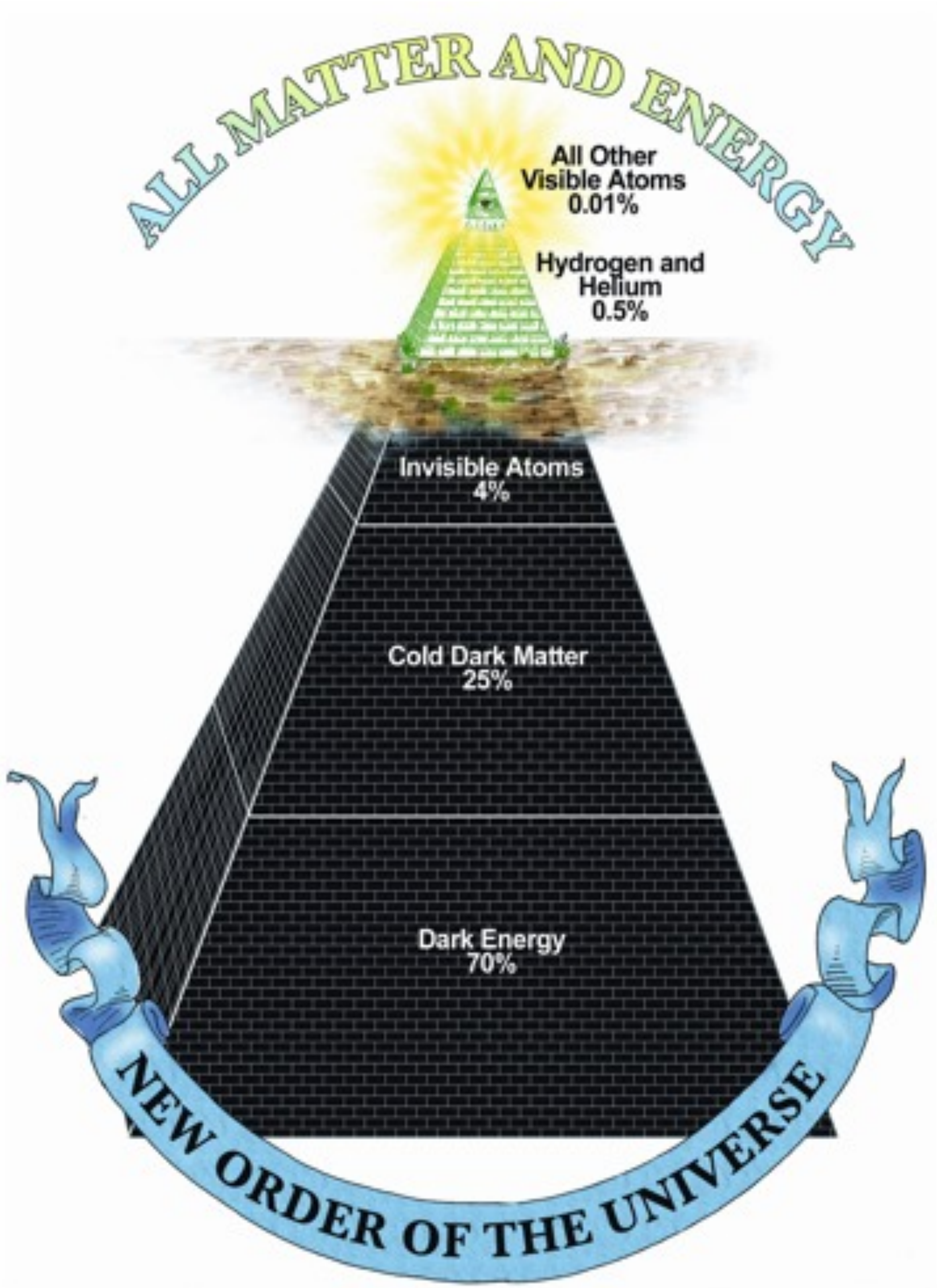
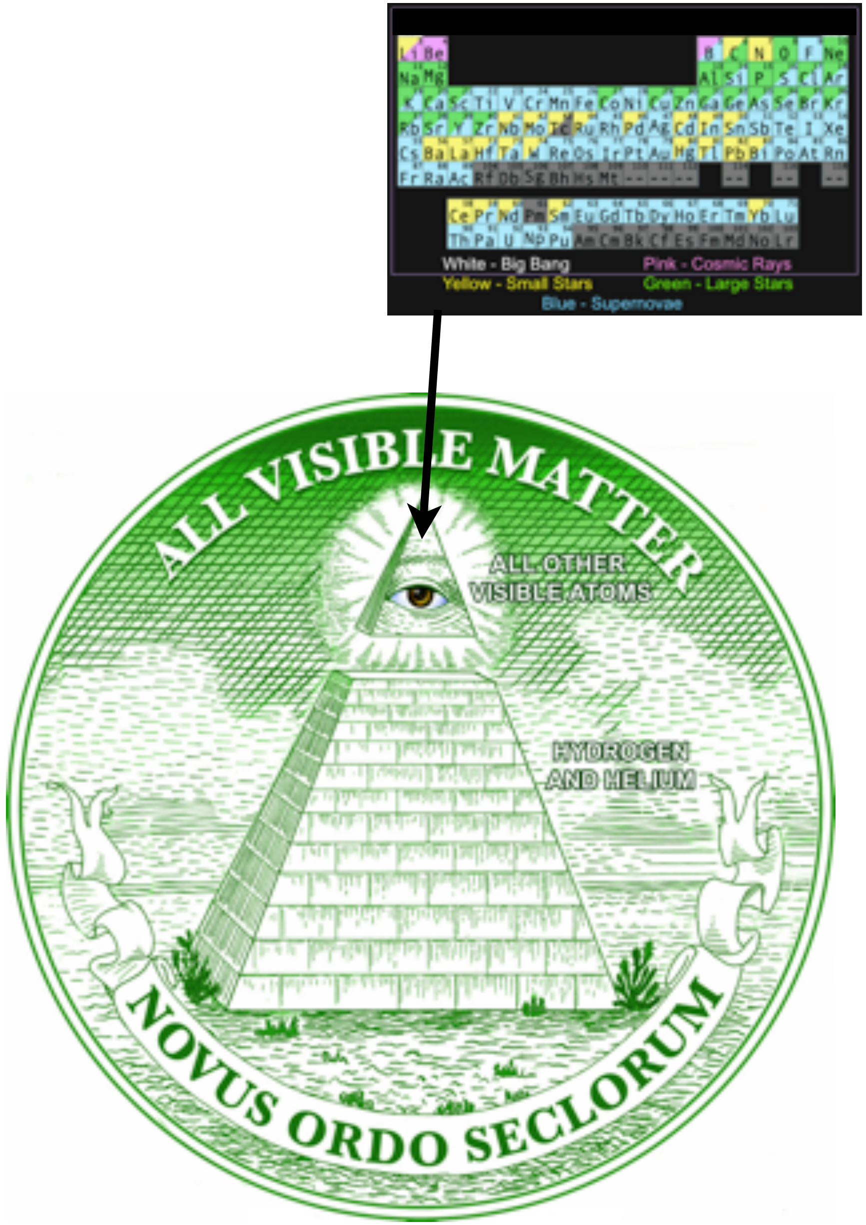


Fig. 3. Upper panel: The TT power spectrum recovered from the 3-year WMAP data (Hinshaw et al. 2006), projected into comoving space assuming a cosmological model with $\Omega_m = 0.25$ and $\Omega_V = 0.75$. For comparison, in the lower panel we plot the baryon oscillations calculated by dividing the SDSS power spectrum with a smooth cubic spline fit (Percival et al. 2007a). Vertical dotted lines show the positions of the peaks in the CMB power spectrum. As can be seen, there is still a long way to go before low redshift observations can rival the CMB in terms of the significance of the acoustic oscillation signal.

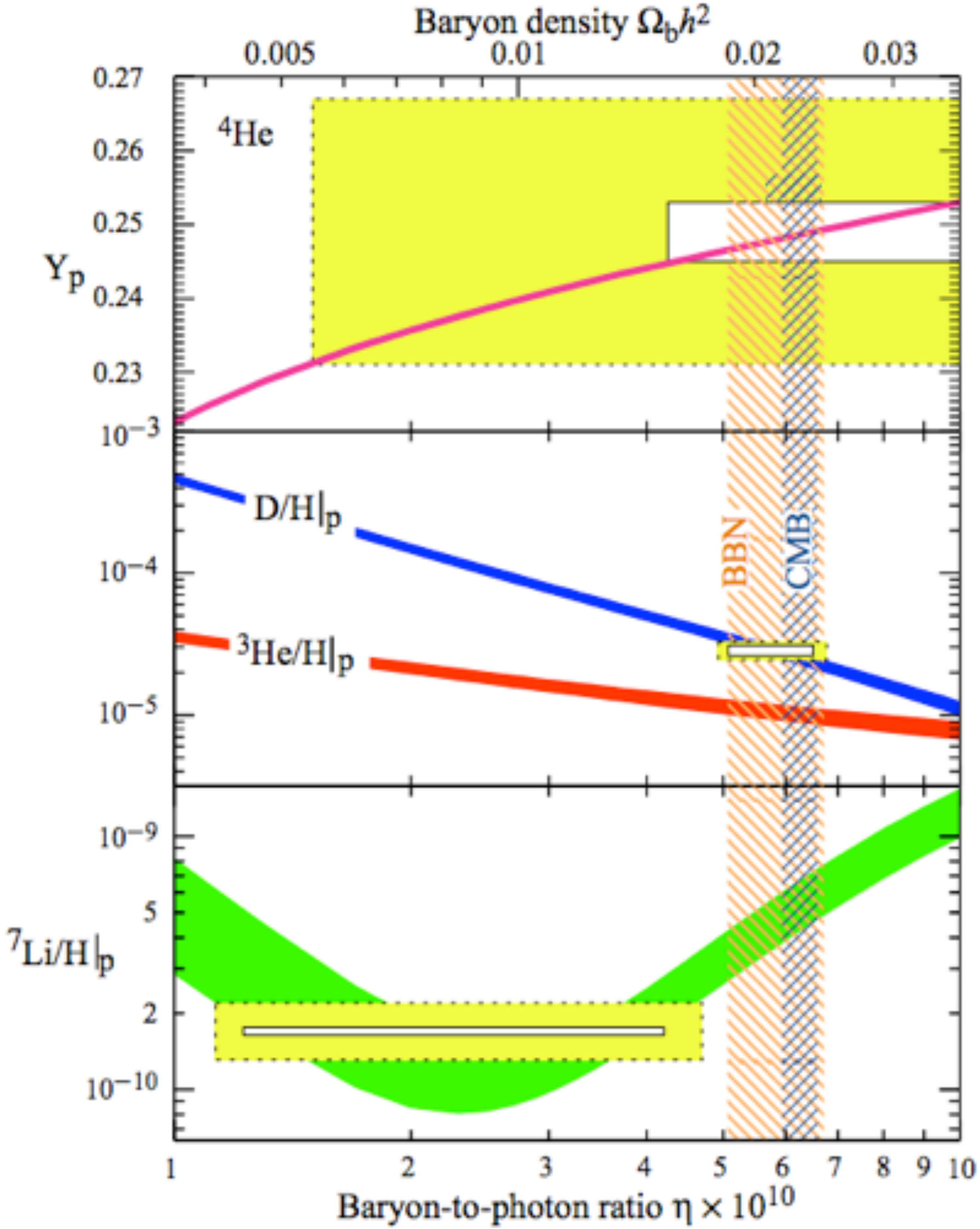





ALL MATTER AND ENERGY
All Other Visible Atoms 0.01%
Hydrogen and Helium 0.5%
Invisible Atoms 4%
Cold Dark Matter 25%
Dark Energy 70%
NEW ORDER OF THE UNIVERSE


White - Big Bang
Pink - Cosmic Rays
Yellow - Small Stars
Green - Large Stars
Blue - Supernovae
ALL VISIBLE MATTER
ALL OTHER VISIBLE ATOMS
HYDROGEN AND HELIUM
NOVUS ORDO SECLORUM

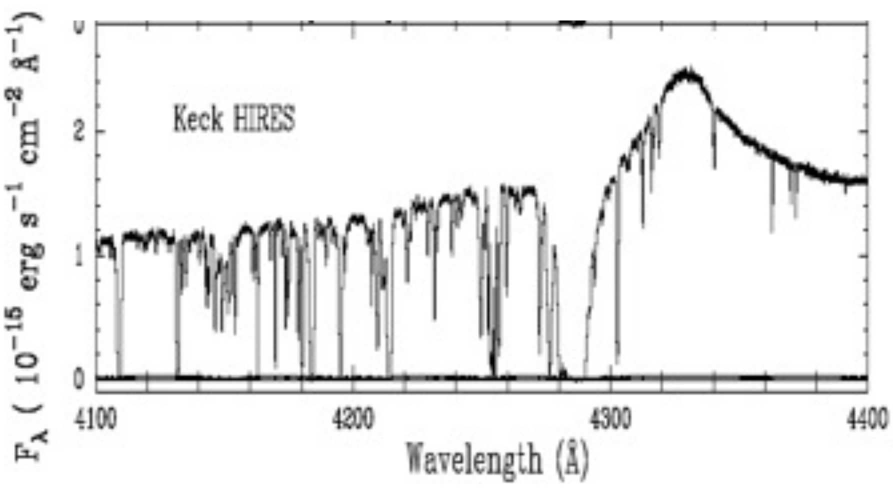
BBN Predicted vs. Measured Abundances of D, ^{3}He, ^{4}He, and ^{7}Li



Particle Data Group, 2012

^{7}Li IS NOW DISCORDANT unless stellar diffusion destroys ^{7}Li

Deuterium absorption at redshift 2.525659 towards Q1243+3047



The Lyα absorption near 4285 Å is from the system in which we measure D/H.

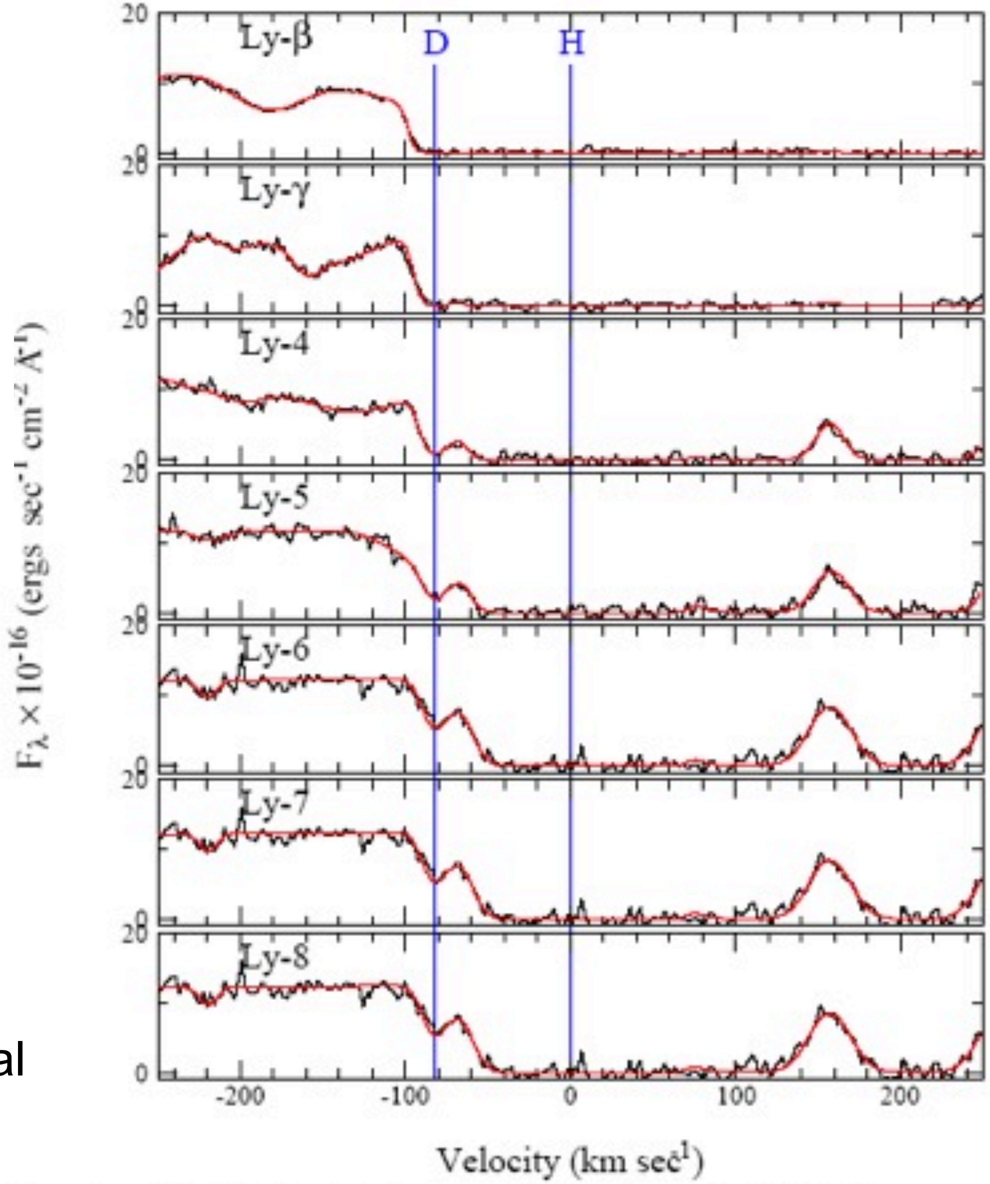


FIG. 7.— The HIRES spectrum of Ly-2 to 8, together with our model of the system, as given in Table 3.

The detection of Deuterium and the modeling of this system seem convincing. This is just a portion of the evidence that the Tytler group presented in this paper. They have similarly convincing evidence for several other Lyman alpha clouds in quasar spectra.

Kirkman, Tytler, Suzuki, O'Meara, & Lubin 2004

BBN is a Prototype for Hydrogen Recombination and DM Annihilation

All three are examples of the universe dropping out of equilibrium!

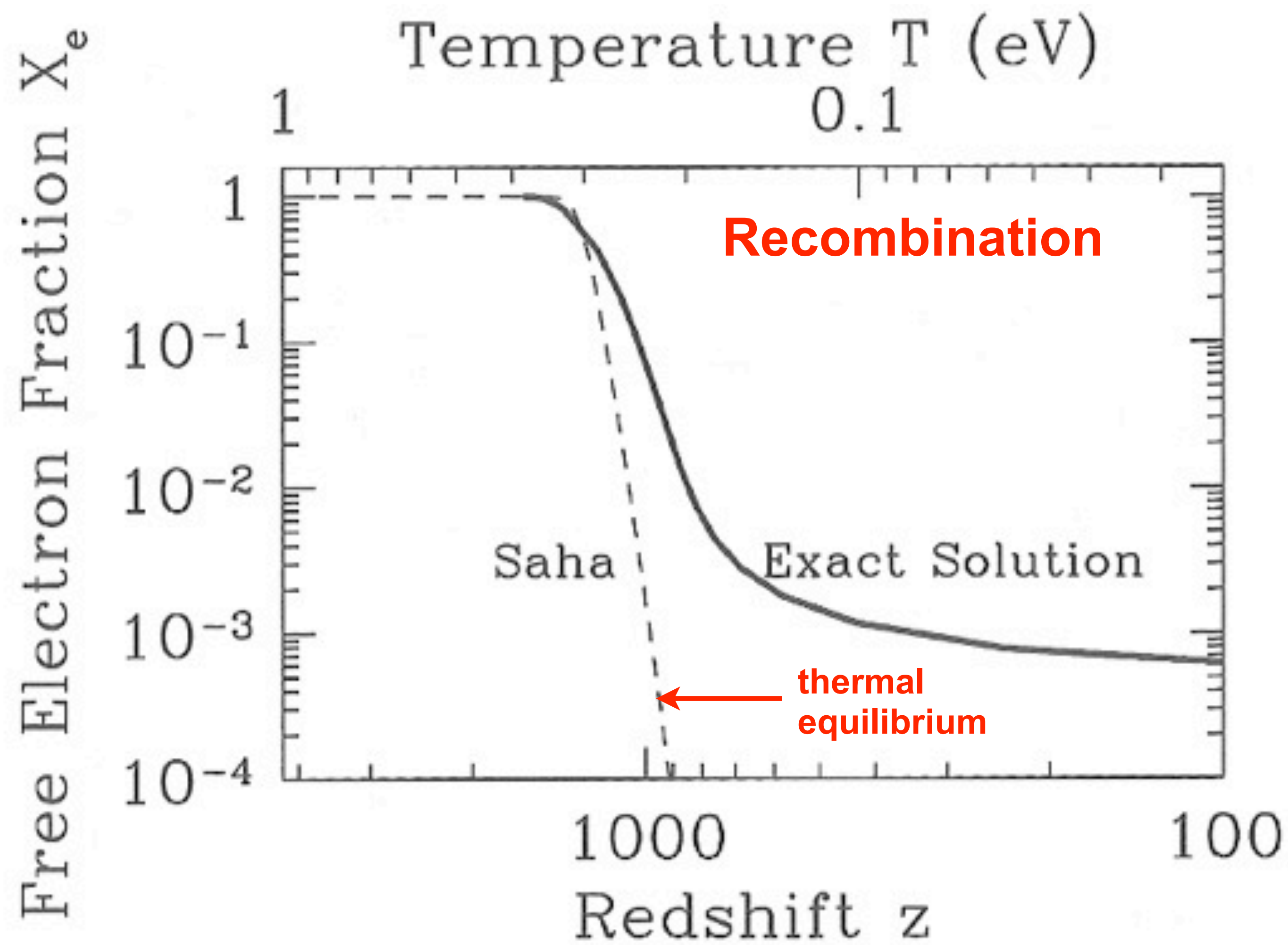
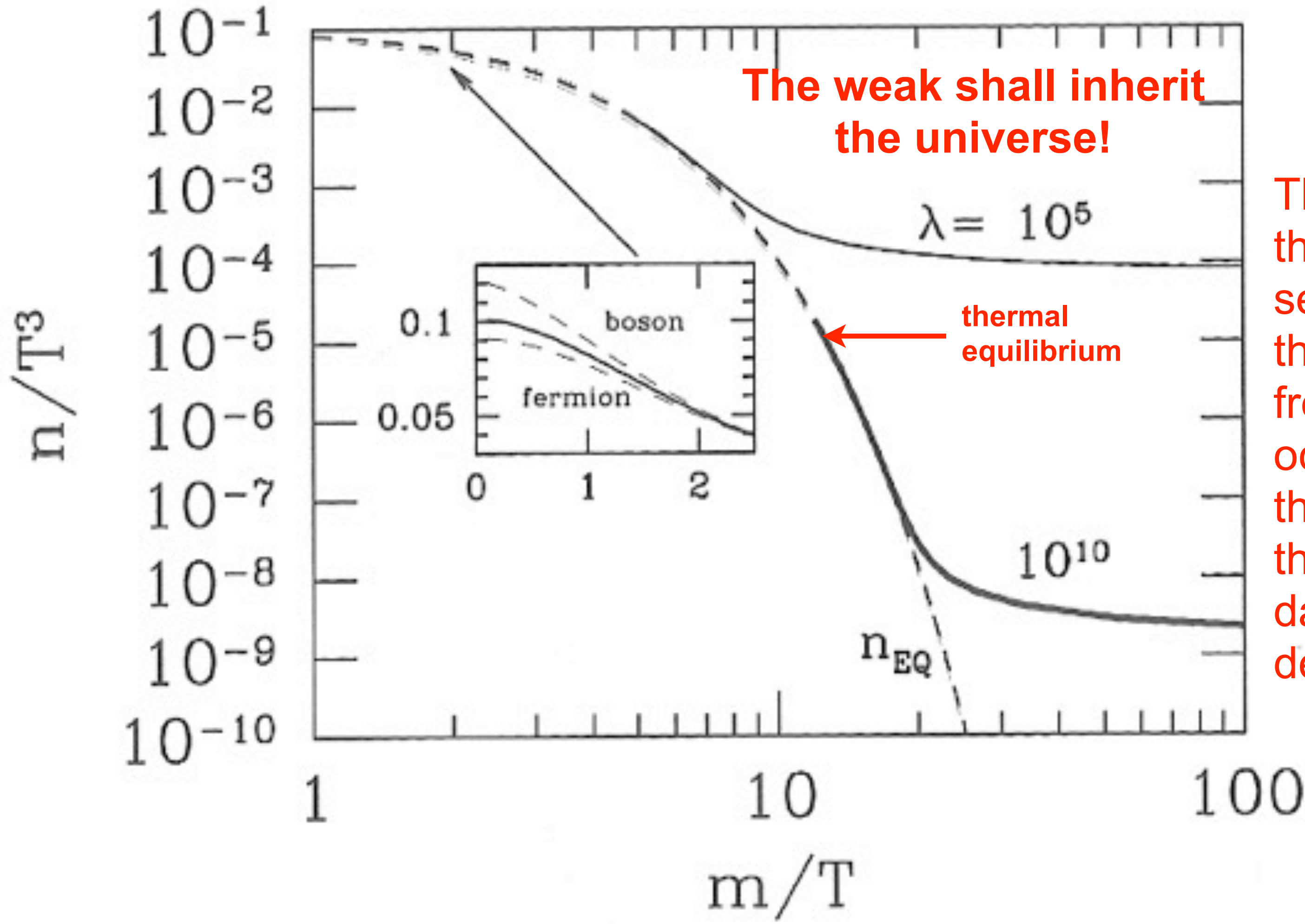


Figure 3.4. Free electron fraction as a function of redshift. Recombination takes place suddenly at $z \sim 1000$ corresponding to $T \sim 1/4$ eV. The Saha approximation, Eq. (3.37), holds in equilibrium and correctly identifies the redshift of recombination, but not the detailed evolution of X_e. Here $\Omega_b = 0.06, \Omega_m = 1, h = 0.5$.

Dodelson, *Modern Cosmology*, p. 72

Dark Matter Annihilation



The weaker the cross section, the earlier freezeout occurs, and the larger the resulting dark matter density.

Figure 3.5. Abundance of heavy stable particle as the temperature drops beneath its mass. Dashed line is equilibrium abundance. Two different solid curves show heavy particle abundance for two different values of λ, the ratio of the annihilation rate to the Hubble rate. Inset shows that the difference between quantum statistics and Boltzmann statistics is important only at temperatures larger than the mass.

Dodelson, *Modern Cosmology*, p. 76

Dark Matter Annihilation

The abundance today of dark matter particles X of the WIMP variety is determined by their survival of annihilation in the early universe. Supersymmetric neutralinos can annihilate with each other (and sometimes with other particles: "co-annihilation").
Dark matter annihilation follows the same pattern as the previous discussions: initially the abundance of dark matter particles X is given by the equilibrium Boltzmann exponential exp(-m_X/T), but as they start to disappear they have trouble finding each other and eventually their number density freezes out. The freezeout process can be followed using the Boltzmann equation, as discussed in Kolb and Turner, Dodelson, Mukhanov, and other textbooks. For a detailed discussion of Susy WIMPs, see the review article by Jungman, Kamionkowski, and Griest (1996). The result is that the abundance today of WIMPs X is given in most cases by (Dodelson's Eqs. 3.59-60)

$$\Omega_X = \left[\frac{4\pi^3 G g_*(m)}{45}\right]^{1/2} \frac{x_f T_0^3}{30\langle\sigma v\rangle\rho_{cr}} = 0.3h^{-2}\left(\frac{x_f}{10}\right)\left(\frac{g_*(m)}{100}\right)^{1/2}\frac{10^{-39}\mathrm{cm}^2}{\langle\sigma v\rangle}.$$

Here $x_f \approx 10$ is the ratio of m_X to the freezeout temperature T_f, and $g_*(m_X) \approx 100$ is the density of states factor in the expression for the energy density of the universe when the temperature equals m_X

$$\rho = \frac{\pi^2}{30}T^4\left[\sum_{i=\mathrm{bosons}} g_i + \frac{7}{8}\sum_{i=\mathrm{fermions}} g_i\right] \equiv g_*\frac{\pi^2}{30}T^4.$$

The sum is over relativistic species *i* (see the graph of *g*(*T*) on the next slide). Note that more X's survive, the weaker the cross section σ. For Susy WIMPs the natural values are $\sigma \sim 10^{-39}$ cm^2, so $\Omega_X \approx 1$ naturally. This is known as the **"WIMP miracle."**

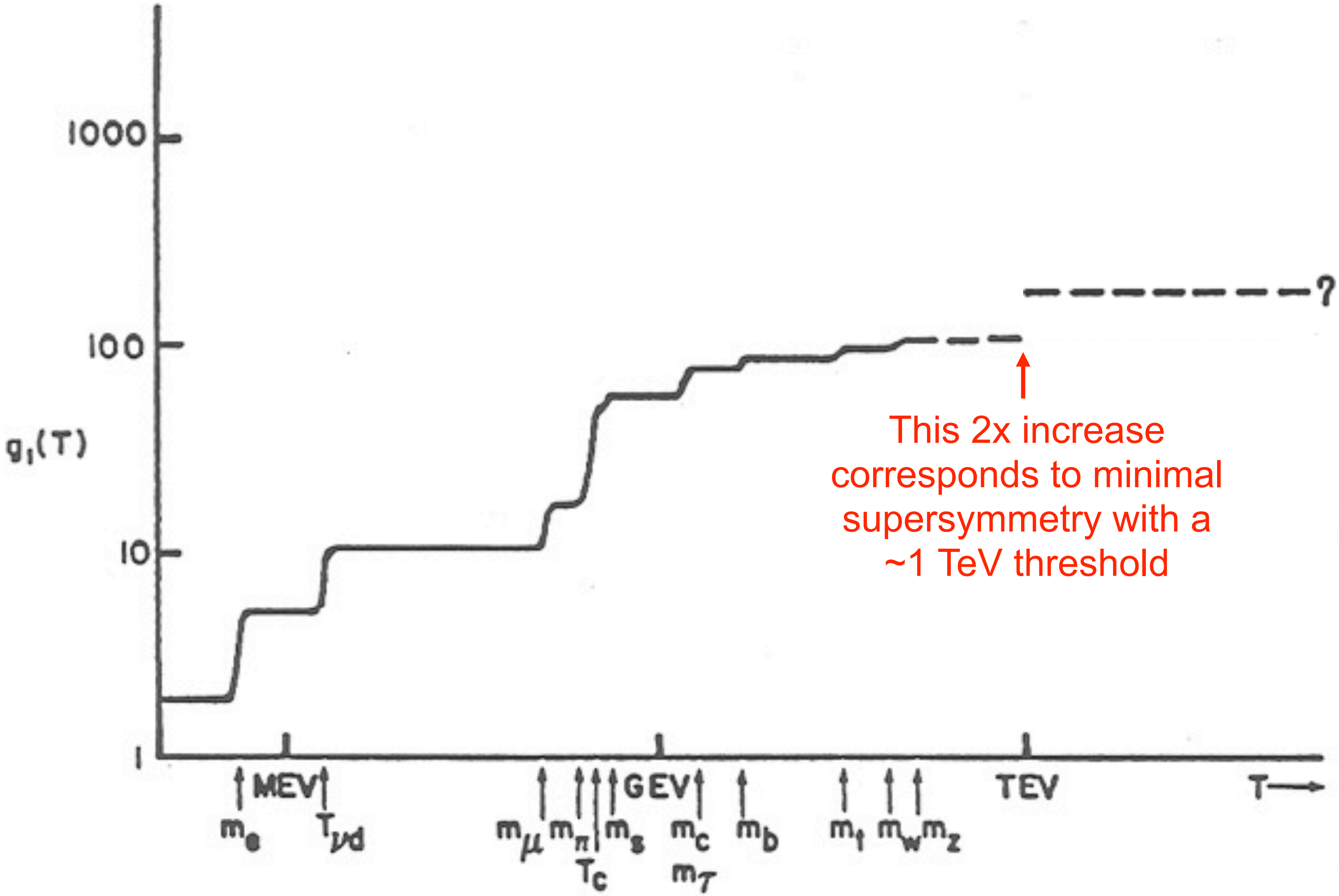


Fig. 1 The effective number of degrees of freedom of thermally interacting relativistic particles as a function of temperature.

Supersymmetry is the basis of most attempts, such as superstring theory, to go beyond the current “Standard Model” of particle physics. Heinz Pagels and Joel Primack pointed out in a 1982 paper that the lightest supersymmetric partner particle is stable because of R-parity, and is thus a good candidate for the dark matter particles – weakly interacting massive particles (**WIMP**s).

Michael Dine and others pointed out that the **axion**, a particle needed to save the strong interactions from violating CP symmetry, could also be the dark matter particle. Searches for both are underway.

Supersymmetric WIMPs

When the British physicist Paul Dirac first combined Special Relativity with quantum mechanics, he found that this predicted that for every ordinary particle like the electron, there must be another particle with the opposite electric charge – the anti-electron (positron). Similarly, corresponding to the proton there must be an anti-proton. Supersymmetry appears to be required to combine General Relativity (our modern theory of space, time, and gravity) with the other forces of nature (the electromagnetic, weak, and strong interactions). The consequence is another doubling of the number of particles, since supersymmetry predicts that for every particle that we now know, including the antiparticles, there must be another, thus far undiscovered particle with the same electric charge but with *spin* differing by half a unit.

Spin	**Matter** (fermions)	**Forces** (bosons)
2		graviton
1		photon, $W^{\pm}, Z^0$ gluons
1/2	quarks u,d,... leptons $e, \nu_e, \ldots$	
0		Higgs bosons axion

Supersymmetric WIMPs

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after doubling

Spin	**Matter** (fermions)	**Forces** (bosons)	**Hypothetical Superpartners**	**Spin**
2		graviton	gravitino	3/2
1		photon, $W^{\pm}, Z^0$ gluons	photino, winos, zino, gluinos	1/2
1/2	quarks u,d,. . . leptons $e, \nu_e, \ldots$		squarks $\tilde{u}, \tilde{d}, \ldots$ sleptons $\tilde{e}, \tilde{\nu}_e, \ldots$	0
0		Higgs bosons axion	Higgsinos axinos	1/2

Note: Supersymmetric cold dark matter candidate particles are underlined.

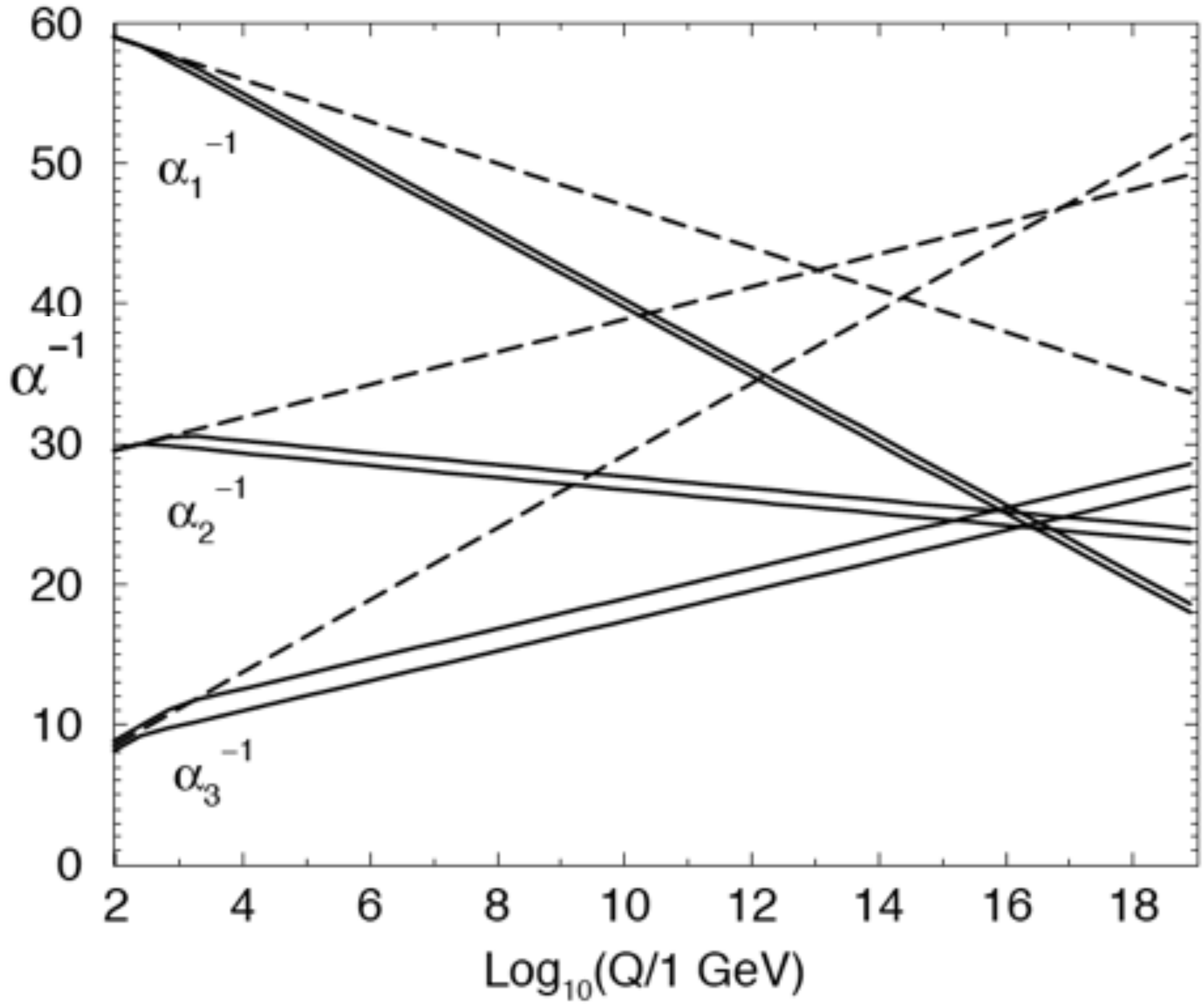
Supersymmetric WIMPs, continued

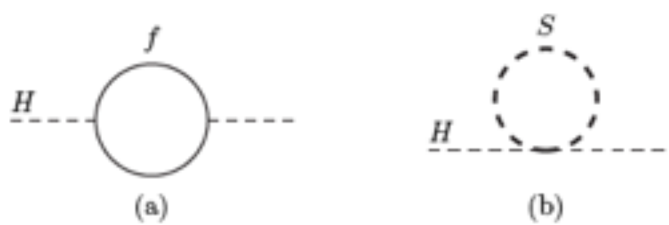
Spin is a fundamental property of elementary particles. Matter particles like electrons and quarks (protons and neutrons are each made up of three quarks) have spin ½, while force particles like photons, W,Z, and gluons have spin 1. The supersymmetric partners of electrons and quarks are called selectrons and squarks, and they have spin 0. The supersymmetric partners of the force particles are called the photino, Winos, Zino, and gluinos, and they have spin ½, so they might be matter particles. The lightest of these particles might be the photino. Whichever is lightest should be stable, so it is a natural candidate to be the dark matter WIMP. Supersymmetry does not predict its mass, but it must be more than 50 times as massive as the proton since it has not yet been produced at accelerators. But it will be produced soon at the LHC, if it exists and its mass is not above ~1 TeV!

SUPERSYMMETRY

The only experimental evidence for supersymmetry is that running of coupling constants in the Standard Model (dashed lines in firgure) does not lead to Grand Unification of the weak, electromagnetic, and strong interactions, while with supersymmetry the three couplings all do come together at a scale just above 10^{16} GeV. The figure assumes the Minimal Supersymmetric Standard Model (MSSM) with sparticle masses between 250 GeV and 1 TeV.

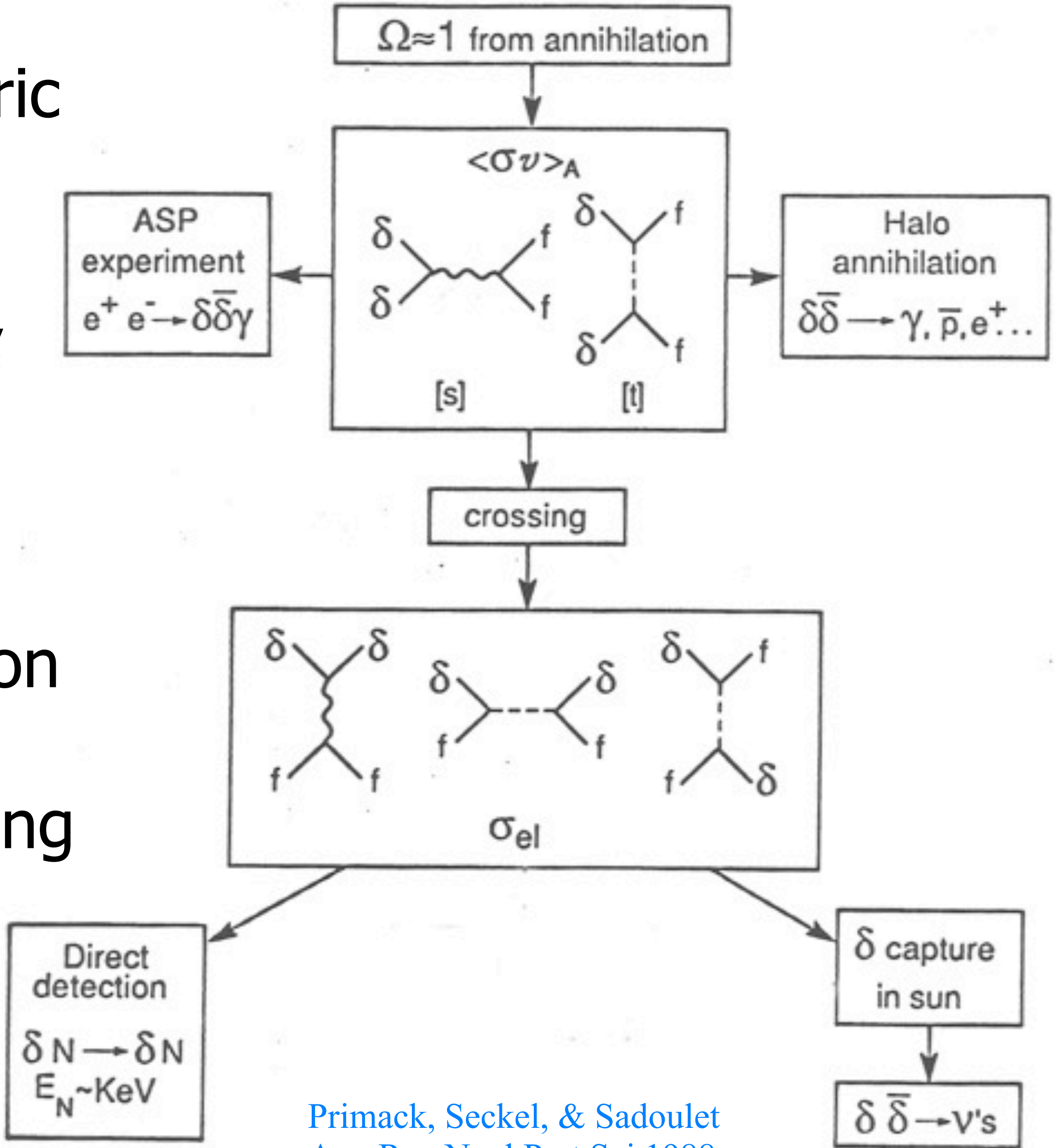
Other arguments for SUSY include: helps unification of gravity since it controls the vacuum energy and moderates loop divergences (fermion and boson loop divergences cancel), solves the hierarchy problem, and naturally leads to DM with $\Omega \sim 1$.





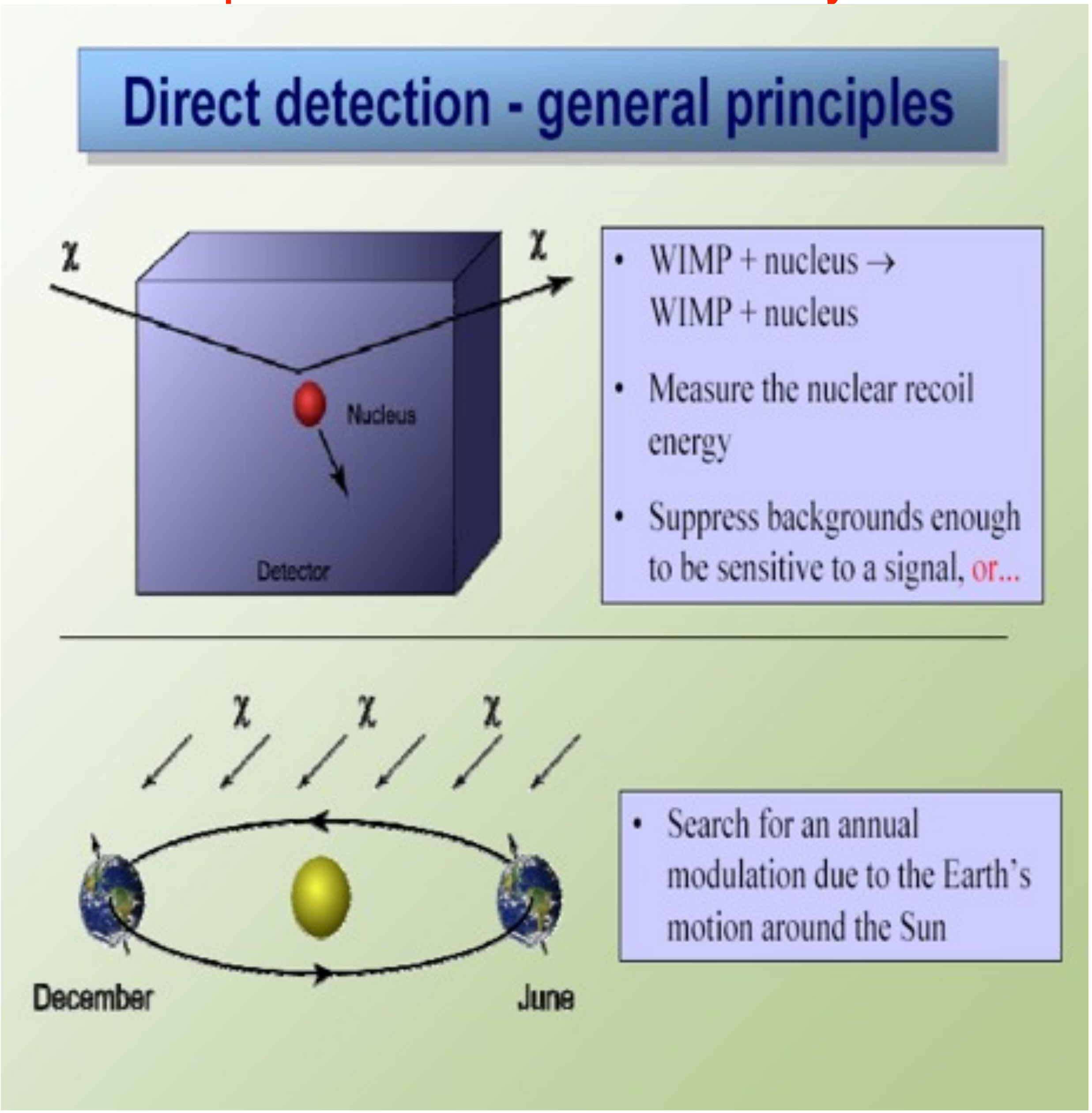
figs from S. P. Martin, A Supersymmetry Primer, arXiv:hep-ph/9709356v5

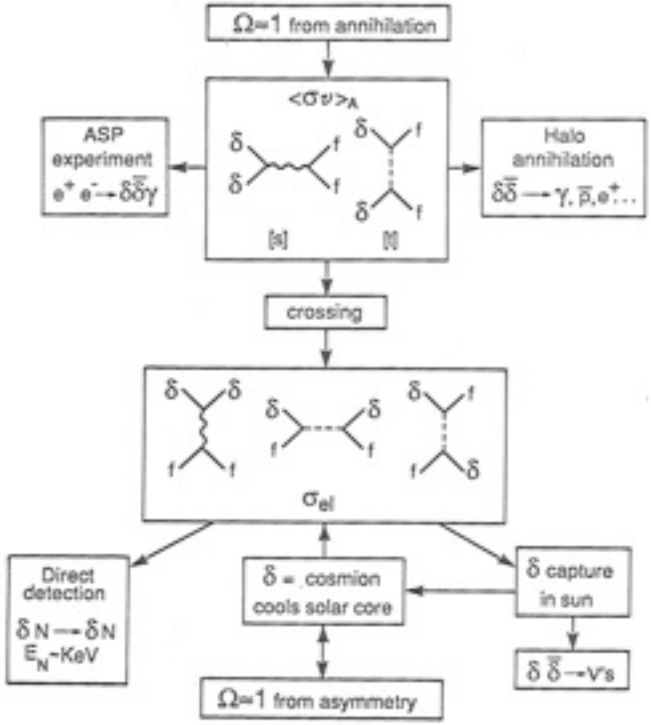
Supersymmetric WIMP (δ) annihilation is related by crossing to WIMP Direct Detection by Elastic Scattering

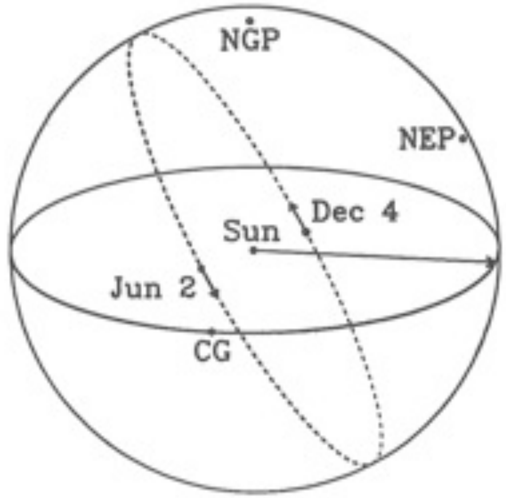


Primack, Seckel, & Sadoulet
Ann Rev Nucl Part Sci 1988

Experiments are Underway for Detection of WIMPs

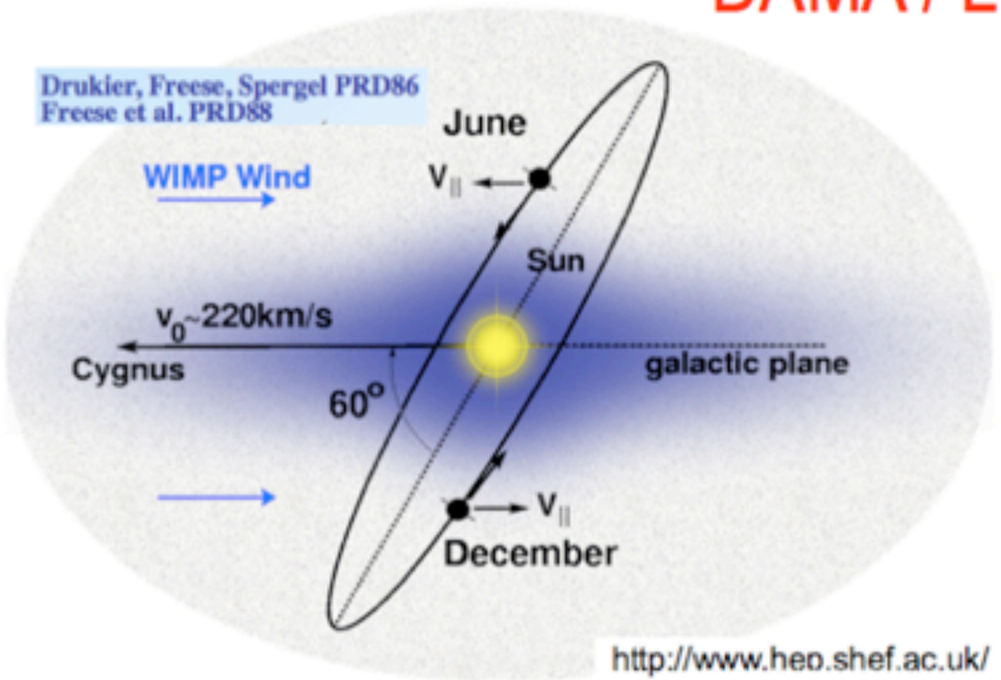




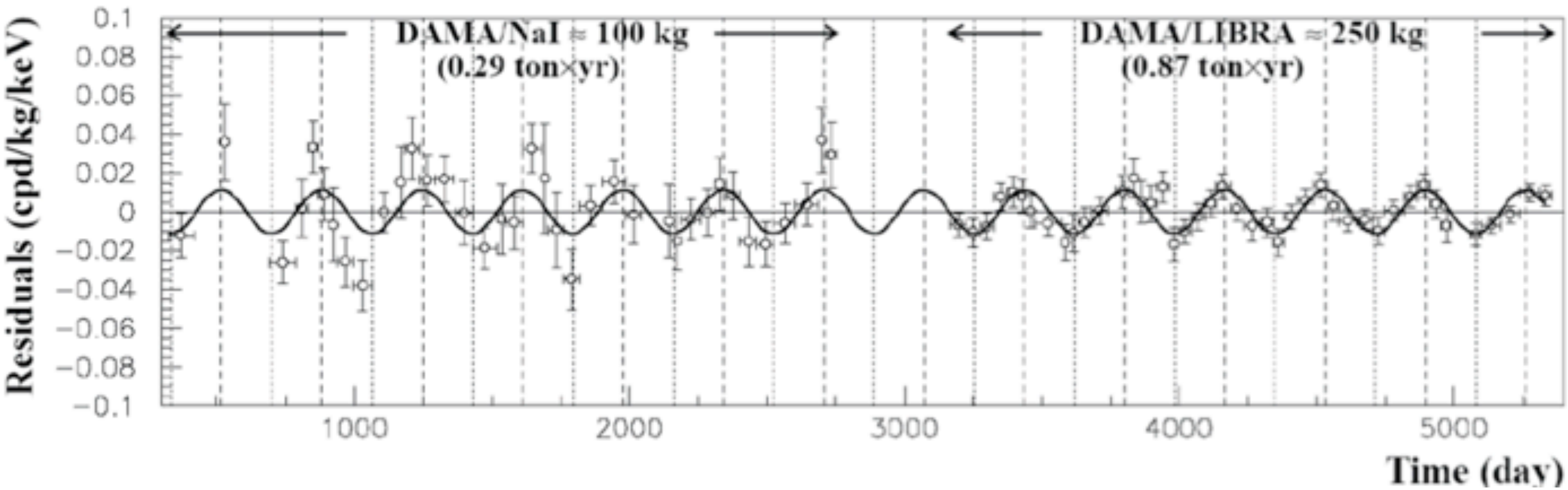


Primack, Seckel, & Sadoulet (1987)

DAMA / LIBRA

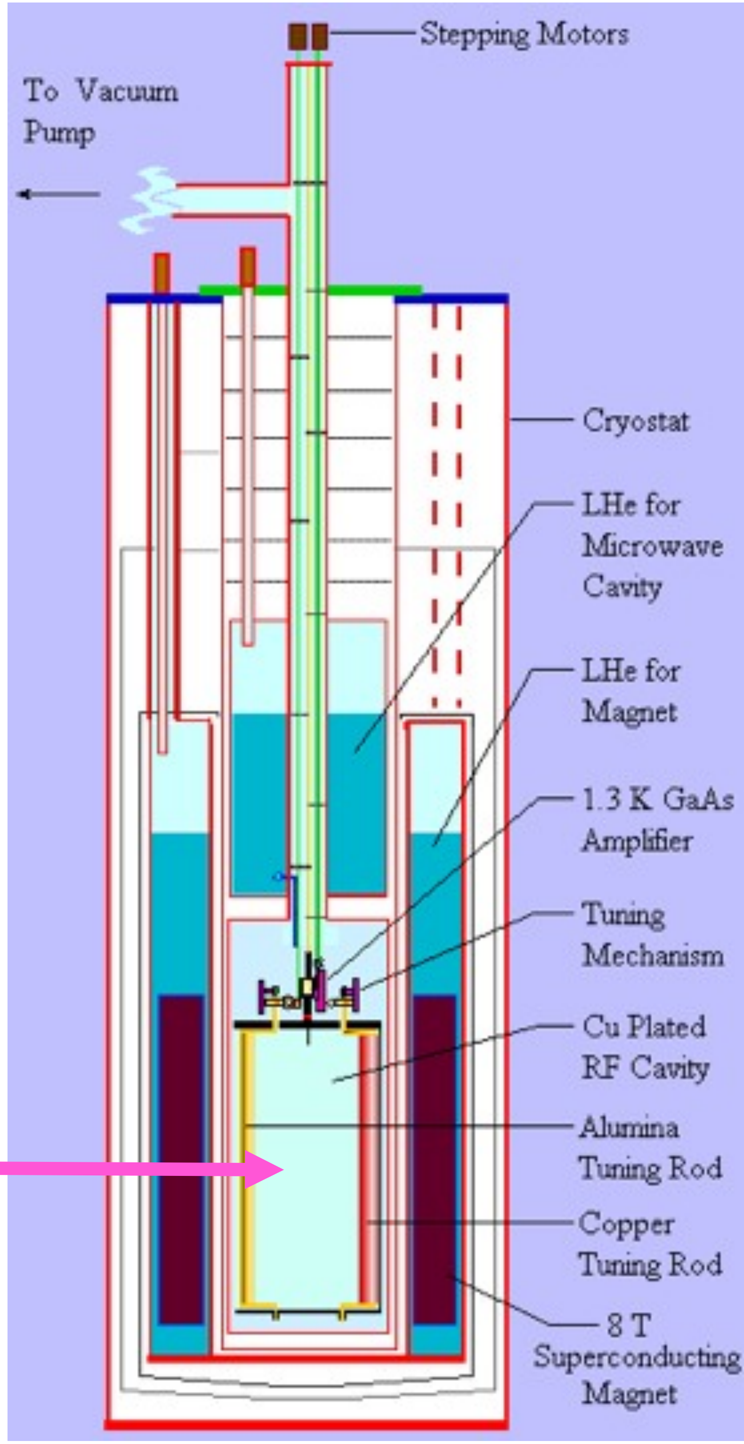


- Annual Modulation
  - Significance is 8.9σ
  - 1-2% effect in bin count rate
  - Appears in lowest energy bins
  - Can another experiment observe this effect?

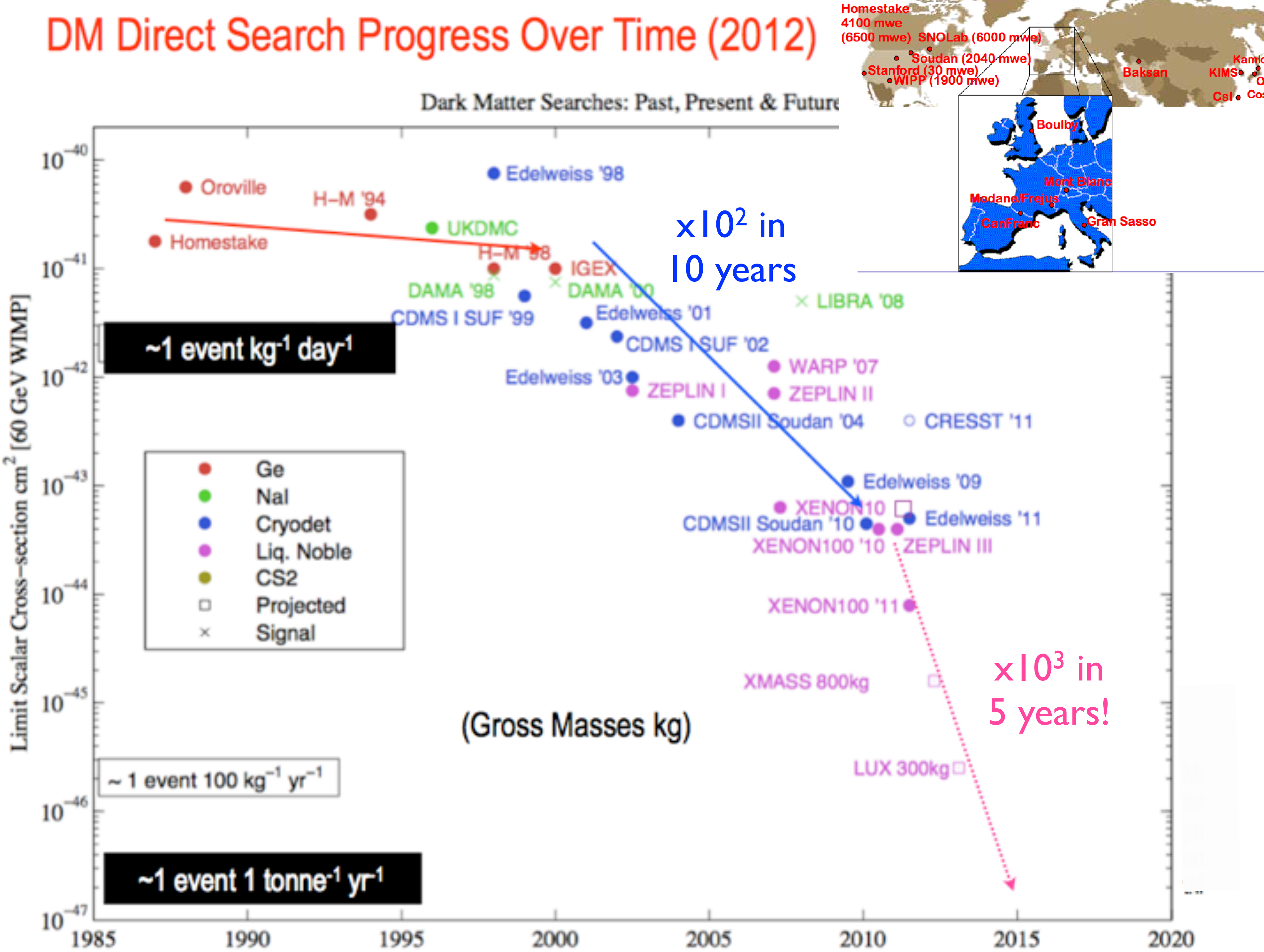


and also AXIONs

The diagram at right shows the layout of the axion search experiment now underway at the University of Washington. Axions would be detected as extra photons in the Microwave Cavity.

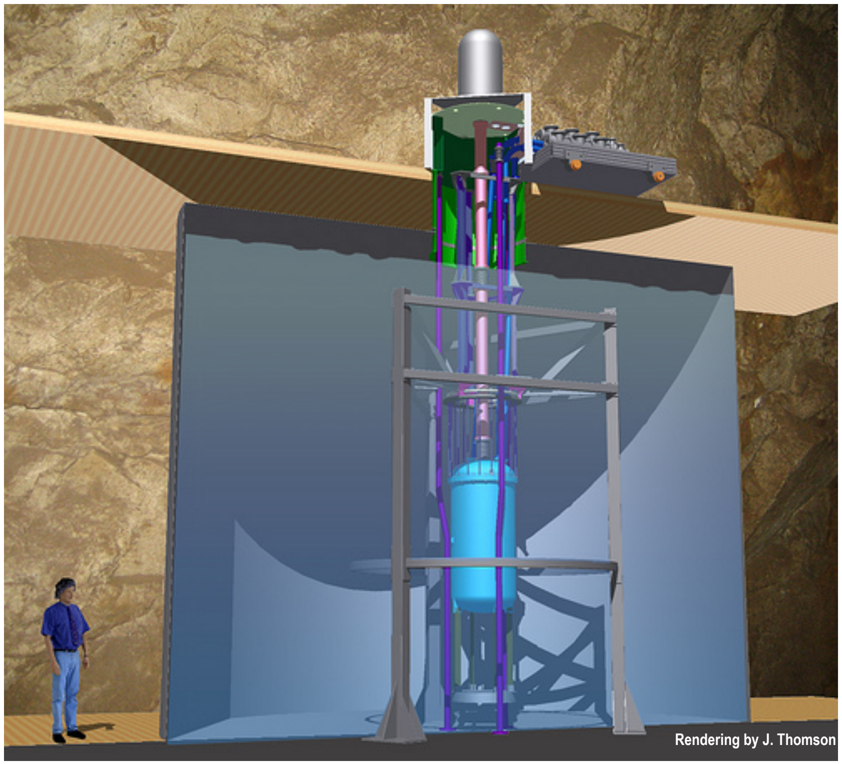


DM Direct Search Progress Over Time (2012)

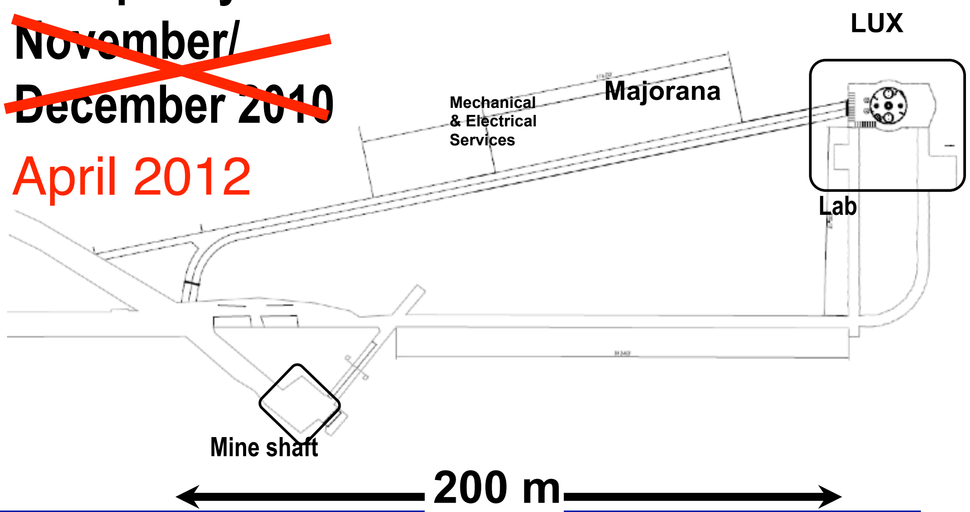


LUX in the Davis Laboratory at the Homestake Mine in South Dakota (4850L)

- Construction/excavation design completed
  - New 300’ access/safety tunnel being excavated
  - Shared with Majorana facility
- Two story, dedicated LUX 55’ x 30’ x 32’ facility being built now



- Beneficial occupancy: ~~November/ December 2010~~ April 2012



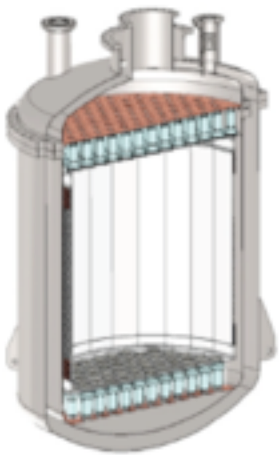
XENON1T: OVERVIEW

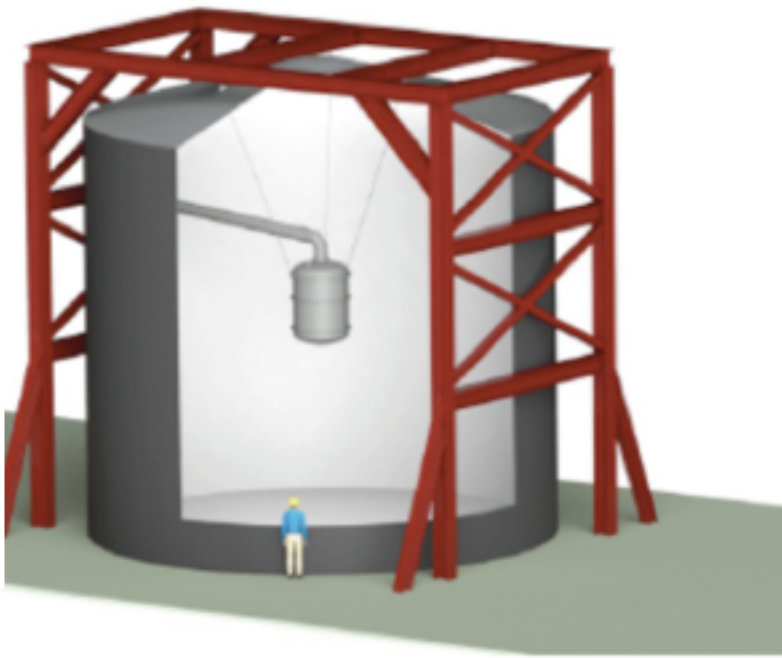
- Detector: 1m drift TPC with 2.2 ton LXe target
- Shield: ~10 m x 10 m Water Cherenkov Muon Veto
- Background: 0.01 mdru (100 lower than XENON100
- Location: approved by INFN for LNGS Hall B
- Capital Cost: ~11 M$ (50% US and 50% non-US)
- Status: Construction start in Fall 2012
- Science Run: projected to start in 2015
- Sensitivity: 2 x 10^{-47} cm^2 at 50 GeV with 2.2 ton-years

COLUMBIA RICE UCLA PURDUE ZURICH COIMBRA

LNGS INFN MPIK BOLOGNA SJTU

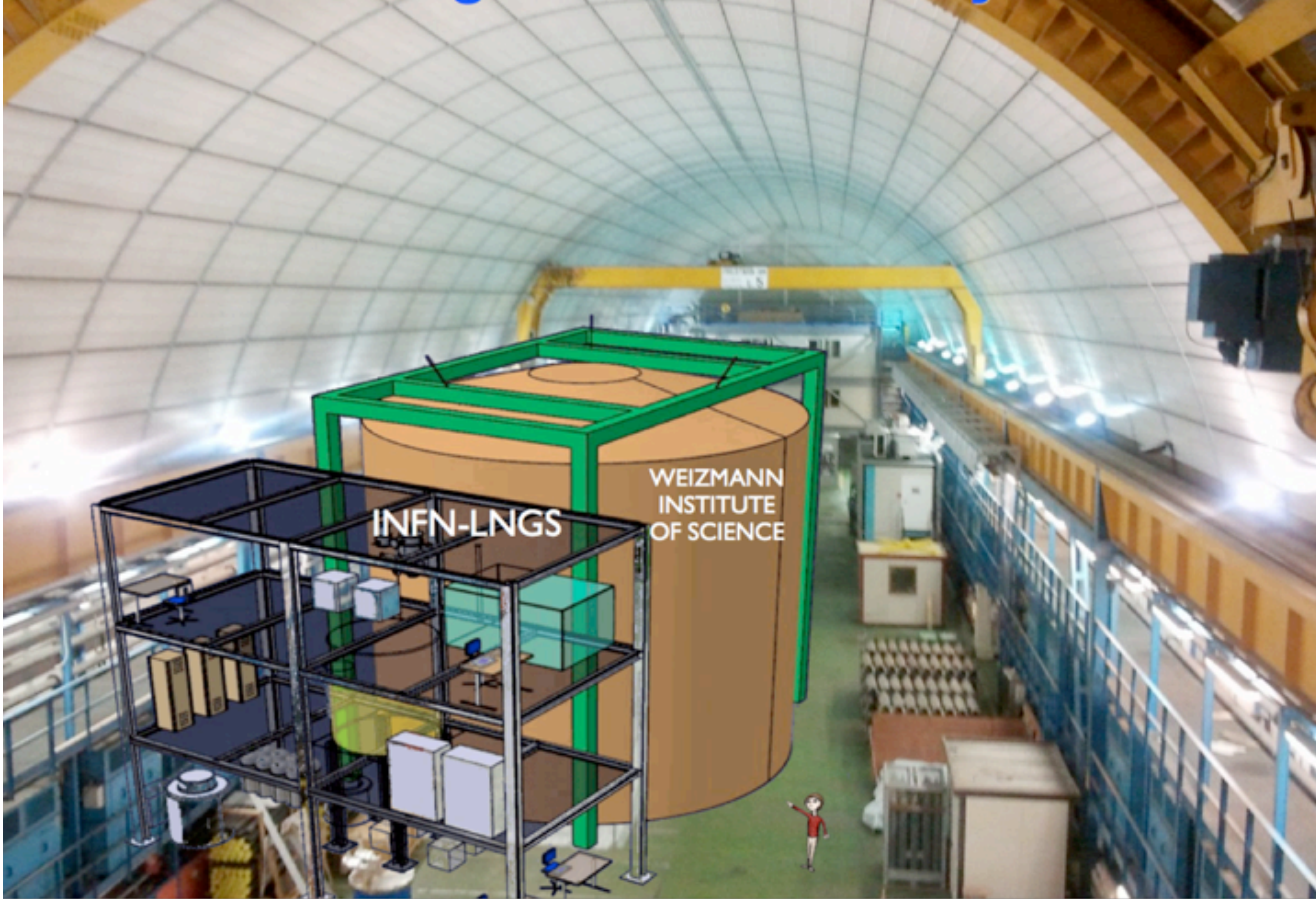
MUENSTER SUBATECH NIKHEF WEIZMANN





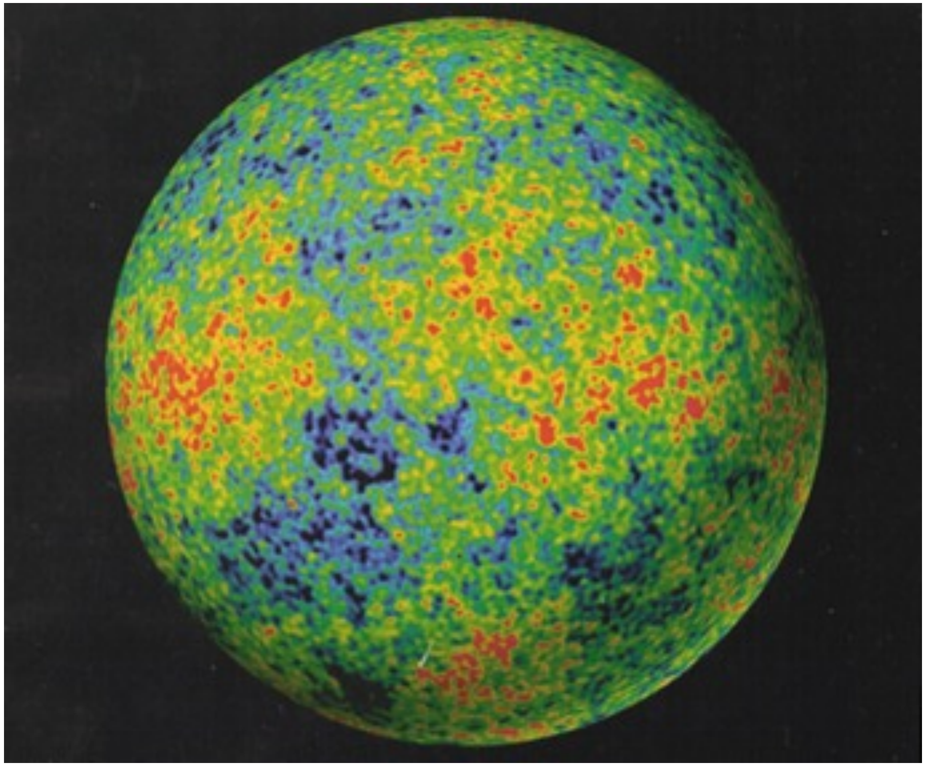
LNGS Underground Laboratory – Hall B

LNGS Underground Laboratory – Hall B



GRAVITY – The Ultimate Capitalist Principle

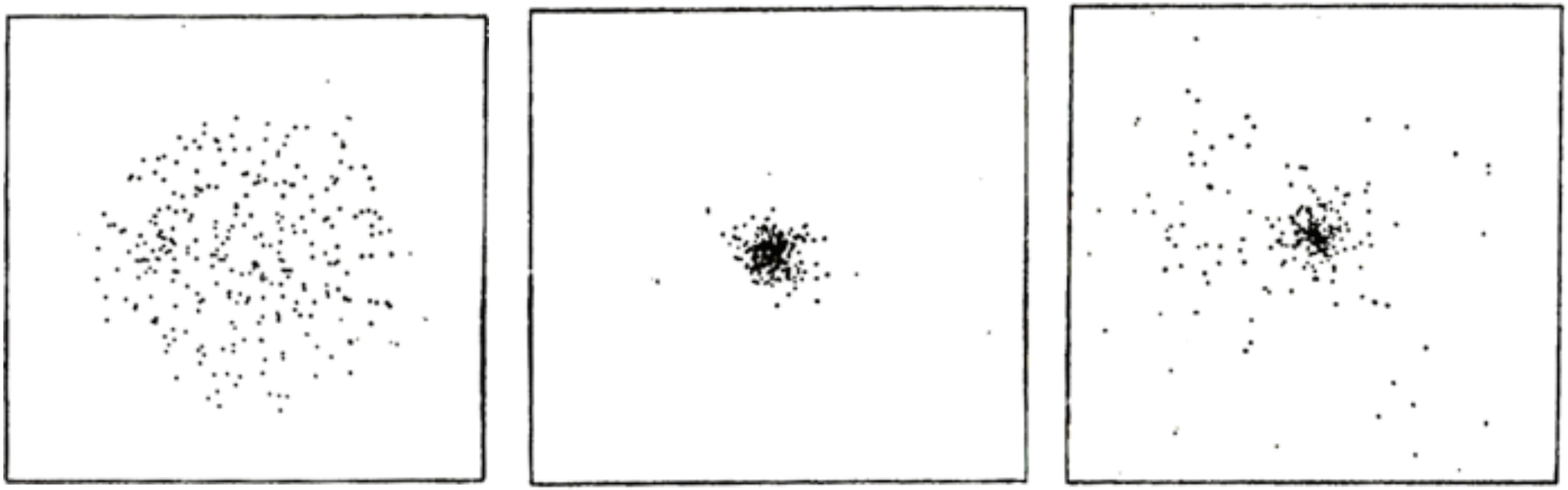
Astronomers say that a region of the universe with more matter is “richer.” Gravity magnifies differences—if one region is slightly denser than average, it will expand slightly more slowly and grow relatively denser than its surroundings, while regions with less than average density will become increasingly less dense. The rich always get richer, and the poor poorer.



Temperature map at 380,000 years after the Big Bang. **Blue** (cooler) regions are slightly denser. From NASA’s **WMAP** satellite, 2003.

The early universe expands ***almost*** perfectly uniformly. But there are small differences in density from place to place (about 30 parts per million). Because of gravity, denser regions expand more slowly, less dense regions more rapidly. Thus gravity amplifies the contrast between them, until…

Structure Formation by Gravitational Collapse



When any region becomes about twice as dense as typical regions its size, it reaches a maximum radius, ***stops expanding***,

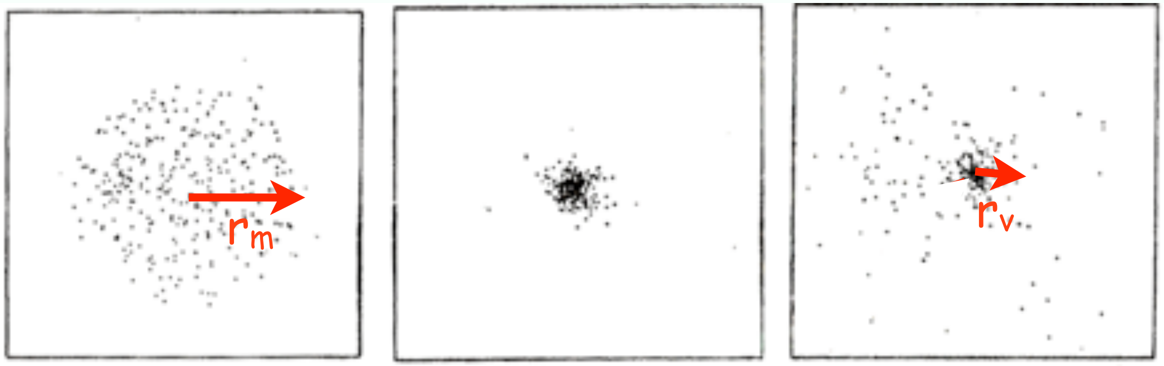
and starts falling together. The forces between the subregions generate velocities which ***prevent*** the material from ***all falling toward the center***.

Through Violent Relaxation the dark matter quickly reaches a ***stable configuration*** that's about half the maximum radius but denser in the center.

Simulation of top-hat collapse: P.J.E. Peebles 1970, ApJ, 75, 13.

Used in my 1984 summer school lectures "Dark matter, Galaxies, and Large Scale Structure," http://tinyurl.com/3bjknb3

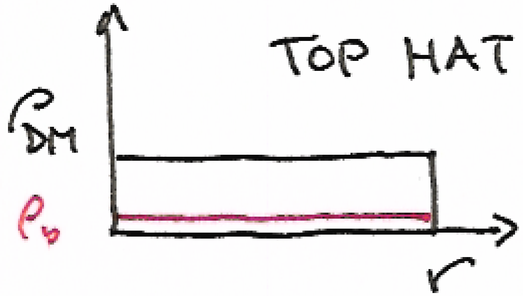
Structure Formation by Gravitational Collapse



TOP HAT
Max Expansion

VIOLENT RELAXATION

VIRIALIZED



Virial Theorem: $\langle K \rangle = -\frac{1}{2}\langle W \rangle$

$-W_m = \frac{C}{r_m}$, so after virialization

$$-\frac{C}{r_m} = E = W + K = \frac{1}{2}\langle W \rangle = -\frac{C}{2 r_v}$$

$$\Rightarrow r_v = \frac{1}{2} r_m, \quad \rho_v = 8\rho_m \approx 50\bar{\rho}(t_m)$$

$$\langle v^2 \rangle \approx \frac{GM}{r_v}$$

VIOLENT RELAXATION: Lynden-Bell 1967, Shu 1978

A survey of entropy in the Universe

*B.Basu and D.Lynden-Bell**

University of Calcutta, Department of Applied Mathematics, Calcutta 700 009, India; and
* Institute of Astronomy, The Observatories, Cambridge CB3 0HA

(Received 1989 November 17)

SUMMARY

Matter emerged from the Big Bang with a large and uniform entropy per baryon. We survey the entropy distribution today and find a large spread, but all recognized objects have significantly smaller entropy per baryon than they had at the Big Bang. This is presumably compensated by high entropy per baryon in intergalactic regions of low baryon density. Most entropy lies in the cosmic microwave background and the corresponding neutrino background. Bekenstein–Hawking entropies per gram for black holes are very large, in stark contrast to the lowest specific entropies of all found in neutron stars.

Formation of structures (planets, stars, galaxies) *reduces* entropy compared with the entropy of the constituents coming out of the Big Bang. A gas cloud radiates energy as it decreases its radius on its way to becoming a star, and as a star it radiates more energy. The total entropy of the star plus radiation increases. But the SuperMassive Black Holes have much more entropy than other constituents of galaxies, including stellar-mass black holes, because $S_{BH} \propto M_{BH}^2$.

A LARGER ESTIMATE OF THE ENTROPY OF THE UNIVERSE

Chas A. Egan and Charles H. Lineweaver

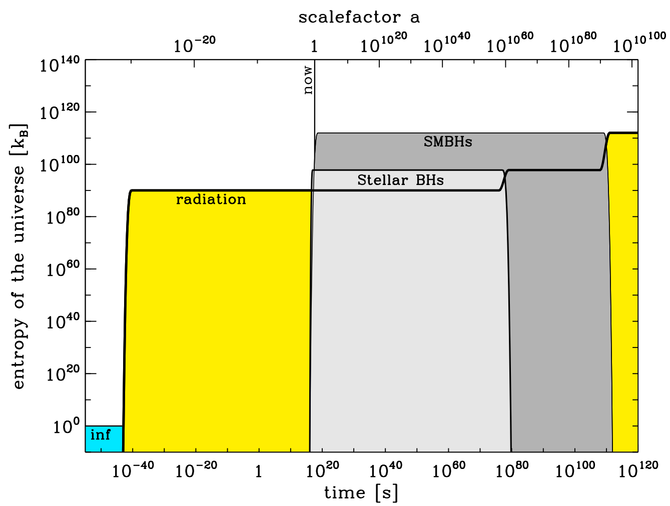
ABSTRACT

The Astrophysical Journal, 710:1825–1834, 2010

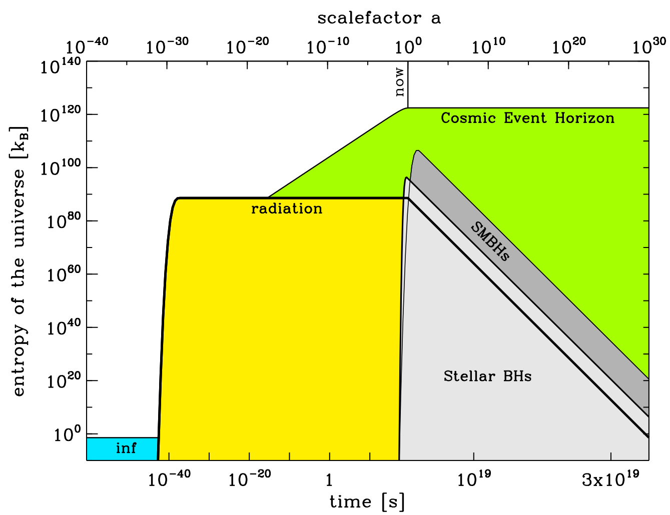
Using recent measurements of the supermassive black hole (SMBH) mass function, we find that SMBHs are the largest contributor to the entropy of the observable universe, contributing at least an order of magnitude more entropy than previously estimated. The total entropy of the observable universe is correspondingly higher, and is $S_{obs} = 3.1 \times 10^{104}$ k. We calculate the entropy of the current cosmic event horizon to be $S_{CEH} = 2.6 \times 10^{122}$ k, dwarfing the entropy of its interior, $S_{CEH\ int} = 1.2 \times 10^{103}$ k.

Component	Entropy S [k]
Cosmic Event Horizon	$2.6 \pm 0.3 \times 10^{122}$
SMBHs	$1.2^{+1.1}_{-0.7} \times 10^{103}$
$*$Stellar BHs ($42 - 140\ M_\odot$)	$1.2 \times 10^{98^{+0.8}_{-1.6}}$
Stellar BHs ($2.5 - 15\ M_\odot$)	$2.2 \times 10^{96^{+0.6}_{-1.2}}$
Photons	$2.03 \pm 0.15 \times 10^{88}$
Relic Neutrinos	$1.93 \pm 0.15 \times 10^{88}$
Dark Matter	$6 \times 10^{86 \pm 1}$
Relic Gravitons	$2.3 \times 10^{86^{+0.2}_{-3.1}}$
ISM & IGM	$2.7 \pm 2.1 \times 10^{80}$
Stars	$3.5 \pm 1.7 \times 10^{78}$
Total	$\mathbf{2.6 \pm 0.3 \times 10^{122}}$

Entropy in a comoving volume (normalized to the present observable universe). N.B. $10^{10^{100}}$ =1googolplex.



Entropy of matter within the Cosmic Event Horizon CEH, and the entropy of the CEH.



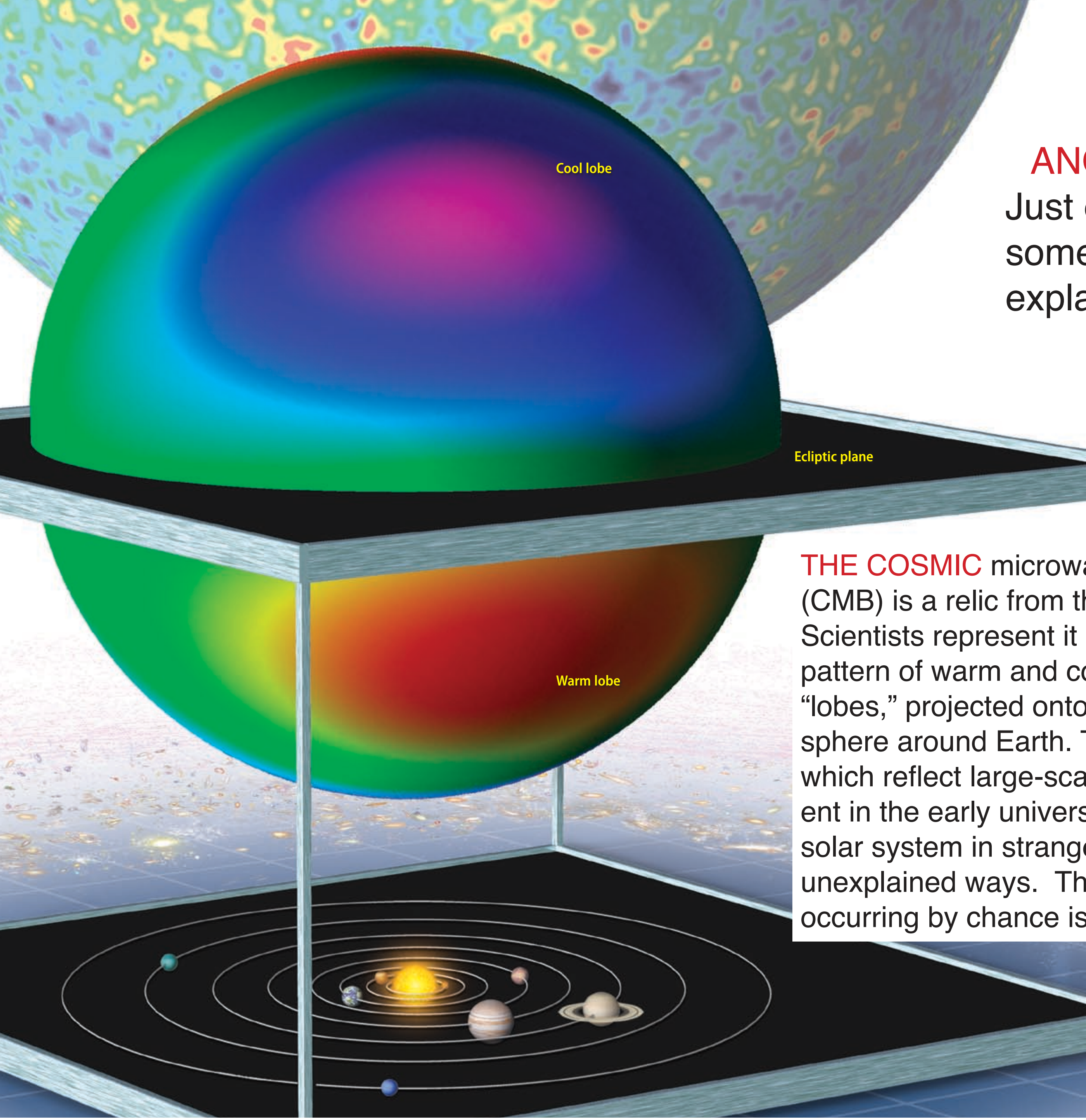
Dark Energy and the Entropy of the Observable Universe

Charles H. Lineweaver and Chas A. Egan 2010AIPC.1241..645L

$$S_{BH} = k_B A/(4\ell_P^2) = k_B \pi R^2/\ell_P^2 \propto M_{BH}^2$$

ANOMALIES

Just chance, or something to be explained?



THE COSMIC microwave background (CMB) is a relic from the early universe. Scientists represent it as a complex pattern of warm and cool spots, or “lobes,” projected onto the celestial sphere around Earth. The patterns, which reflect large-scale structures present in the early universe, line up with the solar system in strange and as-yet-unexplained ways. The likelihood of this occurring by chance is less than 0.1%.

D. Huterer
Astronomy
Dec 2007

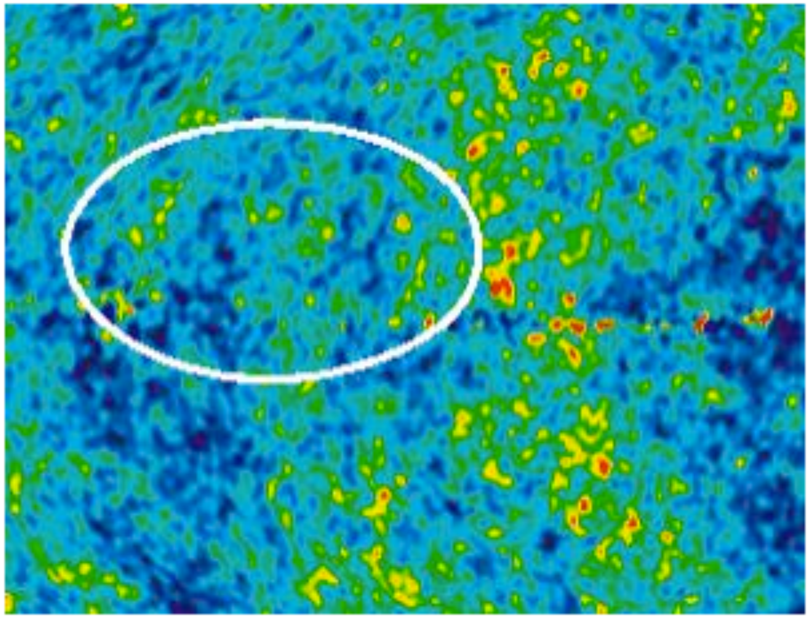
ANOMALIES

Just chance, or something to be explained?

NASA's Wilkinson Microwave Anisotropy Probe team, who have just released their most detailed map yet of the CMB, used Hawking's initials to draw attention to a serious point. With each new round of WMAP data – the latest is based on seven years of data – apparent anomalies called "anisotropies" in the CMB have puzzled physicists. Such patterns have also been used to justify various exotic theories.

One notorious anomaly is the "axis of evil", an apparent alignment in the hot and cold regions where there should be randomness. Another is the "cold spot", a particularly large void in the CMB, which some have proposed is evidence of another universe nestling next to our own.

The WMAP team point out that if something as apparently unlikely as Hawking's initials can be found in the CMB data, then the chances of finding other apparently improbable patterns may also be quite high.



Stephen Hawkings leaves his mark (Image: NASA/WMAP Science Team)