Spin Precession and Avalanches

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In many magnetic materials, spin dynamics at short times are dominated by precessional motion as damping is relatively small. We describe how avalanches evolve under these conditions. The growth front is spread out over a large region and consists of rapidly fluctuating spins often above the ferromagnetic transition temperature. In the limit of no damping the system will transition to an ergodic state if the initial instability is large enough, but otherwise can die out. This dynamic nucleation phenomenon is analyzed theoretically and the implications for real materials are discussed.

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Ferromagnetic systems that are subject to slowly changing external magnetic fields very commonly show avalanchelike responses [1]. This leads to hysteresis, as avalanches occur over a very fast time scale resulting in irreversibility and entropy production.

A large amount of experimental and theoretical work has been devoted to understanding aspects of this behavior, such as Barkhausen noise [1], which demonstrates that there is often a large degree of reproducibility in the mesoscopic dynamics on repeated cycling of the field and also interesting critical properties [2,3]. With advances in experimental techniques, direct tests of the reproducibility of magnetic memory have been undertaken recently which highlighted the prominent role of sample disorder [4].

The theoretical treatments often have relied on simplified models such as the Ising model to understand these complex systems. The dynamics of such models have been purely relaxational, the extreme limit of large damping, whereby a spin is flipped if the energy of the system is decreased by doing so, the excess energy being transferred out of spin degrees of freedom. These models have had a great deal of success in describing many features of disorder ferromagnets, showing fascinating properties, for instance ‘‘return point memory’’ (RPM) [2,3].

However, real magnets are typically dominated by precessional effects on short enough time scales. The Landau-Lifshitz-Gilbert (LLG) equation [5], contains a precessional term and a dissipative one

$$\frac{d\mathbf{s}}{dt} = -\mathbf{s} \times (\mathbf{B} - \gamma \mathbf{s} \times \mathbf{B}), \tag{1}$$

where $\mathbf{s}$ is a microscopic magnetic moment, $\mathbf{B}$ is the local effective field, and γ is a damping coefficient. γ measures the relative importance of damping to precession. It typically ranges from about 0.01 to 1 in real materials [6]. Therefore in many magnetic materials, there should be an interesting short time regime where it makes sense to regard the damping as a perturbation. In fact, it is well known from micromagnetic simulations of the LLG equations that there is a short time scale in which the magnetic response to the applied field change is strongly influenced by the finite level of damping in real materials [7] and this has recently played an important role in the understanding of spin-torque experiments [8–10]. Furthermore, even macroscopic and long-time properties, such as hysteresis loops, are influenced by the level of damping and other materials properties, as we shall explain in more detail below.

Throughout this Letter, we only consider the case of zero thermal noise. This is because we will see that *effective* finite temperature behavior is found even in this case and we want to carefully separate out these two effects.

If we adiabatically lower the external field, taken to be in the z direction, at some point the system will go unstable and have an avalanche. This will involve the nonlinear and possibly chaotic motion of its spins that interact through ferromagnetic and dipolar interactions.

We will first analyze what happens during the avalanche when we set γ to zero. This will describe the dynamics of the system for short time scales. In this case, the dynamics conserve energy and are highly nonlinear. So in the absence of any additional conservation laws, this would appear to imply that the system's equilibrium behavior is ergodic and well described by the microcanonical ensemble (which is equivalent to a system at a finite temperature).

However, there is a reason why ergodicity may be broken, and to our knowledge, this is the first time a phenomenon of this kind has been pointed out. After the system initiates an avalanche, energy is transmitted into neighboring spins, and as usual, some of this will be in the form of spin waves. This will propagate energy away from the avalanche region which will decrease the temperature of the avalanched spins, implying that as time progresses, the avalanched region becomes cooler. So if neighboring spins are not recruited, the avalanche will be extinguished. In this sense, the spin waves act as a damping term even if there is no damping in the LLG equation. For long times, still assuming $\gamma = 0$, the energy will be distributed in all the degrees of freedom of the entire system, which means in the limit of infinite system size, the temperature of the system will have dropped back down to zero. One is then left with a system that has produced only a sub-system-size avalanche and has got trapped in another local minimum.

What we have found numerically is that for zero damping, an avalanche that is initially large enough will propagate through the whole system causing it to go into a state of statistical mechanical equilibrium, often at a high temperature, sometimes even above the ferromagnetic transition temperature. If the initial avalanche is small, the avalanche will usually die out instead, leading to only a finite number of spins changing the sign of s_z.

We now turn to two-dimensional numerical experiments to support these claims and study the case of finite damping. Most real experiments on avalanche dynamics have been effectively two dimensional [3]. Dipolar forces were not included as they complicate the analysis by adding an additional parameter. Their effects will be the subject of future work.

We consider a Hamiltonian that couples nearest neighbor spins on a two-dimensional square lattice and contains an anisotropy term where the orientation of the easy axis is randomized slightly about the z axis and there is disorder in the ferromagnetic coupling.

$$\mathcal{H} = -\sum_{\langle i,j\rangle} J_{ij}\mathbf{s}_i \cdot \mathbf{s}_j - \alpha\sum_i (\mathbf{s}_i \cdot \hat{\mathbf{n}}_i)^2 - B_{\text{ext}}\sum_i s_{i,z}, \quad (2)$$

where the J_{ij}'s are the ferromagnetic coupling constants drawn from a uniform distribution with nonzero positive mean. α is a measure of the anisotropy. We choose the $\hat{n}_i$ to be random but biased towards the z axis (out of plane). See Ref. [11] for details. This system is placed in an external field B_{ext}.

The system was started at high field and the field was lowered adiabatically by evolving the system at fixed B_{ext} until, to a high accuracy, there was no further change in spin variables. To obtain convergence, the damping was made finite, $\gamma = 1$. After this, the field was lowered again. An avalanche was defined to occur when the maximum s_z among all the spins changed by a finite amount $\Delta \equiv 1$. At that point, a successive approximation scheme was initiated to find the precise field at which the transition takes place, further evolution can then proceed using the same procedure. When an avalanche of desired size was detected, the system was restarted with the same external field and preavalanche configuration, but now with a different value of damping and the evolution of the system was recorded.

A common scenario is to find that the whole system will avalanche for sufficiently low damping, but will have a sub-system-size avalanche when the damping is above some critical threshold that depends on the precise configuration right before the avalanche, as well as the level of disorder. This is demonstrated for a 128×128 spin system in Fig. 1. At $\gamma = 0.8$, the whole system avalanches.

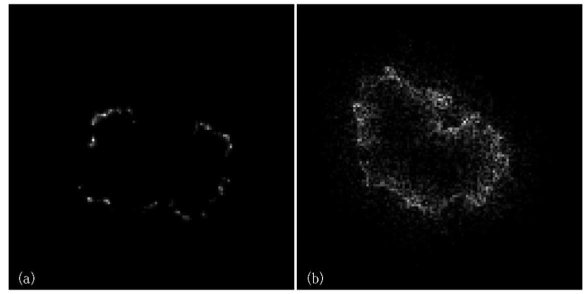

FIG. 1. Snapshots of systems with different damping γ starting with the same configuration, during an avalanche, taken at different times when the avalanches are roughly the same size. (a) For $\gamma = 0.8$ and (b) For $\gamma = 0.01$. The intensity represents the local spin motion.

Figure 1 shows gray scale images of the motion of the system for $\gamma = 0.8$ (a), and $\gamma = 0.01$ (b), during their avalanches. The intensity is proportional to $|d\mathbf{s}/dt|$. However, at $\gamma = 0.9$, the postavalanche cluster of avalanched spins has an approximate diameter of 8 lattice spacings (not shown). In all cases, the system was started in the same configuration right before the avalanche. The motion of the system is confined to the cluster's surface for $\gamma = 0.8$, but is spread out for $\gamma = 0.01$ in a ringlike structure, in which the magnetic system has an elevated effective temperature. In the case of no damping, many islands in front of the avalanche's main boundary form and this elevated temperature range encompasses the entire avalanche region. After completion of the system-size avalanche, the entire system continues to move indefinitely in an ergodic phase.

From these observations, one expects the hysteresis loop to change rather substantially as one varies the damping in these simulations. This has been verified directly.

The effective temperature of the avalanche in this model system is often higher than the transition temperature. It is high because the preavalanche metastable configuration has an energy much higher than the minimum $T = 0$ state. When the system gets out of its trapped static configuration it therefore has a lot of excess energy. By considering the Ising model with low disorder and estimating the critical external field for avalanches to take place, we find that the system can have an energy close to zero, which is consistent with our numerical results of a high post-avalanche temperature [12].

To understand how small damping effects the critical behavior of avalanches, we choose to study systems in three dimensions rather than two, as the latter case is still unclear even for relaxational dynamics [3]. Because of the computational expense of the LLG equations, we instead employed a microcanonical kinetic Ising model that keeps track of the energy loss, details of which will be published elsewhere [14]. Aside from the Ising spin variables, each site has a variable representing the excess energy available for it to flip. Energy was exchanged randomly with neighbors, mimicking energy diffusion, sometimes allowing neighboring spins to gain enough energy to flip. This excess energy was lost at a rate controlled by a damping parameter. We find that the critical properties of this microcanonical kinetic Ising model are indeed affected by damping. For example, the avalanche size distribution exponent, integrated over a hysteresis loop has an exponent that we measured to be 2.0 ± 0.1 for the case of large damping in agreement with previous results [3]. This is expected to be universal for any finite damping and large enough avalanches. However, for low damping, we obtained an effective exponent of 1.4 ± 0.1, seen over more than two decades in 32^3 systems.

We next turn to the mechanism by which this ergodic region spreads. As mentioned above, the effective temperature of the ergodic phase is quite high, with large amplitude motion over a short time scale. The preavalanche configuration is static and when these two regions are connected together, there will be energy transfer between them. One would expect that over a large scale, Fourier's law should hold, so that the temperature in the ergodic region will heat up the metastable region and upon receiving this thermal energy, the metastable region will now have the opportunity to thermally hop into the stable phase.

We will now construct a simple one-dimensional model that attempts to capture the above physics. We use a variable ϕ_i to denote if site i is part of an ergodic region, $\phi_i = 1$, or metastable region, $\phi_i = 0$. There is a temperature field, which starts off being zero in the metastable region and a nonzero constant T_0 for the ergodic sites that seed the avalanche. This temperature corresponds to the energy released per spin when it becomes part of the ergodic region. The equation

$$\frac{\partial T}{\partial t} = D\nabla^2 T + T_0(1 + \eta_i)\frac{\partial \phi_i}{dt} - \nu T \tag{3}$$

describes thermal diffusion with diffusion coefficient D, but adds a source term when the region becomes ergodic. In this case, ∇^2 is a discretized second derivative, $\propto T_{i+1} - 2T_i + T_{i-1}$. We have also included for the sake of generality, a last term, νT, which is related to the damping in the system. This gives a time scale for the temperature to die out. For example, in Fig. 1(a), large ν corresponds to high temperature only on the surface of the cluster, whereas for low ν in Fig. 1(b), it persists over a fairly thick surface layer. However the case of $\nu = 0$ (i.e., no damping) will be the focus of study below. Finally, we added a random component η_i to T_0 to study the effects on disorder on this system.

The probability per unit time that ϕ_i will go from 0 to 1 is $r(t)$, which we can take, for example, to be of the Arrhenius form $r_A = \nu_0 \exp(-1/T_i)$, where T_i is the (dimensionless) temperature on site i.

Together these define a simple model for the disappearance of the metastable region. We are now in a position to analyze under what circumstances an avalanche propagates and when it dies out. We find that for sufficiently large T_0 and initial width w_0, of the ergodic seeded region, the avalanche propagates indefinitely, but dies out if these two quantities are too small. We have studied this numerically for the case of Arrhenius activation $r = r_A$ mentioned above. The results are shown in Fig. 2 ("+" symbols are for $\eta_i = 0$). The + symbols are the 50% probabilities of an avalanche propagating indefinitely; below the line they will die out.

As the avalanche propagates, the temperature at the surface will be T_0 implying that the temperature in the interior will be almost constant with the same value. As the temperature diffuses to sites with $\phi = 0$, there is a finite probability of spins crossing to $\phi = 1$. To estimate the boundary as a function of T_0 and width w, we assume that for small T that $r(T)$ is a rapidly increasing function of T. If the initial ergodic region is a top hat function at temperature T_0, then for large w, there is a long time when the temperature field next to the boundary will approach $T_0/2$. This time τ will scale as w^2. Therefore, the probability p that a site next to the boundary will change to $\phi = 1$ should equal $r(T_0/2)w^2$ [15] for $p \ll 1$. If it does not succeed in crossing during this time τ, the avalanche will die out, otherwise it will continue to propagate. As the avalanche spreads, w increases meaning that p increases so that it is easier to seed new sites. Therefore, we expect the requirement for an avalanche to propagate is that

$$w_0 = \frac{c}{r^{1/2}(T_0/2)}, \tag{4}$$

where c is a constant. The solid line is a best fit in Fig. 2 for the constant c in $w_0 = c\exp(1/T_0)$. Given the simplicity of the estimate, the agreement is excellent.

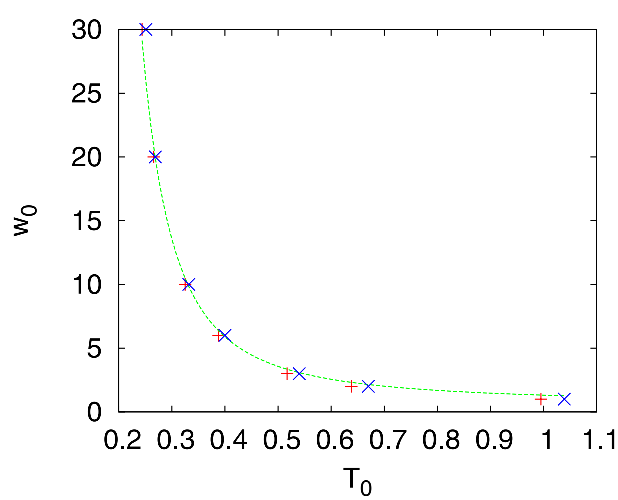

FIG. 2 (color online). The boundary between avalanche propagation and extinction for the one dimensional model discussed in the text $\nu = 0$. The + symbols are the numerical values and the solid line is an analytical fit to the data. The $\times$ symbols are for systems with quenched disorder with $-1 < \eta_i < 1$.

Local one-dimensional models of avalanches with randomness will often not show a transition to propagation because there is a finite probability that at some time it will encounter conditions causing extinction. The difference here is that the temperature field widens with increasing time. So if a site adjacent to the ergodic region fails to thermally hop, in the mean hopping time, the temperature field will take longer to die away with increasing w, and therefore the probability of extinction rapidly goes to zero as the width increases, again for $\nu = 0$. As a result, the model is also quite insensitive to the addition of disorder as shown by the $\times$ symbols in Fig. 2.

Therefore, for a *zero temperature* magnetic system with no spin damping, we expect that an initial disturbance will likely propagate if its size is above some threshold value, causing a transition between a static configuration and one at some effective *finite temperature*. Of course, if there is any coupling to degrees of freedom other than spin, so that $\gamma > 0$, the motion will die out and the spins will stop moving. In such a case, the avalanched spins will lose their energy and cool down to zero temperature. The system is locally being annealed at a finite rate. The finite damping case for Eq. (3), with $\nu > 0$ can also be analyzed. In this case it is quite similar to heat balance models for explosive crystallization [16], which show many interesting properties [17,18]. Instead of the temperature field at the front widening indefinitely, it should be of finite width, leading to a finite probability, per unit time, of the avalanche dying out. Thus one expects that in one dimension, propagation will always terminate eventually. In three dimensions, because the surface area of the front is increasing, we do expect to see infinite sized avalanches for sufficiently small disorder.

We now consider the issue of RPM. For the proof of it to be valid, a no passing rule must be satisfied [2,19]. In the case of strong precession, this rule can be violated, unlike the relaxational dynamics needed to give RPM, where this can never happen. On the other hand, over a large enough scale, it may be unlikely that a coarse grained variable will violate RPM but nevertheless it is possible for this to occur.

In conclusion, we have described how the evolution of avalanches is altered by the more realistic inclusion of precessional motion. For finite but small damping, the growth front becomes spread out over a large region, for which the spins inside can be described, for short times, by an ergodic system of rapidly fluctuating spins at high temperature. For longer times, interior spins slowly anneal to a low temperature state. We have found a new mechanism that describes the growth or termination of avalanches. An initial disturbance can terminate even when there is no damping due to the diffusion of energy away from a growth front, but if the initial disturbance is above a critical size, will likely continue to propagate indefinitely.

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