EM-110A Cheat-sheet (a lot of it might not be needed)

$$\int_V \vec{\nabla}\cdot\vec{g}\; d\tau = \oint_\Sigma \vec{g}\cdot\vec{da} \; ; \; \int_\Sigma (\vec{\nabla}\times\vec{g})\cdot\vec{da} = \oint_C \vec{g}\cdot\vec{dl}$$

$$\vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right) = -4\pi\delta(\vec{r}) \; ; \; \vec{\nabla}f(r) = \hat{r}\,\frac{df}{dr}$$

$$\vec{\nabla}\cdot(f\vec{v}) = f(\vec{\nabla}\cdot\vec{v}) + \vec{\nabla}f\cdot\vec{v} \; ; \; \vec{\nabla}\times(f\vec{v}) = f\,(\vec{\nabla}\times\vec{v}) + \vec{\nabla}f\times\vec{v}$$

Electrostatic fields:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, d\tau(\vec{r}') \; ; \; V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\, d\tau(\vec{r}')$$

Magnetostatic fields:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, d\tau(\vec{r}') \; ; \; \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}\, d\tau(\vec{r}')$$

Forces:

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) \; ; \; d\vec{F} = I\vec{dl}\times\vec{B}$$

Maxwell's equations:

$$\vec{\nabla}\cdot\vec{B} = 0 \; ; \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \; ; \; \left(\vec{\nabla}\cdot\vec{D} = \rho_{free}\right)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \; ; \; \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t} \; ; \; \left(\vec{\nabla}\times\vec{H} = \vec{J}_{free} + \frac{\partial\vec{D}}{\partial t}\right)$$

Constitutive relations:

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E} \; ; \; \vec{P} = \epsilon_0\chi_e\vec{E}$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu\vec{H} \; ; \; \vec{M} = \chi_m\vec{H}$$

Bound charges/currents:

$$\rho_b = -\vec{\nabla}\cdot\vec{P} \; ; \; \sigma_b = \vec{P}\cdot\hat{n} \; ; \; \vec{J}_b = \vec{\nabla}\times\vec{M} \; ; \; \vec{K}_b = \vec{M}\times\hat{n}$$

Discontinuities:

$$\Delta\vec{E}_\parallel = 0 \; ; \; \Delta\vec{B}_\parallel = \mu_0\vec{K}\times\hat{n} \; ; \; \Delta\vec{H}_\parallel = \vec{K}_{free}\times\hat{n}$$

$$\Delta E_\perp = \frac{\sigma}{\epsilon_0} \; ; \; \Delta D_\perp = \sigma_{free} \; ; \; \Delta B_\perp = 0 \; ; \; \Delta V = 0 \; ; \; \Delta\vec{A} = 0$$

Flux:

$$\Phi_{11} = L_1\, I_1 \; ; \; \Phi_{12} = M_{12}\, I_1 \; ; \; \text{emf} = -\frac{d\Phi}{dt}$$

Dipoles:

$$\vec{p} = \alpha\vec{E} \; ; \; \vec{p} = \sum_i q_i\vec{r}_i \; ; \; \vec{m} = \frac{1}{2}\oint \vec{r}\times I\vec{dl} = I\vec{A}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}\ ;\ \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\vec{p}\cdot\hat{r})\,\hat{r} - \vec{p}]\ ;\ \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}[3(\vec{m}\cdot\hat{r})\,\hat{r} - \vec{m}\,]$$

Forces on dipoles:

$$\vec{F}_e = (\vec{p}\cdot\vec{\nabla})\vec{E} = \vec{\nabla}(\vec{p}\cdot\vec{E})\ ;\ \vec{F}_m = \vec{\nabla}(\vec{m}\cdot\vec{B})$$

Expansions:

$$\text{Spherical}\,\phi\,\text{symmetric}:\ V(r,\theta) = \sum_{l=0}^{\infty} P_l(cos\,\theta)(A_l r^l + \frac{B_l}{r^{l+1}})$$

$$P_0(x) = 1\ ;\ P_1(x) = x\ ;\ P_2(x) = (3x^2-1)/2\ ;\ P_3(x) = (5x^3-3x)/2$$

$$\int_{-1}^{1} P_l(x)P_n(x)dx = \frac{2}{2l+1}\,\delta_{n,l}$$

$$\int_0^a sin(\frac{n\pi x}{a})\,sin(\frac{m\pi x}{a})\,dx = \frac{a}{2}\,\delta_{n,m}\ ;\ \int_0^a cos(\frac{n\pi x}{a})\,cos(\frac{m\pi x}{a})\,dx = \frac{a}{2}\,\delta_{n,m}\ \text{for}(n,m)\neq 0$$

Energy:

$$W_e = \frac{1}{2}CV^2 = \frac{1}{2}\int \rho V\,d\tau\ ;\ W_m = \frac{1}{2}LI^2 = \frac{1}{2}\int \vec{J}\cdot\vec{A}\,d\tau$$