EM-110A Midterm exam - Feb 19, 2008

Name:

Take a deep breath, read the questions carefully, ponder on them for 5 minutes, and then start writing. Writing down the solution takes only a few lines, so take your time thinking through the solution first. (numbers in parentheses represent the point worth of each question). You will probably not have enough time to answer all questions, so pick carefully the ones you are more comfortable with.
Good luck,
Keivan Esfarjani

1. Derive the following relations for an arbitrary *scalar* field f:

$$(3) \int_V \vec{\nabla} f \, d\tau = \oint_\Sigma f \vec{da}$$

$$(4) \int_\Sigma \vec{\nabla} f \times \vec{da} = \oint_C f \vec{dl}$$

2. (4) Calculate the electric field created by a straight infinite line of charge density λ.

3. (2) Consider a flat interface between a material of dielectric of constant ϵ and the vacuum. There are some free charges in the vacuum polarizing the plane. Let σ_b be the induced bound charge density on the surface. Draw a diagram, and state the continuity (or discontinuity) of the normal component of the electric field as well as that of the displacement vector $\vec{D}$ across the interface (use explicit notations).

   (2) Using $\vec{D} = \epsilon \vec{E}$, express each of the normal components of the electric field above and below the surface as a function of σ_b.

   (4) In what limit (on ϵ) do we recover a metal from the given dielectric? Take this limit and check whether you get the expected results. What is the expected result for the fields by the way?

   (6) In class I said if you use Gauss's law you find that the magnitude of the electric field is $\sigma_b/2\epsilon_0$ above and below. Can you resolve the apparent paradox here? (In what context is the above statement correct?)

4. (12) Consider a **metallic** sphere of radius a of charge Q surrounded by a dielectric shell of thickness d and dielectric constant ϵ. Given that the electrostatic potential V is zero at infinity, find and plot its value as a function of r, the distance from the center of the sphere. (Hint: to find V, you have to start with D(4), then get E(3), deduce V(3) by integration, and finally plot $V, E(2)$ versus distance)

5. (5) Consider a sphere of radius a with a **surface** charge density $\sigma(r = a, \theta, \phi) = \sigma_0 \, cos(\theta)$. Calculate the multipole moments of this charge distribution. Find the exact potential this distribution creates outside the sphere. Find the potential inside the sphere. (If you can guess the form of the solution and just prove it is the correct solution, it is acceptable too).

6. (6) A dielectric is inserted inside a charged capacitor. Describe **qualitatively** what is the difference in terms of:
   -the force on the dielectric,
   -the work done by the operator to insert the dielectric,
   -the energy of the capacitor
   between the following two situations: a) the capacitor has a constant charge and is unplugged from the battery ; b) the capacitor is connected to a battery of constant voltage V.

7. (4) Consider a square of side a with point charges $+q, -q, +2q, -2q$ on its respective four corners in a clockwise order. What is a good approximation to the potential at a point situated far away from its center ($r >> a$). Draw a picture to clarify your notation.

Cheat sheet

$$\int_V \vec{\nabla} \cdot \vec{g}\; d\tau = \oint_\Sigma \vec{g} \cdot \vec{da}$$

$$\int_\Sigma (\vec{\nabla} \times \vec{g}) \cdot \vec{da} = \oint_C \vec{g} \cdot \vec{dl}$$

$$\vec{\nabla} \cdot (f\vec{v}) = f(\vec{\nabla} \cdot \vec{v}) + \vec{\nabla} f \cdot \vec{v}$$

$$\vec{\nabla} \times (f\vec{v}) = f\,(\vec{\nabla} \times \vec{v}) + \vec{\nabla} f \times \vec{v}$$

$$\Delta E_\perp = \frac{\sigma}{\epsilon_0};\; \Delta D_\perp = \sigma_f$$

$$V(r,\theta) = \sum_{l=0}^{\infty} P_l(cos\,\theta)(A_l r^l + \frac{B_l}{r^{l+1}})$$

$$P_0(x) = 1;\; P_1(x) = x;\; P_2(x) = (3x^2 - 1)/2;\; P_3(x) = (5x^3 - 3x)/2$$

$$\int_{-1}^{1} P_l(x) P_n(x) dx = \frac{2}{2l+1}\,\delta_{n,l}$$

$$\nabla_r^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d}{dr}\right) \text{ (in spherical coordinates)}$$

$$\nabla^2 P_l(cos\,\theta) = -\frac{l(l+1)}{r^2} P_l(cos\,\theta)$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon \vec{E};\; \vec{P} = \epsilon_0 \chi \vec{E}$$

$$\vec{\nabla} \cdot \left(\frac{\vec{r}}{r^3}\right) = -4\pi\delta(\vec{r})$$