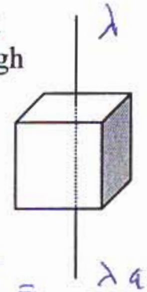
**E&M – 110 A: Quiz #2**

NAME:

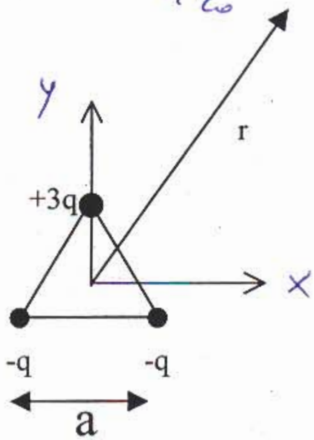
1) For the *infinite* wire of *uniform* charge density λ, going vertically through the *center* of the cube of length a, calculate the flux of the electric field through each of the 6 faces of the cube.




$$\Phi_{tot} = \oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} = \lambda a.$$

$$\Phi_{top} = \Phi_{bottom} = 0 \Rightarrow \Phi_{\text{each lateral side}} = \frac{\Phi_{tot}}{4} = \frac{\lambda a}{4\varepsilon_0}$$

2) For this configuration of charge placed on an equilateral triangle, calculate the first two multipole (what are they called?) moments, taking the origin of the coordinates system at the center of the triangle. What is a good approximation to the electrostatic potential created by this charge distribution at a far point r, using your multipole expansion?




Monopole: $Q = \sum_i q_i = 3q - q - q = q$

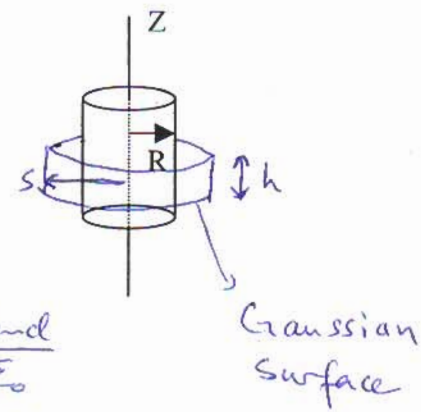
Dipole $\vec{p} = \sum_i q_i \vec{r}_i$

$$= 3q\,\hat{y} \times \frac{a}{\sqrt{3}} - q\frac{a}{\sqrt{3}}\left(\hat{x}\frac{\sqrt{3}}{2} - \hat{y}\,\frac{1}{2}\right) - q\frac{a}{\sqrt{3}}\left(-\hat{x}\frac{\sqrt{3}}{2} - \hat{y}\,\frac{1}{2}\right)$$

$$= \frac{4qa}{\sqrt{3}}\,\hat{y}$$

$V \simeq \frac{Q}{4\pi\varepsilon_0}\frac{1}{r} + \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3} + \ldots$ with $Q, \vec{p}$ given above

3) Consider an *infinitely long* cylinder of radius R with *uniform* charge density ρ. Calculate the electric field inside and outside the cylinder. Plot on the same figure the variations of the charge density and the electric field versus distance from the axis s.




Use Gauss's law $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\varepsilon_0}$

for a cylinder of height h, radius s.

$\oint \vec{E} \cdot d\vec{a} = h \times 2\pi s \, E(s)$ since $\vec{E}(\vec{r}) = E(s)\,\hat{s}$ (radial)

$$\frac{Q_{encl}}{\varepsilon_0} = \begin{cases} \rho \times \frac{h\pi R^2}{\varepsilon_0} & \text{for } s > R \\ \rho \times \frac{h\pi s^2}{\varepsilon_0} & \text{for } s < R \end{cases}$$

$$\Rightarrow \quad 2\pi s h \, E(s) = \frac{\rho h \pi}{\varepsilon_0} \begin{cases} R^2 & \text{if } s > R \\ s^2 & \text{if } s < R \end{cases}$$

$$E(s) = \frac{\rho}{2\varepsilon_0} \begin{cases} R^2/s & \text{if } s > R \\ s & \text{if } s < R \end{cases}$$