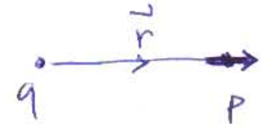
EM-110A Quiz #3 - Feb 26, 2008



Name:

1. A point charge q is situated a large distance r from a neutral atom of polarizability α. Find the force of attraction between them.

Field created by q : $\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}$ induces a dipole $\vec{p} = \alpha\vec{E}$ (radial)

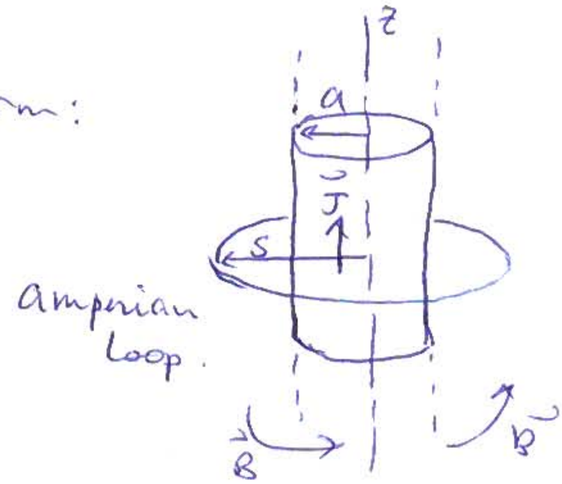
This dipole created field $\vec{E}_2 = -\vec{\nabla}V = -\vec{\nabla}\left(\frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}\right) = \frac{p}{4\pi\varepsilon_0} \times \frac{-2}{r^3}\hat{r}$ $(\vec{p}\cdot\vec{r} = pr)$

which gives d force qE_2 on the charge $F = qE_2 = \underset{\text{attractive}}{(-)}\, 2\alpha\left(\frac{q}{4\pi\varepsilon_0}\right)^2\frac{1}{r^5}$

2. A steady current flows down a long cylindrical wire of radius a and of axis oriented along the z axis. Assume the current density $\vec{J} = J\hat{z}$ is uniform within the cylinder. Using Ampere's law, calculate the magnetic field inside and outside the cylinder.

Ampère law in integral form:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}.$$



Since $\vec{B}$ lines make loops around the z axis $\vec{B}(\vec{r}) = B(s)\,\hat{\varphi}$

$$\oint \vec{B}\cdot d\vec{l} = \int_0^{2\pi} B(s)\,\hat{\varphi}\cdot s\,d\varphi\,\hat{\varphi} \qquad (d\vec{l} = s\,d\varphi\,\hat{\varphi})$$

$$= 2\pi s\, B(s)$$

$$I_{encl} = J \times \begin{cases} \pi a^2 & \text{for } s > a \\ \pi s^2 & \text{for } s < a \end{cases}$$

$$\Rightarrow B(s) = \begin{cases} \frac{\mu_0 J a^2}{2s} & \text{for } s > a. \\ \frac{\mu_0 J s}{2} & \text{for } s < a. \end{cases}$$

3. Consider a charged capacitor with surface charge density σ and $-\sigma$ on its two plates. The inter-plate separation is $2a$ and there are two dielectric materials of constants ϵ_1 and ϵ_2 each of thickness a between the two plates.

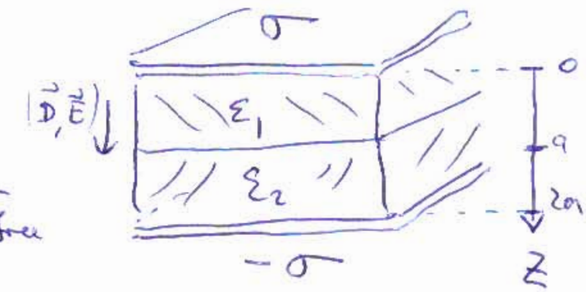
   a) Find the displacement vector $\vec{D}$ in each slab.

   b) Deduce the electric field in each slab.

   c) Find the polarization vector $\vec{P}$ in each slab.

   d) Plot the electrostatic potential drop as a function of distance, assuming $\epsilon_1 = 2\epsilon_0$ and $\epsilon_2 = 3\epsilon_0$.

Since free charge σ is only on capacitor plates we start with D : $\vec{\nabla}\cdot\vec{D} = \sigma_{free}$



$\oint \vec{D}\cdot d\vec{a} = Q_f$ gives $\vec{D} = \sigma\hat{z}$ between the two plates

$D = 0$ outside the capacitor.

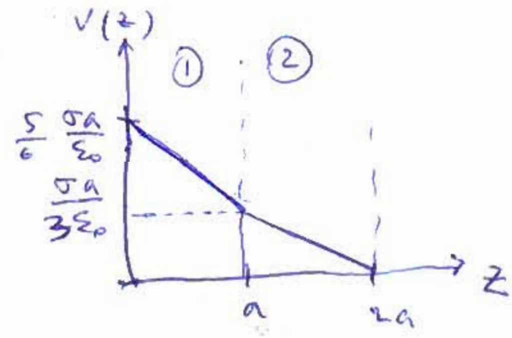
$$D = \varepsilon E \Rightarrow \begin{cases} \vec{E}_1 = \dfrac{\sigma}{\varepsilon_1}\hat{z} \\ \vec{E}_2 = \dfrac{\sigma}{\varepsilon_2}\hat{z} \end{cases}$$

$$\vec{P} = \vec{D} - \varepsilon_0\vec{E} = (\varepsilon - \varepsilon_0)\vec{E} = \begin{cases} \dfrac{\varepsilon_1 - \varepsilon_0}{\varepsilon_1}\sigma\,\hat{z} \\ \dfrac{\varepsilon_2 - \varepsilon_0}{\varepsilon_2}\sigma\,\hat{z} \end{cases}$$

If $\varepsilon_1 = 2\varepsilon_0$; $\varepsilon_2 = 3\varepsilon_0$

$$V_1(z) = -\frac{\sigma}{2\varepsilon_0} z + A$$

$$V_2(z) = -\frac{\sigma}{3\varepsilon_0} z + B$$



Constants A & B are found from continuity at $z = a$, and one of the boundaries (say $V(z=2a) = 0$)

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J};\ \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0};\ \vec{\nabla}\cdot\vec{D} = \rho_{\text{free}}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E};\ \vec{P} = \epsilon_0\chi\vec{E};\ \vec{p} = \alpha\vec{E}$$