EM-110A Sample test

Below are a series of "hardest" typical questions you could get in your final exam. Other questions that I might ask, but are too trivial, will not be listed here. We will try to cover all the lectured material. For each of the problems draw a clear and if necessary many clear pictures, make sure you understand the questions asked, and then try to figure out the method to solve them.

1. Consider two cocentric metallic shells of radii R_1 and R_2 ($R_1 < R_2$) of charge Q_1 and Q_2 respectively. A material of dielectric constant ϵ is placed between the sheets. Calculates all the fields ($\vec{D}, \vec{E}, \vec{P}, V$ (in this order) at all points in space, and plot $V(r)$ as a function of r, the distance from the center of the sphere, assuming $Q_1 = Q$; $Q_2 = -2Q$; $R_2 = 2R_1$. Also calculate all bound volume and surface charges, and show them on your diagram.

2. For the following distribution of surface charge on the surface of a sphere of radius R, $\sigma(\theta, \phi) = \sigma_0 \, cos\, 2\theta$, calculate the electrostatic potential inside and outside the sphere.

3. For a cubic box of length a, the potential on 5 of its facets is zero and on the sixth facet the potential is $V(x, y, z = a) = cos^2 \frac{\pi(x+y)}{a}$. Calculate the electrostatic potential inside the cube.

4. Calculate the magnetic field created by a straight portion of current I of length L at an arbitrary point in space. (Take one end of the wire at the origin, and the other end on the Z axis of coordinates $(0, 0, L)$. The field point would be $\vec{r} = (x, y, z)$.

   Using this result, calculate the force between two such wires placed at a distance D from each other, the currents cwrunning parallel to each other.

   If the top end of the wires is hanging from a fixed point, calculate the final angle of each of the wires with the vertical direction, assuming their mass is M and the magnetic force is much smaller than the gravitational force.

5. A *uniform* current density $\vec{J} = J\hat{z}$ is flowing through an infinite cylinder of radius R_1. A magnetic material of susceptibility χ and of thickness D is covering this cylinder. Finally a surface current density K is flowing through a metallic cylinder of radius $R_2 = R_1 + D$, coaxial with the first one, in the opposite direction as $\vec{J}$. ($\vec{K} = -K\hat{z}$; $K > 0$) Calculate all the fields $\vec{H}, \vec{B}, \vec{M}, \vec{A}$ (in this order) and bound volume and surface currents in this problem.

6. Brush up on Faraday and Lenz's law, and try to understand the examples mentioned in class and in the book (Eddy currents, magnetic levitation, ...)

7. A metallic ring in put on top of a solenoid. Initially, the current is zero, then it is turned on to create a magnetic field inside the solenoid. Find the direction of the force acting on each porion dl of the ring. Deduce the net force on the ring. Describe what happens when the current is turned on?

8. Know your definitions of paramagnetism, diamagnetism and ferromagnetism. Know the difference between linear and non-linear materials (dielectric and magnetic).

9. A conducting circular disk of radius a and thickness d in the (x,y) plane sits in a region of uniform ibut time-dependent magnetic field $B = B_0 \, e^{-t/\tau} \hat{z}$.

   Find the current density $\vec{J}$ induced in the conductor, assuming the conductivity of the disk is σ ? (Neglect effects of the magnetic field produced by the induced current $\vec{J}$ itself.) Plot it as a function of the distance from center of the disk.

   Is there any force on the disk? and if so in what direction?

10. A current I_1 is flowing in a circular loop, through a current source. How much extra work does the current source need to do if we bring another current loop I_2 from infinity to a final position where the mutual inductance of the two circuits would be M? Explain physically why the current source 1 needs to do extra work. Is this work positive or negative? ($M > 0$)