Solutions to homework set #7

Chap6 - Pb 6)

$$m \frac{d\vec{v}}{dt} = -\frac{m}{\tau}\vec{v} + q\vec{E} \quad \text{with} \quad \vec{E} = \vec{E}_0\, e^{i\omega t}$$

The forced solution after long times has the form $\vec{v} = \vec{v}_0\, e^{i(\omega t + \varphi)}$ with a dephasing... (the short time behavior is the relaxation of the initial condition; we have seen it decays to zero like $e^{-t/\tau}$).

$$\left(i\omega m + \frac{1}{\tau} m\right) \vec{v}_0\, e^{i\varphi} = q\vec{E}_0 \qquad \vec{v}_0 = \frac{q\vec{E}_0\, e^{-i\varphi}}{m\left(\frac{1}{\tau} + i\omega\right)}$$

$$\vec{J} = qn\vec{v} = \frac{\frac{nq^2}{m}\vec{E}_0\, e^{i\omega t}}{\left(\frac{1}{\tau} + i\omega\right)} \qquad (q^2 = e^2)$$

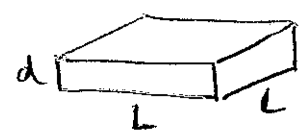
$$\sigma = \frac{\vec{J}}{\vec{E}} = \frac{ne^2\tau}{m}\,\frac{1}{1+i\omega\tau} = \frac{\sigma_0}{1+i\omega\tau} = \sigma_0\,\frac{1-i\omega\tau}{1+(\omega\tau)^2}$$

The imaginary part reflects the phase difference between $\vec{v}$ (or $\vec{j}$) and $\vec{E}$ or the applied bias V.

$$\tan\varphi = -\omega\tau = \frac{\operatorname{Im}\sigma}{\operatorname{Re}\sigma}$$

(the sign depends on the choice of $e^{-i\omega t}$ or $e^{i\omega t}$ in the $\vec{E}_0 e^{i\omega t}$

Pb 10) : $R_{sq} = \rho/d$



$$\tau \approx d/v_F \qquad \sigma = \frac{ne^2\tau}{m} = \frac{ne^2 d}{m v_F}$$

$$R_{sq} = \frac{\rho}{d} = \frac{1}{\sigma d} = \frac{m v_F}{ne^2 d^2}$$

; If monatomic metal sheet, then we have $1\bar{e}$/atom $\to 1\bar{e}/d^3$ where d = lattice parameter

$\Rightarrow nd^3 = 1$; on the other hand for a half-filled band $k_F = \frac{\pi}{2d} \Rightarrow m v_F = \frac{\hbar\pi}{2d}$

Pb 10

$$m v_F = \hbar k_F = \frac{\hbar \pi}{2d} \Rightarrow R_{sq} = \frac{m v_F}{n e^2 d^2} = \frac{\hbar \pi}{e^2 2d} \times \frac{d^3}{d^2} = \frac{\pi \hbar}{2e^2} \approx \frac{\hbar}{e^2}$$

Chap 8 pb 1)

$E_g = 0.23$ eV $\quad m^* = 0.015\, m_e$

$\varepsilon = 18$

a) $E_{ionization} = |E_{Ryd}| \times \left(\frac{m^*}{m}\right) \frac{1}{\varepsilon^2} = 13.6 \text{ eV} \times 0.015 \times \frac{1}{(18)^2} = 0.63$ meV

b) $a_B^* = a_B \dfrac{\varepsilon}{\left(\frac{m^*}{m}\right)} = 0.53 \text{Å} \times \dfrac{18}{0.015} = 636 \text{ Å} \approx 64 \text{ nm}$

c) Overlap occurs if $a_B^{*3} \times \frac{4\pi}{3} \sim \frac{1}{N_D} \Rightarrow N_D \sim \dfrac{3}{4\pi (a_B^*)^3} = 9.3\, 10^{-10} \text{ Å}^{-3}$

$= 9.3\, 10^{20} \text{ m}^{-3}$

Pb 2)

$N_D = 10^{13} / \text{cm}^3 = 10^{19} / \text{m}^3$

$E_d = 1 \text{ meV} = \frac{300 K}{25} \sim 12 \text{ K} > 4 \text{K} \Rightarrow \tilde{n} \approx \sqrt{N_c N_d}\, e^{-\frac{E_d}{2kT}}$

$m^* = 0.01\, m_e$

a) $n\,(T = 4T) = \left[N_c N_D\right]^{\frac{1}{2}} e^{-E_d/2kT}$ with $N_c = 2\left(\frac{2\pi m kT}{h^2}\right)^{3/2}$

b) Hall coefficient $= -\dfrac{1}{ne} = -1.37$ SI

$N_c = 3.77 \cdot 10^{19} \text{ m}^{-3}$

$\Rightarrow \tilde{n} = 4.55 \cdot 10^{18} \text{ m}^{-3}$

Pb 3 Hall effect with two carrier types:

The equation of motion for $\bar{e}$ & holes in steady state

$$m_e \frac{d\vec{v}_e}{dt} = 0 = -m_e \frac{\vec{v}_e}{\tau_e} - e(\vec{E} + \vec{v}_e \times \vec{B}) \to -m\frac{v_e}{\tau_e} - e(E + i v_e B)$$

$$0 = -m_h \frac{\vec{v}_h}{\tau_h} + e(\vec{E} + \vec{v}_h \times B) \to -m\frac{v_h}{\tau_h} + e(E - i v_h B)$$

defines the relationship between the drift velocities and the applied fields. $(\vec{E}, \vec{B})$

$$v_e = \frac{-\frac{eE}{m_e}}{\frac{1}{\tau_e} + i\omega_e} \qquad v_h = \frac{\frac{eE}{m_h}}{\frac{1}{\tau_h} + i\omega_h} \qquad \omega_e = \frac{eB}{m_e} \quad \omega_h = \frac{eB}{m_h}$$

where the complex notation was used ($\vec{E}$, $\vec{v}$ vectors are represented with two complex numbers): $\vec{E} \to E_x + iE_y$, $\vec{V} \to v_x + iv_y$

$$J = -e n_e v_e + e n_h v_h$$

Then we use the fact that there is no transverse current ($J_y = 0$ or $\mathrm{Im}\, J = 0$) to get a relationship between E_x & E_y.

$$R_H = \frac{E_y}{JB}.$$

$$\text{So: } J = e^2 E\left[\frac{n_h \tau_h / m_h}{1 + i\omega_h \tau_h} + \frac{n_e \tau_e / m_e}{1 - i\omega_e \tau_e}\right]$$

$$\mathrm{Im}\, J = 0 = e^2\left[E_y\left[\left(\frac{n\tau/m}{1+\omega^2\tau^2}\right)_h + \left(\frac{n\tau/m}{1+\omega^2\tau^2}\right)_e\right] + E_x\left[\left(\frac{(n\tau/m)(-\omega\tau)}{1+\omega^2\tau^2}\right)_h + \left(\frac{(n\tau/m)\omega\tau}{1+\omega^2\tau^2}\right)_e\right]\right]$$

$$\Rightarrow E_x = -E_y \left[(n\tau/m)_h + (n\tau/m)_e\right] \Big/ \left[\left(\frac{n\tau^2\omega}{m}\right)_e - \left(\frac{n\tau^2\omega}{m}\right)_h\right]$$

neglecting $(\omega\tau)^2$ terms.

$$J_x = e^2 E_x \left[\left(\frac{n\tau/m}{1+\omega^2\tau^2}\right)_h + \left(\frac{n\tau/m}{1+\omega^2\tau^2}\right)_e \right] + e^2 E_y \left[(\omega n\tau^2/m)_h - (n\omega\tau^2/m)_e \right]$$

$$= e^2 E_y \left[\left(\frac{n\omega\tau^2}{m}\right)_h - \left(\frac{n\omega\tau^2}{m}\right)_e + \frac{\left[\left(\frac{n\tau}{m}\right)_h + \left(\frac{n\tau}{m}\right)_e\right]^2}{-\left(\frac{n\omega\tau^2}{m}\right)_e + \left(\frac{n\omega\tau^2}{m}\right)_h} \right]$$

Finally $R_H = \frac{E_y}{J_x B} = \frac{-\left(\frac{n\omega\tau^2}{m}\right)_e + \left(\frac{n\omega\tau^2}{m}\right)_h}{\left[\left(\frac{n\omega\tau^2}{m}\right)_e - \left(\frac{n\omega\tau^2}{m}\right)_h\right]^2 + \left[\left(\frac{n\tau}{m}\right)_h + \left(\frac{n\tau}{m}\right)_e\right]^2} \times \frac{1}{B}$

taking $\omega\tau \ll 1$, the first 2 terms in the denominator are negligible,

$$R_H = \frac{-\left(ne\frac{\tau^2}{m^2}\right)_e + \left(ne\frac{\tau^2}{m^2}\right)_h}{\left[\left(\frac{n\tau}{m}\right)_e + \left(\frac{n\tau}{m}\right)_h\right]^2}$$

or using $b = \frac{\mu_e}{\mu_h} = \frac{\tau_e/m_e}{\tau_h/m_h}$ and $n_h = p$, $n_e = n$

we have $R_H = \frac{1}{e}\frac{(-nb^2 + p)}{(nb+p)^2}$ QED.

Band Structure of a 2D square lattice

free electrons: dispersion is $\frac{\hbar^2 k^2}{2m}$

in the plane wave basis, for each $k \in$ FBZ the Hamiltonian matrix in the PW basis (G) is diagonal

$$H(k) = \begin{array}{c} \begin{array}{cccccc} G=0 & G_1 & G_2 & -G_1 & -G_2 & G_1+G_2 \cdots \end{array} \\ \begin{pmatrix} \frac{\hbar^2 k^2}{2m} & & & & \\ & \frac{\hbar^2 (k+G_1)^2}{2m} & & 0 & \\ & & \frac{\hbar^2 (k+G_2)^2}{2m} & & \\ & 0 & & \frac{\hbar^2 (k-G_1)^2}{2m} & \\ & & & & \ddots \end{pmatrix} \end{array}$$

$\vec{G}_1 = \frac{2\pi}{a}\hat{x}$

$\vec{G}_2 = \frac{2\pi}{a}\hat{y}$

So different bands, (corresponding to different eigenvalues of the above matrix) are obtained by folding the free $\bar{e}$ dispersion $\frac{\hbar^2 k^2}{2m}$ inside the FBZ (shift k outside FBZ to a k inside by adding appropriate G vector)