Name & Student ID:

Quiz # 2 , December 1, 2008 Thermodynamics (5D)

(5 points) A mass m_2 of ice at $T = 0C$ is dropped into a glass containing a mass m_1 of water at temperature $T > 0C$. The whole system is assumed to be thermally isolated.

* Discuss what is the final state of the system (m_1', m_2', T_f depending on the parameters of the problem such as L, latent heat of melting of ice, and the heat capacities of water and ice (neglect the heat capacity of the glass container).

If m_2 is too large or T is too small, we'll have at the end ice + water. However if m_1 is large and T is large, all ice will melt. To be more quantitative:

Either 1) Ice remains ($\Rightarrow T_f = 0\,°C$)

$$(m_2 - m_2')L = m_1 C(T-0) \Rightarrow \boxed{m_2' = m_2 - \frac{m_1 C T}{L}.}$$

$(m_2 - m_2')L$: heat necessary to melt a mass of $m_2 - m_2'$ of ice

$m_1 C(T-0)$: heat necessary to bring the temperature of water from T down to $T_f = 0\,°C$

conservation of mass: $m_1 + m_2 = m_1' + m_2' \Rightarrow \boxed{m_1' = m_1 + \frac{m_1 C T}{L}.}$

Or 2) All ice melts ($\Rightarrow \boxed{m_2' = 0 \quad m_1' = m_1 + m_2}$)

$$m_2 L + m_2 C(T_f - 0) = m_1 C(T - T_f)$$

$m_2 L$: heat to melt m_2 mass of ice

$m_2 C(T_f - 0)$: heat to bring m_2 mass of water from 0 to T_f

$m_1 C(T - T_f)$: heat to bring mass m_1 of water from T to T_f

$$\Rightarrow \boxed{T_f = \frac{m_1 C T - m_2 L}{(m_1 + m_2) C}}$$

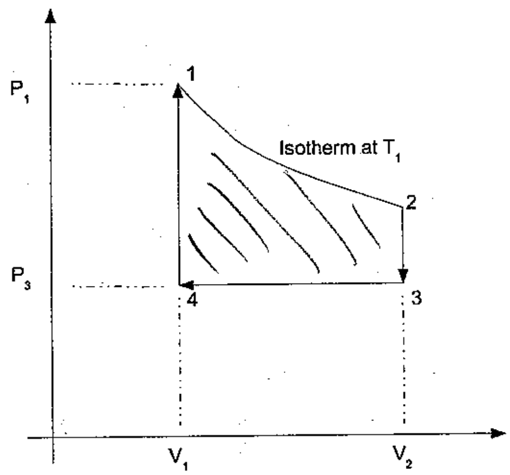
Limiting case $m_2' = 0 \Rightarrow$ $\boxed{\frac{m_2}{m_1} \begin{matrix} \leq \\ > \end{matrix} \frac{CT}{L}}$

$\leq$: All ice melts (case 2)

$>$: ice remains (case 1)

(5 points) n moles of an ideal gas undergo the cycle displayed in the figure, starting from state 1. Calculate the total work done by the gas (assumed ideal) and the total heat **absorbed by** the gas during one full cycle. Results must be given as a function of n, P_1, P_3, V_1, V_2 and T_1.

* What kind of machine are we dealing with here? heat pump, engine, other...? Explain why?



- $W = \int P\,dV$ = area inside the cycle (hashed)

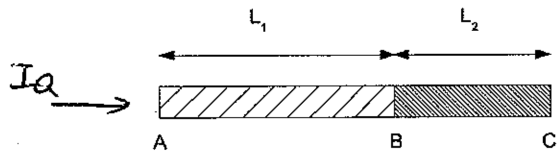
$$= nRT_1 \int_{V_1}^{V_2} \frac{dV}{V} - P_3 (V_2 - V_1) = nRT_1 \ln \frac{V_2}{V_1} - P_3 (V_2 - V_1)$$

- $\Delta E^{int}_{cycle} = 0 \Rightarrow Q = W$

- $Q > 0 \quad W > 0 \Rightarrow$ give heat to do work $\Rightarrow$ engine

(3 points) Write the equation relating the heat current I_Q to the temperature difference ΔT applied across a bar of thermal conductivity κ, cross section area A, and length L.

* If two bars of the same cross section A but respective lengths L_1 and L_2, and thermal conductivities κ_1 and κ_2 are attached in series, express the steady state temperatures T_B and T_C as a function of T_A, L_1, L_2, A, κ_1, κ_2 and the heat current I_Q flowing through the system (neglect radiative and convective heat losses).



$$I_Q = -KA\frac{\Delta T}{L}.$$

* Same heat current flows through both bars.

$$I_Q = +K_1 A \frac{(T_B - T_A)}{L_1} = +K_2 A \frac{(T_C - T_B)}{L_2}$$

$$\Rightarrow \quad T_B = T_A + \frac{L_1}{K_1 A} I_Q$$

$$T_C = T_B + \frac{L_2}{K_2 A} I_Q = T_A + \underbrace{\left(\frac{L_1}{K_1 A} + \frac{L_2}{K_2 A}\right)}_{\text{equivalent series resistance}} I_Q$$

Good luck,
Keivan Esfarjani

Eventually needed formulas:

$\frac{dQ}{dt} = -\kappa A \frac{dT}{dx}$ $\quad Q = mL \quad Q = mc\Delta T \quad PV = nRT \quad \Delta E_{int} = Q - W \quad W = \int PdV$