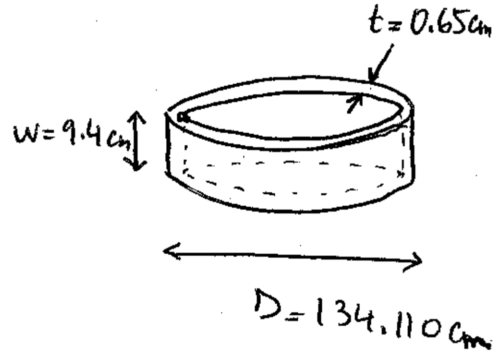
Outline of the Solutions to Homework # 1

17.27 (Barrel)

$D_B = 134.122$ cm at $20^\circ C$

$D_{iron} = 134.110$ cm at $20^\circ C$.

$w = 9.4$ cm $t = 0.65$ cm



a) $D_B = D_{iron}\left(1 + \frac{\beta}{3}\Delta T\right)$

$$\Rightarrow \Delta T = \frac{3}{\beta_{iron}}\left(\frac{D_B - D_{iron}}{D_{iron}}\right) = \frac{3}{35.10^{-6}}\left(\frac{0.012}{134.1}\right) \simeq 7.67\ ^\circ C.$$

$$T_f = 20 + 7.67 = 27.\ ^\circ C$$

if you add 0.008 cm to D_B : $\Delta T = \frac{3}{35.10^{-6}} \times \frac{0.02}{134.1} = 12.7\ ^\circ C.$

$$T_f = 20 + 12 = 32\ ^\circ C.$$

b) When it cools to $20^\circ C$

$$\frac{\Delta D}{D} = \alpha \Delta T = \frac{\beta}{3}\Delta T = \frac{0.012}{134.1} \simeq 9.10^{-5}$$

Tension = cross section × pressure

$$= (t \times w)\ E\ \frac{\Delta D}{D}$$

pressure $= E \cdot \frac{\Delta D}{D}$

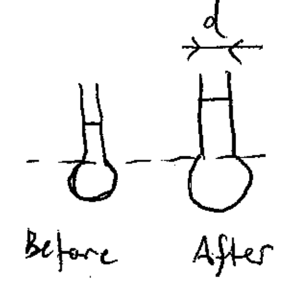
E ↓ Young Modulus of iron

17.72

ΔV_{hg} of mercury is given by: $\left(V_{bulb} + \frac{\pi d^2}{4} h_o\right) \times \beta_{hg} \Delta T$

the volume of pyrex glass is changed by $\Delta V_{pyrx} = V_o\ \beta_{pyrx}\ \Delta T$

Net increase in volume of mercury is $(V_o)(\beta_{hg} - \beta_{pyrx})\ \Delta T$



If the change in the bulb was neglected, we would have $\Delta h = h\, \alpha\, \Delta T$

with $\alpha = \frac{1}{3}(\beta_{hg} - \beta_{pyrx})$: coeff of linear expansion (relative to pyrex)

there is however an extra raise in height due to the expansion of Mercury in the bulb:

$$\Delta V_b = V_b (\beta_{hg} - \beta_{pyrx}) \Delta T = \frac{\pi d^2}{4} \Delta h'$$

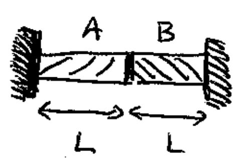
(d = diameter of column)

$\Rightarrow$ The net increase in height is $\Delta h + \Delta h' = \left(\frac{h}{3} + \frac{4 V_b}{\pi d^2}\right)(\beta_{hg} - \beta_{py})\Delta T$

If we ignore tube volume compared to bulb volume

then $h \frac{\pi d^2}{4} \ll V_b \Rightarrow \Delta h + \Delta h' = \frac{4 V_b}{\pi d^2} (\beta_{hg} - \beta_{pyrx}) \Delta T$

Hot Rods



- After heating combined Length is $L + L = 2L$ but due to their different hardness, their expansion at constant total length will be different
- Since both expand $\rightarrow$ there will be a compressive stress in both rods. equilibrium implies it has to be the same for both rods

$\frac{\Delta L_i}{L} = \alpha_i \Delta T$ (for each rod i) $\rightarrow \Delta L_{tot} = L(\alpha_A + \alpha_B)\Delta T$

Just like springs put in series, the Young modulus of 2 rods in series is $Y = \frac{Y_A Y_B}{Y_A + Y_B}$ $\Rightarrow$ Total stress $= \left|\frac{F}{A}\right| = Y_{tot} \frac{\Delta L_{tot}}{L} = \left(\frac{Y_A Y_B}{Y_A + Y_B}\right)(\alpha_A + \alpha_B)\Delta T$