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Physics 5D Fall 2008

Assignment 1

Due at 2:00pm on Tuesday, October 14, 2008

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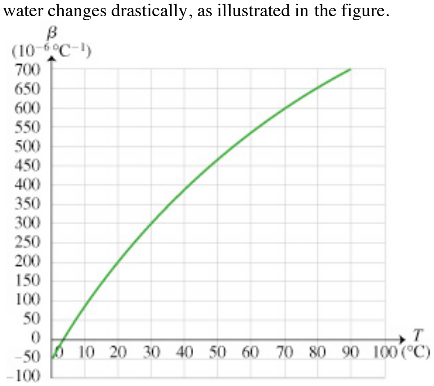
Coefficient of Volume Expansion for Water Conceptual Question

Description: Conceptual question on the expansion characteristics of liquid water.

The anomalous expansion characteristics of liquid water are crucial to many biological systems. Rather than an approximately constant value for the coefficient of volume expansion, the value for water changes drastically, as illustrated in the figure.



Part A

Below what temperature T does water shrink when heated?

Hint A.1 Coefficient of volume expansion

The fractional change in volume of a substance when subject to a change in temperature depends on the amount of the temperature change and the substance. The coefficient of volume expansion represents the fractional change in volume per degree of temperature change. A large coefficient of expansion means that the substance expands by a relatively large amount.

Hint A.2 Volume expansion relation

The dependence of the change in volume of a substance (ΔV) on the initial volume (V_0), the temperature change (ΔT), and the coefficient of volume expansion (β) is

$$\Delta V = \beta V_0 \Delta T$$

Hint A.3 Negative coefficient of volume expansion

Since the relationship for volumetric expansion is

$$\Delta V = \beta V_0 \Delta T$$

if β is less than zero, the change in volume will be negative when the change in temperature is positive.

Express your answer numerically in degrees Celsius.

ANSWER: T = 4 °C

Part B

If the temperature of water at 30 $^\circ C$ is raised by 1 $^\circ C$, the water will expand. At approximately what initial temperature T will water expand by twice as much when raised by 1 $^\circ C$?

Hint B.1 Coefficient of volume expansion

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Hint B.2 Volume expansion relation

The dependence of the change in volume of a substance (ΔV) on the initial volume (V_0), the temperature change (ΔT), and the coefficient of volume expansion (β) is

$$\Delta V = \beta V_0 \Delta T$$

Hint B.3 Comparing expansion rates

The coefficient of volume expansion represents the fractional change in volume per 1 $^\circ\mathrm{C}$ of temperature change. The larger the coefficient of expansion, the more the substance expands. Specifically, if the coefficient is twice as large, the water will expand twice as much.

Express your answer numerically in degrees Celsius.

ANSWER: T = 71 $^\circ\mathrm{C}$

Part C

The relative density of water is determined by the rate at which it expands (and contracts) with changes in temperature. At approximately what other temperature T does water have the same density as at 1 $^\circ\mathrm{C}$?

Part C.1 Determine when liquid water is most dense

As a substance expands, it becomes less dense. Since at very low temperatures water shrinks when heated, this means it is actually becoming denser as you raise its temperature. Based on this idea, at what temperature T is liquid water most dense?

Express your answer numerically in degrees Celsius.

ANSWER: T = 4 $^\circ\mathrm{C}$

As you move, in temperature, away from 4$^\circ\mathrm{C}$, liquid water becomes progressively less dense, at a rate that is proportional to the coefficient of volume expansion.

ANSWER: T = 7 $^\circ\mathrm{C}$

The Incredible Shrinking (and Expanding) Bridge

Description: A mix of qualitative and computational questions introducing the concepts of thermal linear and volume expansion.

Learning Goal: To understand thermal linear expansion in solid materials.

Most materials expand when their temperatures increase. Such *thermal expansion*, which is explained by the increase in the average distance between the constituent molecules, plays an important role in engineering. In fact, as the temperature increases or decreases, the changes in the dimensions of various parts of bridges, machines, etc., may be significant enough to cause trouble if not taken into account. That is why power lines are always sagging and parts of metal bridges fit loosely together, allowing for some movement.

It turns out that for relatively small changes in temperature, the linear dimensions change in direct proportion to the temperature. For instance, if a rod has length L_0 at a certain temperature T_0 and length L at a higher temperature T, then the change in length of the rod is proportional to the change in temperature and to the initial length of the rod:

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$$L - L_0 = \alpha L_0 (T - T_0),$$

or

$$\Delta L = \alpha L_0 \Delta T.$$

Here, α is a constant called the *coefficient of linear expansion*; its value depends on the material. A large value of α means that the material expands substantially as the temperature increases; smaller values of α indicate that the material tends to retain its dimensions. For instance, quartz does not expand much; aluminum expands a lot. The value of α for aluminum is about 60 times that of quartz!

In this problem, you will answer some basic questions related to the concept of thermal expansion.

Part A

A square is cut out of a copper sheet. The square is heated uniformly. As a result, it turns into

ANSWER:

- (•) a square with a larger area
- ( ) a square with a smaller area
- ( ) a rectangle with a larger area
- ( ) a rectangle with a smaller area

Since all dimensions change by the same ratio, the square retains its shape. The area increases, and so does the thickness of the sheet.

Part B

A square is cut out of a copper sheet. Two straight scratches on the surface of the square intersect forming an angle θ. The square is heated uniformly. As a result, the angle between the scratches

ANSWER:

- increases
- decreases
- stays the same
- The answer depends on whether θ is an acute or obtuse angle.

Since all dimensions change by the same ratio, every "detail" of the square retains its shape. The scratches will extend in length, but they will still form the same angle.

Part C

A square is cut out of a copper sheet. A circular hole is drilled in the square. The square is heated uniformly. As a result, the diameter of the hole

ANSWER:

- increases
- decreases
- stays the same
- The answer depends on the size of the hole.

There is a popular misconception that the hole would get smaller because "the material around it expands." However, both experiment and logic dictate otherwise. One way to think about it is to imagine what would happen to the "disk" cut out of that hole. When heated, the disk would expand, of course, and so should "the empty space" left by the absence of that disk. Another way to visualize the expansion is to think of the material surrounding the hole as a large number of thin rings concentric with the hole itself. When heated, all of these rings would expand, including the innermost one, which "traces" the edge of the hole.

The next few questions refer to the Golden Great Bridge, built on planet Tehar in a galaxy far, far away. The bridge-building technology on Tehar is not very well developed: The bridge is just a long slab of pure gold with the opposite ends resting on the shores of the river.

In the spring, when the air temperature is 100 $^\circ\mathrm{C}$, the length of the bridge is 160.0 m. Answer the questions below knowing that the value of α for gold is $1.42 \times 10^{-5}\,^\circ\mathrm{C}^{-1}$.

Part D

Compared to its length in the spring, by what amount $\Delta L_{\rm winter}$ does the length of the bridge decrease during the Teharian winter when the temperature hovers around -150$^\circ\mathrm{C}$?

Express your answer as a positive value in meters.

ANSWER: $\Delta L_{\rm winter}$ = 0.568 m

Part E

Compared to its length in the spring, by what amount $\Delta L_{\rm summer}$ does the length of the bridge increase during the Teharian summer when the temperature hovers around 700$^\circ\mathrm{C}$?

Express your answer numerically in meters.

ANSWER: $\Delta L_{\rm summer}$ = 1.36 m

Part F

The Teharians want to build another bridge next to the Golden Great Bridge. In the spring, it would have the same length as the Golden Great Bridge. However, in the summer, it should increase its length by 2.30 m. What substance should be used to build the new bridge? Use the following table of coefficients of linear expansion.

Material	α $[(\mathrm{C}^\circ)^{-1}]$
Aluminum	2.4×10^{-5}

Brass	2.0×10^{-5}
Copper	1.7×10^{-5}
Glass	$0.4 - 0.9 \times 10^{-5}$
Quartz	0.04×10^{-5}

ANSWER:
- (•) Aluminum
- ( ) Brass
- ( ) Copper
- ( ) Glass
- ( ) Quartz

Air Bubble Rising in a Lake

Description: Calculate the ratio of the volume of a bubble at the surface of a lake to its volume at the bottom of that lake. Use this to determine the safety of a diver ascending while holding his breath.

A diver named Jacques observes a bubble of air rising from the bottom of a lake (where the absolute pressure is 3.50 atm) to the surface (where the pressure is 1.00 atm). The temperature at the bottom is 4.0 $^\circ\mathrm{C}$, and the temperature at the surface is 23.0 $^\circ\mathrm{C}$.

Part A

What is the ratio of the volume of the bubble as it reaches the surface (V_s) to its volume at the bottom (V_b)?

Hint A.1 How to approach the problem

Use the ideal gas equation to calculate the ratio of the volume V_s at the surface to the volume V_b at the bottom of the lake. Be careful about the units when performing the calculations.

Hint A.2 Mass of air in the bubble

Since the bubble is surrounded entirely by the water, it can be assumed that no air can enter or leave the bubble during its ascent, so the number of moles n must be a constant.

Part A.3 Find an expression for the volume ratio

Find a symbolic expression for the volume ratio V_s/V_b.

Hint A.3.a Using the ideal gas law

Recall that the ideal gas law is

$$pV = nRT,$$

or

$$V = \frac{nRT}{p},$$

where p is the pressure, V is the volume, n is the number of moles of gas, R is the ideal gas constant, and T is the temperature. Set up two copies of this equation for the two situations: at the surface, where the variables are V_s, p_s, and T_s, and at the bottom, where they are V_b, p_b, and T_b. If you divide the first of these equations by the second, then you will have the ratio V_s/V_b of the volumes. We are assuming that n will stay constant, because gas does not escape or enter the bubble as it rises, and R is always a constant, so both should cancel out of your expression.

Express your answer in terms of the temperature T_b and pressure p_b at the bottom of the lake and the temperature T_s and pressure p_s at the surface.

ANSWER:

$\frac{V_s}{V_b}$ =
- ( ) $\frac{p_s T_s}{p_b T_b}$
- ( ) $\frac{p_b T_b}{p_s T_s}$
- (•) $\frac{p_b T_s}{p_s T_b}$

$\frac{p_s T_b}{p_b T_s}$

Recall that the temperatures T_b and T_s are expressed in kelvins in the ideal gas equation, so you will need to convert them from degrees Celsius before performing the calculations.

ANSWER: $\frac{V_s}{V_b} = 3.74$

Part B

Would it be safe for Jacques to hold his breath while ascending from the bottom of the lake to the surface?

ANSWER:
- yes
- no (selected)

If Jacques were holding his breath, then air would be unable to enter or leave his lungs. As he ascends to the surface, the air in his lungs would expand, like the air in the bubble, and his lungs would have to stretch outward to hold this increased volume, which would be extremely unsafe.

In fact, even if he does not hold his breath, if he ascends too quickly after a particularly long or deep dive, the nitrogen dissolved in his bloodstream could form into small bubbles, which can be equally dangerous to any diver. This condition is known as decompression sickness, or more commonly as the bends.

Problem 17.2

Description: (a) How many atoms are there in a m copper penny?

Part A

How many atoms are there in a 3.5 g copper penny?

Express your answer using two significant figures.

ANSWER: $N_{\mathrm{Cu}} = \frac{\frac{m}{63.546}}{1.66} \cdot 10^{27}$ atoms

Volume of Copper

Description: Calculate the volume of a given number of moles of copper.

Part A

What is the volume V of a sample of 3.30 mol of copper? The atomic mass of copper (Cu) is 63.5 $\mathrm{g/mol}$, and the density of copper is $8.92 \times 10^3\ \mathrm{kg/m^3}$.

Hint A.1 **How to approach the problem**

Since the number of moles of copper is known, calculate its mass, and then use the mass and the density of copper to find the volume of the copper.

Part A.2 **What is the mass of the copper?**

Calculate the mass m of 3.30 mol of copper.

Express your answer in grams.

ANSWER: $m = n \cdot 63.5$ g

Hint A.3 **Using the density in calculations**

Recall that density is defined as the ratio of the mass to volume: $\rho = M/V$. Be careful with the units, since the volume should be expressed in cubic centimeters.

Express your answer in cubic centimeters.

ANSWER: $V = \frac{n \cdot 63.5}{8.92}$ $\mathrm{cm^3}$

Problem 17.8

Description: A concrete highway is built of slabs l long (20 degree(s) C). (a) How wide should the expansion cracks between the slabs be (at 15 degree(s) C) to prevent buckling if the range of temperature is - 30 degree(s) C to T?

A concrete highway is built of slabs 13 m long (20 °C).

Part A

How wide should the expansion cracks between the slabs be (at 15 °C) to prevent buckling if the range of temperature is - 30 °C to 37°C?

Express your answer using two significant figures.

ANSWER: $\Delta l = 12 \cdot 10^{-6} l\,(T - 15)$ m

Problem 17.12

Description: At a given latitude, ocean water in the so-called "mixed layer" (from the surface to a depth of about 50 m) is at approximately the same temperature due to the mixing action of waves. Assume that because of global warming, the temperature of the mixed ...

At a given latitude, ocean water in the so-called "mixed layer" (from the surface to a depth of about 50 m) is at approximately the same temperature due to the mixing action of waves. Assume that because of global warming, the temperature of the mixed layer is everywhere increased by 0.8 °C, while the temperature of the deeper portions of the ocean remains unchanged.

Part A

Estimate the resulting rise in sea level. The ocean covers about 70 % of the Earth's surface.

Express your answer using one significant figure.

ANSWER: $\Delta d = 210 \cdot 10^{-3} \cdot 50T$ mm

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Problem 17.27

Description: A barrel of diameter db at T is to be enclosed by an iron band. The circular band has an inside diameter of d at T. It is w wide and t thick. (a) To what temperature must the band be heated so that it will fit over the barrel? (b) What will be the...

A barrel of diameter 134.467 cm at 10°C is to be enclosed by an iron band. The circular band has an inside diameter of 134.450 cm at 10°C. It is 9.7 cm wide and 0.60 cm thick.

Part A

To what temperature must the band be heated so that it will fit over the barrel?

Express your answer using two significant figures.

ANSWER: $T = T + \frac{\frac{g \cdot 10^6}{12}}{d}$ °C

Part B

What will be the tension in the band when it cools to 10°C?

Express your answer using two significant figures.

ANSWER: $T = \frac{tw \cdot 10^{11} g}{d}$ N

Problem 17.72

Description: The tube of a mercury thermometer has an inside diameter of d. The bulb has a volume of V. (a) How far will the thread of mercury move when the temperature changes from T_1 to T_2? Take into account expansion of the Pyrex glass. (b) Determine a...

The tube of a mercury thermometer has an inside diameter of 0.150 mm. The bulb has a volume of 0.280 cm^3.

Part A

How far will the thread of mercury move when the temperature changes from 10.5 $^\circ\mathrm{C}$ to 34.0 $^\circ\mathrm{C}$? Take into account expansion of the Pyrex glass.

ANSWER: $\delta L = \frac{4V}{\pi d^2}(T_2 - T_1)(180 - 9)\cdot 10^{-6}$ cm

Part B

Determine a formula for the change in length of the mercury column in terms of relevant variables. Ignore tube volume compared to bulb volume.

ANSWER: **Answer Key:**

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The formula is: delta_L = 4*(V_bulb)/(pi*(d)^2*(T_2 - T_1)*(beta_Hg - beta_glass).
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Problem 17.15

Description: An aluminum sphere is d in diameter. (a) What will be its change in volume if it is heated from t_1 to t_2?

An aluminum sphere is 8.90 cm in diameter.

Part A

What will be its change in volume if it is heated from 20 $^\circ\mathrm{C}$ to 200 $^\circ\mathrm{C}$?

Express your answer using two significant figures.

ANSWER: $\Delta V = 75\cdot 10^{-6}\frac{\frac{4\pi}{3}d^3}{8}(t_2 - t_1)$ cm^3

5.0

Hot Rods

Description: Two rods with different thermal properties are heated with the constraint that their combined length must not change. Find the thermal stress that is produced in the rods.

Two circular rods, both of length L and having the same diameter, are placed end to end between rigid supports with no initial stress in the rods.

The coefficient of linear expansion and Young's modulus for rod A are α_A and Y_A respectively; those for rod B are α_B and Y_B respectively. Both rods are "normal" materials with $\alpha > 0$.

The temperature of the rods is now raised by ΔT.

Part A

After the rods have been heated, which of the following statements is true?

Choose the best answer.

ANSWER:

- ○ The length of each rod is still L.
- ◉ The length of each rod changes but the combined length of the rods is still $2L$.

The length of the combined rod remains the same, but because the rods have different expansion coefficients, the lengths of the individual rods change. In other words, $\Delta L_A + \Delta L_B = 0$ even though $\Delta L_A \neq 0$ and $\Delta L_B \neq 0$.

Part B

After the rods have been heated, which of the following statements is true?

Choose the best answer.

ANSWER:

- ○ The stress in each rod remains zero.
- ◉ A compressive stress arises that is the same for both rods.
- ○ A compressive stress arises that is different for the two rods.
- ○ A tensile stress arises that is the same for both rods.
- ○ A tensile stress arises that is different for the two rods.
- ○ A compressive stress arises in one rod and a tensile stress arises in the other rod.

Stress is a force per unit area. By Newton's 3rd law, the force on rod A due to rod B is the same as that on rod B due to rod A. Since the rods have the same diameter, their cross-sectional area is the same. Therefore, the stress on each rod must be the same.

Part C

What is the stress F/A in the rods after heating?

Hint C.1 How to approach the problem

You know that the length of the two rods remains equal to $2L$ during the heating process. Find the change in length of each rod as a function of the stress on the rods, then set the total change in length to zero and solve for the stress.

Hint C.2 Change in length for each rod

The change in length for each rod is due to both thermal as well as compressive stresses. The corresponding formulas are

$$\Delta L_{\rm thermal} = \alpha L \Delta T$$

and

$$\Delta L_{\rm compressive\ stress} = L\frac{F}{AY}.$$

The sum gives the net change in length for each rod.

Express the stress in terms of α_A, α_B, Y_A, Y_B, and ΔT.

ANSWER: $\frac{F}{A} = \frac{-\left(\alpha_A+\alpha_B\right)\Delta T}{\frac{1}{Y_A}+\frac{1}{Y_B}}$

Another way of thinking about this is that the combination of rods has a net thermal expansion coefficient

$$\alpha = \alpha_1 + \alpha_2$$

and a net Young's modulus given by

$$Y = \frac{Y_1 Y_2}{Y_1 + Y_2}.$$

Summary	2 of 11 items complete (18.18% avg. score) 20 of 110 points