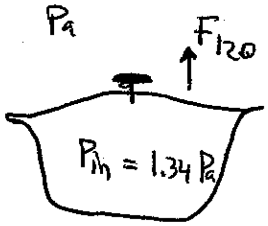
Outline of Solutions to Homework #2

Pressure cooker: You have to take into account the external (atmospheric) pressure.

A) Heating:

$$\frac{F_{20}}{A} = P_a = 1 \text{ atm} \qquad \frac{F_{120}}{A} = P_{in}\left(= \frac{nRT}{V}\right) - P_a$$



n and V are the same at 20 & 120 °C.

$$\frac{P_a}{T_0} = \frac{P_{in}}{T}$$

$P_a = 1$ atm

$T_0 = 20\ °C = 273+20 = 293\ °K$

$T = 120\ °C = 393\ °K.$

* Remember to convert temperatures to Kelvin when using the ideal gas law.

$$P_{in} = \frac{393}{293} \text{ atm} = 1.34 \text{ atm} \rightarrow \frac{F_{120}}{A} = 0.34 \text{ atm} = 3.4 \cdot 10^4 \text{ Pa.}$$

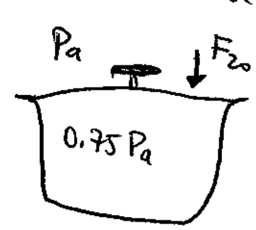
(pascal)

B) After it's cooked: again after lid is closed n, V do not change $\Rightarrow \frac{P}{T} = $ constant

$$\frac{P_a}{393} = \frac{P_f}{293} \Rightarrow P_f = \frac{293}{393} P_a \simeq 0.75 P_a \quad (P_a = 1 \text{atm})$$

It is lower than the atmospheric pressure, so there is a downward force on the lid

$$\frac{F_{20}}{A} = P_a - 0.75 P_a = 0.25 P_a$$



Up, Up & Away (Hot air balloon)

A) Density of hot air inside balloon → It can be calculated using the IGL : $PV = nRT$

$\frac{n}{V}$ = # of moles / unit volume

need to convert # of moles to mass

$n \times \mathcal{N}_A \times m_{\text{air molecules}}$ = mass of hot air inside balloon.

$$\rho_h = \frac{n \mathcal{N}_A\, m_{molec}}{V_1} \qquad \frac{n}{V_1} = \frac{P}{RT} = \frac{\rho_h}{\mathcal{N}_A\, m_{molec}} \Rightarrow \rho_h = \frac{P \mathcal{N}_A m_{mol}}{RT}$$

Pressure above balloon is lower than the pressure below; this pressure difference overcomes gravity and makes the balloon float.

Using $T \times \rho$ = constant (same molecular mass and pressure at the bottom of balloon & outside) $\Rightarrow \rho_h = \rho_c \frac{T_c}{T_h}$

Knowing the densities inside & outside we can get the pressure difference (Buoyancy force) & balance it with the weight of the balloon.

B) $$\text{Weight} = mg = (m_{air} + m_{balloon})\, g = (n \mathcal{N}_A\, m_{molec} + m_{balloon})\, g$$
$$= (\rho_h V_1 + m_b)\, g = \left(\frac{\rho_c T_c}{T_h} V_1 + m_b\right) g$$

C) $$F_{buoyancy} = \text{Weight of displaced cold air}$$
$$= g \times (V_1 + V_2)\, \rho_c$$

D) Min temp. for floating : $F_{buoy} = \text{Weight} \Rightarrow \frac{\rho_c T_c}{T_h} V_1 + m_b = (V_1 + V_2)\rho_c$

$$\Rightarrow T_h = T_c \frac{V_1}{V_1+V_2-\frac{m_b}{\rho_c}}$$

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17.50 Air bubble in Lake.

Use ideal gas law at the bottom & (surface) top of the lake:

$P_b V_b = nRT_b \rightarrow P_b = \frac{nRT_b}{V_b} = 1\text{ atm}$ + pressure of 37 m column of water on top of the bubble.

$P_t V_t = nRT_t \quad P_t = 1\text{ atm}$

$$\frac{P_b V_b}{T_b} = \frac{P_t V_t}{T_t} \Rightarrow V_t = \frac{P_b}{P_t}\frac{T_t}{T_b} \times V_b$$

$P_b = P_t + 37\, g\, \rho_{water} \quad (= \rho g h) \qquad \rho_{water} = 1000\ kg/m^3$

Also need to convert T(°C) to Kelvin

$$\Rightarrow V_t = 1.51\ cm^3 \times \frac{[37 \times 9.8 \times 1000 + 1.01\,10^5]}{1.01\ 10^5} \times \frac{273+18.5}{273+5.5}$$

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17.71 Iron cube floating in mercury

at $T = 0°C = 273°K$ it floats.

A) if $T = 25°C = 298°K$, both volume-expand.

if comparing with respect to mercury surface, then its expansion does not matter. Iron expansion implies a lower density and therefore it would sink a little more.

B) $Mass(Fe) = \rho^{Fe} V_0$; Also Archimedes' ~~law~~ law $\Rightarrow Mass(Fe) = \rho^{hg} V_{subm}$

Mass is the same, equate the 2 right hand sides at $T = 0°C$ & $T = 25°C$.

$T = 0°C \quad \rho^{Fe} V_o = \rho^{hg} V_s = \text{Mass (Fe)}$

At $T = 25°C$

$V_o \to V_o (1 + \beta_{Fe} \Delta T)$

$V_s \to V_{s'}$?

$\rho_{hg} \to \rho_{hg} / (1 + \beta_{hg} \Delta T)$ → mercury becomes less dense.

$\rho^{hg}_{(0°C)} V_s = \rho^{hg}_{(25°C)} V_{s'} = \text{Mass (Fe)} \Rightarrow V_{s'} = V_s \times (1 + \beta_{hg} \Delta T)$

(The submerged volume has increased → in agreement with iron sinking further)

fraction of volume submerged $= \dfrac{V_s}{V_o}$ (at T=0)

(at T=25°C) $\dfrac{V_s'}{V_o'} = \dfrac{V_s (1 + \beta_{hg} \Delta T)}{V_o (1 + \beta_{Fe} \Delta T)} = \dfrac{V_s}{V_o}(1 + x)$

$x = \dfrac{1 + \beta_{hg} \Delta T}{1 + \beta_{Fe} \Delta T} - 1 = \dfrac{1 + 180 \times 25 \cdot 10^{-6}}{1 + 35 \times 25 \cdot 10^{-6}} - 1$

$= \dfrac{1.0045}{1.000875} - 1 = 0.0036 = 0.36\ \%$