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Physics 5D Fall 2008

Assignment 2

Due at 2:00pm on Tuesday, October 21, 2008

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Average Spacing of Gas Molecules

Description: For an ideal gas, if it is assumed that each molecule is uniformly spaced and surrounded by a cube, find the length of an edge of that cube, given the temperature and pressure of the gas.

Consider an ideal gas at 27.0 degrees Celsius and 1.00 atmosphere pressure. Imagine the molecules to be uniformly spaced, with each molecule at the center of a small cube.

Part A

What is the length L of an edge of each small cube if adjacent cubes touch but don't overlap?

Hint A.1 **How to approach the problem**

Calculate the volume of one mole of the gas, being careful about the units, and then use this result to find the volume of one molecule, or rather, the volume of the imaginary cube that is assumed to surround each molecule. Then use the volume per molecule to calculate the length of a side of the cube.

Part A.2 **Calculate the volume per mole**

What is the volume $V_{\rm mole}$ of one mole of the gas?

Hint A.2.a **Ideal gas equation**

The ideal gas equation states that $pV = nRT$. For this question, since you want to find the volume of one mole of the gas, $n = 1$. Also, be careful of the units that you use for the gas constant R.

Express your answer numerically in cubic meters using three significant figures.

ANSWER: $V_{\rm mole}$ = .0246 **2.46×10^{-2}** $\rm m^3$

Part A.3 **Calculate the volume per molecule**

Calculate the volume per molecule $V_{\rm molecule}$ of the gas.

Hint A.3.a **How to approach the problem**

Recall that one mole of a gas has Avogadro's number, i.e., N_A, molecules. So above you found the volume explored by N_A molecules. Use this information to find the volume surrounding one molecule.

Express your answer in cubic meters using three significant figures.

ANSWER: $V_{\rm molecule}$ = $4.09\cdot10^{-26}$ **4.09×10^{-26}** $\rm m^3$

Note that we used Avogadro's number at this step in our calculations to find the volume surrounding each molecule.

Part A.4 **The edge length of a cube**

Recall that the volume $V_{\rm c}$ of a cube is related to its edge length l by $V_{\rm c} = l^3$. Which of the following is the correct expression for l in terms of $V_{\rm c}$?

ANSWER:

- $V_{\rm c}$
- $\frac{V_{\rm c}}{3}$

- V_c^3
- $3V_c$
- $V_c^{1/3}$

Express your answer numerically in meters.

ANSWER: L = $3.45 \cdot 10^{-9}$ m

3.45×10^{-9}

For carbon dioxide gas, the physical volume of a molecule (based on the van der Waals equation constant $4.27 \times 10^5 \ \mathrm{m^3/mol}$) is approximately $7.09 \times 10^{-29} \ \mathrm{m^3}$. This implies that the linear dimension of the molecule is only about $4.14 \times 10^{-10} \ \mathrm{m}$.

Comparing this number to the result of your calculation above, you can see that the size of the molecule is less than one eighth of the length of the cube edge that surrounds the molecule, which is also the average distance separating one molecule from the next. A small molecular size, as compared to the distance between molecules, is a necessary assumption in the kinetic-molecular model of an ideal gas.

Pressure Cooker

Description: Calculate the force on the lid of an air-filled pressure cooker after it is sealed and heated. Then the release valve is opened (so that inside and outside pressure equilibrates). Find the new force after the cooker is resealed and cooled.

A pressure cooker is a pot whose lid can be tightly sealed to prevent gas from entering or escaping.

Part A

If an otherwise empty pressure cooker is filled with air of room temperature and then placed on a hot stove, what would be the magnitude of the net force F_{120} on the lid when the air inside the cooker had been heated to $120^\circ\mathrm{C}$? Assume that the temperature of the air outside the pressure cooker is $20^\circ\mathrm{C}$ (room temperature) and that the area of the pressure cooker lid is A. Take atmospheric pressure to be p_a.

Treat the air, both inside and outside the pressure cooker, as an ideal gas obeying $pV = Nk_\mathrm{B}T$.

Part A.1 Calculate the pressure inside

Calculate the pressure p_in of the air inside the cooker after heating.

Part A.1.a What stays constant when the cooker is heated?

Which of the following quantities remain constant (inside the pressure cooker) as the pressure cooker is heated?

a. pressure p
b. volume V
c. number of particles N
d. the quantity k_B
e. temperature T

Choose the best answer.

ANSWER:

- d only
- c and d
- a and b and c
- b and c and d
- a and b and c and d
- All of the quantities listed remain constant while the cooker is heating.

While the cooker is heating, only the temperature and pressure in the cooker will change.

Rewrite the ideal gas law with all the constant quantities on one side:

$$\frac{p}{T} = \frac{Nk_B}{V}.$$

Since Nk_B/V is constant, p_1/T_1, that is, p/T before heating must be equal to $p_2/T_2 = p/T$ after heating, or mathematically

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}.$$

Recall, however, that the ideal gas law as written here only holds for temperatures measured in kelvins. Also $p_2 = p_{in}$ (after heating).

Express the pressure inside in terms of p_a.

ANSWER: $p_{in} = p_a \frac{393}{293}$

Hint A.2 **Relating pressure and force**

By definition, $p = F/A$.

Part A.3 **Determine the role of the outside pressure**

Remember that there is pressure on *both* sides of the lid, so the *net* force on the lid is a combination of the downward force due to the pressure of the outside air and the upward force due to the pressure of the inside air.

Calculate the difference in pressure $p_{in} - p_{out}$ between the air outside and that inside of the cooker (after heating).

Express the difference in pressure in terms of p_a.

ANSWER: $p_{in} - p_{out} = p_a \frac{100}{293}$

Express the force in terms of given variables.

ANSWER: $F_{120} = p_a \frac{100}{293} A$

$A \cdot p_a \cdot 0.34$

Part B

The pressure relief valve on the lid is now opened, allowing hot air to escape until the pressure inside the cooker becomes equal to the outside pressure p_a. The pot is then sealed again and removed from the stove. Assume that when the cooker is removed from the stove, the air inside it is still at $120^\circ C$.

What is the magnitude of the net force F_{20} on the lid when the air inside the cooker has cooled back down to $20^\circ C$?

Hint B.1 **How to approach the problem**

To determine the force on the cooker lid once it is cool, you need to know the final pressure in the cooker. You can find the final pressure by following the same procedure used in Part A of this problem.

Part B.2 **What stays constant when the cooker is opened?**

Which of the following quantities remain constant (inside the pressure cooker) while the pressure release valve is open?

a. pressure p
b. volume V
c. number of particles N
d. the quantity k_B
e. temperature T

ANSWER:
- c only
- d only
- c and d

- (selected) b and d and e
- c and d and e
- a and b and c and d
- All listed quantities remain constant.

While the value is open, only the pressure and number of particles in the cooker will change.

If you wanted to, you could use the ideal gas law at this point to compute how many gas particles leave the pressure cooker before the pressure inside and outside are equalized. However, you don't need to do that to solve this problem.

Part B.3 **Calculate the pressure inside**

Calculate the pressure $p_{\rm in}$ of the air inside the cooker after it cools.

Part B.3.a **What stays constant when the resealed cooker cools?**

Which of the following quantities remain constant (inside the pressure cooker) while the resealed cooker cools?

a. pressure p
b. volume V
c. number of particles N
d. the quantity $k_{\rm B}$
e. temperature T

ANSWER:

- a only
- a and d
- a and b and c
- (selected) b and c and d
- a and b and c and d
- All of the quantities listed remain constant.

While the cooker is cooling, only the pressure and temperature in the cooker will change. As in Part A, this means that the ratio $p/T = Nk_{\rm B}/V$ is constant during the cooling process. Thus, you can use the same technique as in Part A (setting p/T before the cooker cools equal to p/T after the cooker cools) to find the final pressure in the cooker.

Express the pressure inside in terms of $p_{\rm a}$.

ANSWER: $p_{\rm in} = p_a\frac{293}{393}$

Express the magnitude of the net force in terms of given variables.

ANSWER: $F_{20} = p_a\frac{100}{393}A$

$A \cdot p_a \cdot 0.25$

The Oxygen Room

Description: Calculate the amount of oxygen needed to fill a room to a given temperature and pressure.

A room with dimensions $7.00\ {\rm m} \times 8.00\ {\rm m} \times 2.50\ {\rm m}$ is to be filled with pure oxygen at 22.0 $^\circ{\rm C}$ and 1.00 ${\rm atm}$. The molar mass of oxygen is 32.0 ${\rm g/mol}$.

Part A

How many moles $n_{\rm oxygen}$ of oxygen are required to fill the room?

Hint A.1 **How to approach the problem**

Use the ideal gas equation to calculate the number of moles of oxygen in the room. Be careful about the units when doing the calculations.

Hint A.2 **Using the correct units**

No matter which definition for the gas constant R is used, the temperature must be expressed in kelvins. Also, be sure to convert either the volume from cubic meters to liters or the pressure from atmospheres to pascals, *but not both*, to use one of the standard forms of the gas constant,

$$R = 8.3144\ \frac{\mathrm{J}}{\mathrm{mol \cdot K}} = 0.08206\ \frac{\mathrm{liters \cdot atm}}{\mathrm{mol \cdot K}}.$$

Express your answer using three significant figures.

ANSWER: n_{oxygen} = $5.78 \cdot 10^3$ mol
5780

You could have converted the pressure to $1.013 \times 10^5\ \mathrm{Pa}$ and used $R = 8.3144\ \mathrm{J/(mol \cdot K)}$, or you could have converted the volume to 1.40×10^5 liters and used $R = 0.08206\ \mathrm{(liters \cdot atm)/(mol \cdot K)}$. When solving these problems, use whichever conversions (and the appropriate value for the gas constant) are easiest. However, no matter which method you use, always be sure to convert the temperature from degrees Celsius to kelvins.

Part B

What is the mass m_{oxygen} of this oxygen?

Express your answer in kilograms to three significant figures.

ANSWER: m_{oxygen} = 185 kg
185

Up, Up, and Away

Description: Use ideal gas law to find the density of hot air in a balloon, given the density and temperature of the cold air outside of it. Then find the minimum temperature needed for the balloon to float using Newton's Laws and Archimede's Principle.

Hot air balloons float in the air because of the difference in density between cold and hot air. Consider a balloon in which the mass of the pilot basket together with the mass of the balloon fabric and other equipment is m_{b}. The volume of the hot air inside the balloon is V_1 and the volume of the basket, fabric, and other equipment is V_2. The absolute temperature of the cold air outside the balloon is T_{c} and its density is ρ_{c}. The absolute temperature of the hot air at the bottom of the balloon is T_{h} (where $T_{\mathrm{h}} > T_{\mathrm{c}}$). The balloon is open at the bottom, so that the pressure inside and outside the balloon is the same here. Assume that we can treat air as an ideal gas. Use g for the magnitude of the acceleration due to gravity.

Part A

What is the density ρ_{h} of hot air inside the balloon? Assume that this density is uniform throughout the balloon.

Part A.1 Find density in terms of temperature and pressure

Use the ideal gas law, $pV = nRT$, to find an expression for the density ρ of an ideal gas in terms of its temperature and pressure.

Part A.1.a Find density in terms of mass and volume

Derive an expression for the density ρ of an gas in terms of the volume occupied by the gas, V, the number of moles of gas particles, n, and the mass per mole, m.

ANSWER: $\rho = \frac{mn}{V}$

Express the density in terms of T, p, R, and m, the mass of one mole of gas.

ANSWER: $\rho = \frac{mp}{RT}$

Hint A.2 How to use your general density equation

You now have an expression for the density of a gas in terms of temperature, mass, and pressure. Use this result to write one equation for the density of the hot air (at the bottom of the balloon) and another for the density of the cold air. Then divide the two expressions (remembering that the pressure at the bottom of the balloon and mass per mole of the hot air are the same as those of the cold air) to find a relationship between the temperatures and densities alone. Finally, use the assumption that the density is constant throughout the balloon, so the density you will have found is the density everywhere inside the balloon.

Express the density in terms of T_h, T_c, and ρ_c.

ANSWER: $\rho_h = \frac{\rho_c T_c}{T_h}$

Part B

What is the total weight W of the balloon plus the hot air inside it?

Express your answer in terms of quantities given in the problem introduction and/or ρ_h.

ANSWER: $W = (m_b + \rho_h V_1)\, g$

$\left(m_b + \frac{\rho_c T_c}{T_h} V_1\right) g$

Part C

What is the magnitude of the buoyant force F_B on the balloon?

Hint C.1 **How to approach the problem**

According to Archimedes' principle, the buoyant force is equal to the weight of the (cold) air displaced by the balloon and the (hot) air inside it.

Part C.2 **Volume of displaced air**

What is the volume of cold air displaced by the balloon and its contents?

ANSWER: Volume = $V_1 + V_2$

Express your answer in terms of g, ρ_c, V_1, and V_2.

ANSWER: $F_B = g\rho_c (V_1 + V_2)$

Part D

For the balloon to float, what is the minimum temperature T_{min} of the hot air inside it?

Hint D.1 **How to approach the problem**

The balloon will just begin to float when the magnitude of the buoyant force is equal to the magnitude of the force of gravity (the balloon's weight). So, to find the minimum required temperature, set the weight of the balloon equal to the buoyant force and solve for the resulting temperature of the hot air.

Use the result of Part A to eliminate ρ_h from your answer, if necessary.

Express the minimum temperature in terms of T_c, V_1, V_2, m_b, and ρ_c.

ANSWER: $T_{min} = T_c \frac{V_1}{(V_1 + V_2) - \frac{m_b}{\rho_c}}$

The answer, a bit more elegantly displayed, is

$$T_{min} = T_c \frac{V_1}{(V_1 + V_2) - m_b/\rho_c}.$$

This equation implies that if the mass of the balloon alone is too large, $m_b > \rho_c(V_1 + V_2)$, then the balloon cannot get off the ground at any temperature.

For a given volume and balloon mass (sometimes known as the *payload*), the larger the balloon volume, the lower the temperature required for the balloon to float.

Mathematically, it is possible that $T_{min} < T_c$, but this makes little physical sense. You can show that this would only be the case if the average density of the payload is less than that of the cold air; if this were the case, no balloon would be needed!

[Print]

Problem 17.50

Description: An air bubble at the bottom of a lake d m deep has a volume of v cm^3. (a) If the temperature at the bottom is tb degree(s) C and at the top tt degree(s) C, what is the volume of the bubble just before it reaches the surface?

An air bubble at the bottom of a lake 34.0 m deep has a volume of 1.51 $\mathrm{cm^3}$.

Part A

If the temperature at the bottom is 5.2 $^\circ\mathrm{C}$ and at the top 20.0 $^\circ\mathrm{C}$, what is the volume of the bubble just before it reaches the surface?

ANSWER:

$$V_{\mathrm{surface}} = \frac{\frac{\frac{v\left(1.013\cdot 10^5+10^3\cdot 9.80d\right)(273.15+tt)}{1.013}}{10^5}}{273.15+tb} \quad \mathrm{cm^3}$$

Problem 17.55

Description: The lowest pressure attainable using the best available vacuum techniques is about 10^-12 N/m^2. (a) At such a pressure, how many molecules are there per (cm)^3 at t degree(s) C?

The lowest pressure attainable using the best available vacuum techniques is about $10^{-12}\ \mathrm{N/m^2}$.

Part A

At such a pressure, how many molecules are there per $\mathrm{cm^3}$ at 2 $^\circ\mathrm{C}$?

ANSWER:

$$n = \frac{\frac{\frac{10^{-12}}{1.38}\cdot 10^{23}}{273+t}}{1000000} \quad \frac{\mathrm{molecules}}{\mathrm{cm^3}}$$

Problem 17.31

Description: If V of a gas initially at STP is placed under a pressure of P, the temperature of the gas rises to T. (a) What is the volume?

If 4.50 $\mathrm{m^3}$ of a gas initially at STP is placed under a pressure of 3.50 atm, the temperature of the gas rises to 50.0 $^\circ\mathrm{C}$.

Part A

What is the volume?

ANSWER:

$$V = \frac{\frac{V}{P}(273+T)}{273} \quad \mathrm{m^3}$$

Problem 17.34

Description: If n of helium gas is at T and a gauge pressure of P. (a) Calculate the volume of the helium gas under these conditions. (b) Calculate the temperature if the gas is compressed to precisely half the volume at a gauge pressure of 1.00 atm.

If 14.00 mol of helium gas is at 16.0 $^\circ\mathrm{C}$ and a gauge pressure of 0.300 atm.

Part A

Calculate the volume of the helium gas under these conditions.

Express your answer using four significant figures.

ANSWER:

$$V = \frac{\frac{\frac{n\cdot 8.314(T+273.15)}{P+1}}{1.013}}{10^5} \quad \mathrm{m^3}$$

Part B

Calculate the temperature if the gas is compressed to precisely half the volume at a gauge pressure of 1.00 atm.

Express your answer using two significant figures.

ANSWER: $T = \dfrac{T+273.15}{P+1} - 273.15 \quad {}^\circ\mathrm{C}$

Problem 17.42

Description: A tire is filled with air at T_1 to a gauge pressure of 250 kPa. (a) If the tire reaches a temperature of T_2, what fraction of the original air must be removed if the original pressure of 250 kPa is to be maintained?

A tire is filled with air at 15 $^\circ\mathrm{C}$ to a gauge pressure of 250 kPa.

Part A

If the tire reaches a temperature of 31 $^\circ\mathrm{C}$, what fraction of the original air must be removed if the original pressure of 250 kPa is to be maintained?

Express your answer using two significant figures.

ANSWER: $\left(1 - \dfrac{T_1}{T_2}\right)\cdot 100 \quad \%$

Problem 17.71

Description: An iron cube floats in a bowl of liquid mercury at 0 degree(s) C. (a) If the temperature is raised to T, will the cube float higher or lower in the mercury? (b) By what percent will the fraction of volume submerged change?

An iron cube floats in a bowl of liquid mercury at 0 $^\circ\mathrm{C}$.

Part A

If the temperature is raised to 27 $^\circ\mathrm{C}$, will the cube float higher or lower in the mercury?

ANSWER:

- ○ at the same level
- ◉ lower
- ○ higher

Part B

By what percent will the fraction of volume submerged change?

Express your answer using two significant figures.

ANSWER: $\dfrac{V_{\mathrm{Hg\ displaced}}/V_{\mathrm{Fe}} - V_{0\ \mathrm{Hg\ displaced}}/V_{0\ \mathrm{Fe}}}{V_{0\ \mathrm{Hg\ displaced}}/V_{0\ \mathrm{Fe}}} = \left(\dfrac{1+180\cdot 10^{-6}T}{1+35\cdot 10^{-6}T} - 1\right)\cdot 100 \quad \%$

Summary	3 of 10 items complete (28.5% avg. score) 28.5 of 100 points