• Outline of the Solutions to Homework #3.

18.43 If you look at the $P(V)$ diagram (fig 18-4 p.482) the critical point is shown as C. There you have $\frac{\partial P}{\partial V} = 0$; it is furthermore an inflexion point i.e. $\frac{\partial^2 P}{\partial V^2} = 0$.

Since we have $P(n,V,T) = \frac{nRT}{V-bn} - \frac{an^2}{V^2}$ (*)

$$(1)\quad \frac{\partial P}{\partial V} = 0 = -\frac{nRT}{(V-nb)^2} + \frac{2an^2}{V^3}$$

$$(2)\quad \frac{\partial^2 P}{\partial V^2} = 0 = \frac{2nRT}{(V-nb)^3} - \frac{6an^2}{V^4}$$

Solving these two equations for T, V [for fixed n, take for example $n=1$, as really $\frac{V}{n}$ matters in eq. (*) we find the critical volume & temperature. (V_c, T_c)

Plugging (V_c, T_c) back in eq (*) gives us $P_c = P(1, V_c, T_c)$

$(1) \Rightarrow T = \frac{2a}{V^3}\frac{(V-b)^2}{R}$; substituting this T in (2) we can get V : $0 = \frac{4a}{V^3\,(V-b)} - \frac{6a}{V^4} \Rightarrow 3(V-b) = 2V$

$\Rightarrow \boxed{V_c = 3b} \Rightarrow \boxed{T_c = \frac{2a}{27b^3}\frac{4b^2}{R} = \frac{8a}{27bR}} \Rightarrow P_c = \frac{8a}{27b}/2b - \frac{a}{9b^2}$

$\Rightarrow \boxed{P_c = \frac{a}{27b^2}}$

Knowing T_c & P_c one can then solve for a&b

$\boxed{b = \frac{R}{8}\frac{T_c}{P_c}}$; $\boxed{a = \frac{27}{64}\frac{R^2T_c^2}{P_c}}$

18.57

$$\rho = \frac{1}{cm^3} = 10^{+6}/m^3 \; ; \; T = 2.7\,°K$$

$$v_{rms} = \sqrt{\frac{3kT}{m}} \quad \text{(does not involve the density)}$$

$$P = \frac{nRT}{V} \qquad \frac{n}{V} = \text{number of moles per unit volume } (m^3)$$

10^6 atoms per m^3 corresponds to $\frac{10^6}{\mathcal{N}_A}$ moles per m^3

$$\Rightarrow \quad P = \frac{10^6}{\mathcal{N}_A} \cdot k_B \mathcal{N}_A \cdot T = 10^6 \, k_B T \, .$$

18.60

v_{avg} (not v_{rms}) is given by $\sqrt{\frac{8kT}{\pi m}}$

$$\Rightarrow \quad T = \frac{\pi m v_{avg}^2}{8 k_B}$$

$$m_{O_2} = 32 \text{ (amu)} = 32 \times 1.66 \cdot 10^{-27} \text{ kg}$$

$$m_{He} = 4 \text{ (amu)} = 4 \times 1.66 \; 10^{-27} \text{ kg.}$$