

PHYSICS 110A

Homework 1

Due in class, Tuesday, January 13.

1. Find the gradients of the following functions

(a) $f(x, y, z) = x^2 + y^3 + z^4$.

(b) $f(x, y, z) = e^x \sin(y) \ln(z)$.

2. Show that

(a) $\nabla(r^2) = 2\mathbf{r}$.

(b) $\nabla(1/r) = -\hat{\mathbf{r}}/r^2$.

(c) What is the *general formula* for $\nabla(r^n)$?

3. Calculate the divergence of the following vector functions

(a) $x^2 \hat{\mathbf{x}} + 3xz^2 \hat{\mathbf{y}} - 2xz \hat{\mathbf{z}}$.

(b) $xy \hat{\mathbf{x}} + 2yz \hat{\mathbf{y}} + 3zx \hat{\mathbf{z}}$.

4. Calculate the curls of the vector functions in Qu. 3.

5. Calculate the Laplacian of the following functions:

(a) $T_a = x^2 + 2xy + 3z + 4$.

(b) $T_b = \sin x \sin y \sin z$.

(c) $\mathbf{v} = x^2 \hat{\mathbf{x}} + 3xz^2 \hat{\mathbf{y}} - 2xz \hat{\mathbf{z}}$.

6. (a) Prove that the divergence of a curl is always zero.

(b) Prove that the curl of a gradient is always zero.

7. Griffiths problem 1.32. (Verification of divergence theorem.)

8. Griffiths problem 1.33. (Verification of Stokes' theorem.)

9. Evaluate the following integrals

(a) $\int_0^6 (3x^2 - 2x - 1) \delta(x - 3) dx$.

(b) $\int_0^3 x^3 \delta(x + 1) dx$.

(c) $\int_{-3}^{\infty} \ln(x + 3) \delta(x + 2) dx$.

10. Show that

$$x \frac{d}{dx} (\delta(x)) = -\delta(x).$$

Hint: Use integration by parts.