PHYSICS 110A

Homework 10

Will not be graded. Solutions will be posted shortly on the class website.

I will be out of town during finals week but will have extended office hours on Friday March 13: 11:00–12:00, 1:00-2:00, and 3:00-4:00. The TA will give his usual office hour and discussion section on Monday March 16. His email address is medling@physics.ucsc.edu.

The final exam will be in class on Thursday March 19, 7:30–10:30 pm.

1. A fat wire of radius a carries a time-independent current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel plate capacitor, as shown in Griffiths Fig. 7.43.

   (a) Find the magnetic field in the gap a distance s ($< a$) from the axis.

   (b) Compare this with the magnetic field in the wire at the same distance from the axis.

2. Suppose
$$\mathbf{E}(\mathbf{r},t) = -\frac{1}{4\pi\epsilon_0}\,\frac{q}{r^2}\,\theta(vt-r)\,\hat{\mathbf{r}}\,; \quad \mathbf{B}(\mathbf{r},t) = 0,$$
   where $\theta(x) = 1$ if $x > 0$ and $\theta(x) = 0$ if $x \leq 0$. Show that these fields satisfy all of Maxwell's equations and determine ρ and $\mathbf{J}$. Describe the physical situation that gives rise to these fields.

3. Maxwell's equations in matter are given by Griffiths Eq. 7.55. Determine the speed of electromagnetic waves in a region with no *free* charges or currents.
   Hint: You should start with the equations for $\boldsymbol{\nabla} \times \mathbf{H}$ and $\boldsymbol{\nabla} \times \mathbf{E}$ and have to eliminate one of the variables.

   Ans: $\boxed{v = \dfrac{c}{\sqrt{\epsilon_r \mu_r}},}$ $= \dfrac{1}{\epsilon\mu}$ where $\epsilon = \epsilon_0\epsilon_r, \mu = \mu_0\mu_r$.