PHYSICS 110A

Homework 2

Due in class, Tuesday, January 20.

- **My office hour:** Fridays, 11:00 am–12:00 noon. (ISB 212).
- **TA's office hour:** (Scott Medling) Mondays, 11:00 am–12:00 (noon) (ISB 266).

1. (a) Compute the divergence of the vector function

$$\mathbf{v}(\mathbf{r}) = r^n \, \hat{\mathbf{r}}$$

   for general n.
   Answer: $\boldsymbol{\nabla} \cdot (r^n \, \hat{\mathbf{r}}) = (n+2) r^{n-1}$, unless $n = -2$ in which case it is $4\pi\delta^{(3)}(\mathbf{r})$. For $n < -2$ the divergence is undefined at the origin.

   (b) Compute the *curl* of $r^n \, \hat{\mathbf{r}}$.
   Answer: $\boldsymbol{\nabla} \times (r^n \, \hat{\mathbf{r}}) = 0$.

2. (a) Four equal charges, q, are situated at the corners of a square. What is the net force on a test charge Q at the center?

   (b) Suppose *one* of the charges is removed. What is the force on Q? Explain your reasoning.

   (c) Now 5 equal charges, q, are placed at the corners of a regular pentagon. What is the net force on a test charge Q at the center?

   (d) If one of the 5 charges is removed, what is the force on Q?

3. (a) Find the electric field (magnitude and direction) a distance z above the midpoint between two equal charges q a distance d apart (Fig. 2.4 in Griffiths). Check that your result is consistent with what you'd expect when $z \gg d$.

   (b) Repeat part (a), only this time make the right-hand charge $-q$ instead of q.

4. Find the electric field a distance z above the center of a circular loop of radius R (Fig. 2.9 of Griffiths) which carries a uniform line charge λ.

5. Suppose the electric field in some region is found to be $\mathbf{E} = kr^3 \, \hat{\mathbf{r}}$, in spherical coordinates (k is a constant).

   (a) Find the charge density ρ.

   (b) Find the total charge contained in a sphere of radius R, centered at the origin. (Do it two different ways.)

6. Use Gauss's law to find the electric field inside and outside a charged sphere of radius R with uniform charge density ρ.

7. Two spheres, each of radius R and carrying uniform charge densities $+\rho$ and $-\rho$ respectively, are placed so they partially overlap (see Fig. 2.28 in Grifiths). Call the vector from the positive center to the negative center $\mathbf{d}$. Show that the field in the region of overlap is constant and find its value.
Hint: Use the answer to Qu. 6.

8. The electric field at $\mathbf{r}$ due to charge distribution ρ is given by Coulomb's law

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho(\mathbf{r}')}{\mathscr{r}^2}\,\hat{\boldsymbol{\mathscr{r}}}\,d\tau',$$

where $\boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$. Calculate $\boldsymbol{\nabla} \times \mathbf{E}$ directly and check that you get zero.
Hint: Use the answer to Qu. 1b.

9. One of these is an impossible electrostatic field. Which one?

   (a) $\mathbf{E} = x\,y\,\hat{\mathbf{x}} + 2\,y\,z\,\hat{\mathbf{y}} + 3\,x\,z\,\hat{\mathbf{z}}$,

   (b) $\mathbf{E} = y^2\,\hat{\mathbf{x}} + (2\,x\,y + z^2)\,\hat{\mathbf{y}} + 2\,y\,z\,\hat{\mathbf{z}}$.

   For the *possible* one, find the potential. Verify your answer by computing $-\boldsymbol{\nabla} V$.