PHYSICS 110A

Homework 5

Due in class, Tuesday, February 10.

MIDTERM: The midterm will be in class on Thursday February 12. The material will be everything up to that covered in this homework. The exam will be closed book, but you may bring in one set of notes that you have prepared yourself (no photocopies). No electronic devices may be used during the exam.

1. A point charge q is situated a large distance r from a neutral atom of polarizability α. Find the force of attraction between them.
   Note: You will need the expression for the electric field set up by a dipole.

2. As discussed in class the energy of a "pure" dipole in an electric $\mathbf{E}$ is given by $U = -\mathbf{p} \cdot \mathbf{E}$. Using this, and the expression for the electric field of a dipole, see Griffiths Eq. (3.104) or Qu. 8 of HW 4, show that the interaction energy of two dipoles separated by a distance r is
$$U = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} \left[\mathbf{p}_1 \cdot \mathbf{p}_2 - 3(\mathbf{p}_1 \cdot \hat{\mathbf{r}})\,(\mathbf{p}_2 \cdot \hat{\mathbf{r}}\right] .$$

3. A sphere of radius R carries a "frozen-in" polarization
$$\mathbf{P}(\mathbf{r}) = k\mathbf{r},$$
   where k is a constant and $\mathbf{r}$ is the vector from the center.
   (a) Calculate the bound charges σ_b and ρ_b.
   (b) Verify that the total bound charge, including both σ_b and ρ_b, is zero.
   (c) Find the electric field inside and outside the sphere.

4. A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a frozen-in polarization
$$\mathbf{P}(\mathbf{r}) = \frac{k}{r}\,\hat{\mathbf{r}},$$
   where k is a constant and r is the distance from the center, (Griffiths Fig. 4.18).
   Note: There are no free charges.
   Find the electric field in all three regions by two different methods:
   (a) Locate all the bound charge and use the integral form of Gauss's theorem for $\mathbf{E}$ (Griffiths Eq. (2.13)).
   (b) Use the integral form of Gauss's law for D, (Griffiths Eq. (4.23)), and the relation $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$.

   Note: The second method is faster since avoids calculating the bound charges.

5. The space between a parallel plate capacitor is filled with *two* slabs of dielectric. Each slab has a thickness a so the total distance between the slabs is $2a$, see Griffiths Fig. 4.24. The top slab, slab 1, has a dielectric constant of 2, and the lower slab, slab 2, has a dielectric constant of 1.5. The free charge densities on the top plate is σ and on the bottom plate is $-\sigma$.

   (a) Find the electric displacement $\mathbf{D}$ in each slab.

   (b) Find the electric field $\mathbf{E}$ in each slab.

   (c) Find the polarization $\mathbf{P}$ in each slab.

   (d) Find the potential difference between the plates.

   (e) Find the location and position of all bound charge.

   (f) Now that you know the charge (free and bound) recalculate the electric field in each slab (without using your knowledge of $\mathbf{D}$) and confirm your answer to 5b.

6. A long cylinder of radius a of dielectric material with susceptibility χ_e, is placed with its axis *perpendicular* to a uniform electric field $\mathbf{E}_0$. Calculate the electric field inside the cylinder.
   Hint: Use cylindrical polar coordinates. The solution of Laplace's equation in these coordinates which you found in Qu. 5 of HW 4 will be useful.

7. A spherical conductor of radius a carries a charge Q (see Griffiths Fig. 4.29). It is surrounded by a material of susceptibility χ_e out to a radius b ($> a$). Find the energy of this configuration.

8. **** Not to be done. *** A point charge q is embedded at the center of a sphere of dielectric material (with susceptibility χ_e and radius R) Find the electric field, the polarization, and the bound charge densities ρ_b and σ_b. What is the total bound charge on the surface? Where is the compensating negative bound charge located?