PHYSICS 112
Homework 5
Due in class, Tuesday February 14

Remember MIDERM:
In class on Thursday 10:00–11:10am. This will be followed by a short break and lecture.

1. **Symmetry of filled and vacant orbitals**
   Consider the Fermi-Dirac distribution $f(\epsilon)$. Let $\epsilon = \mu + \delta$, so δ is the difference between the energy and the chemical potential. Show that
   $$f(\mu + \delta) = 1 - f(\mu - \delta)\,,$$
   so the probability that an orbital ϵ above the chemical potential is occupied, is equal to the probability that an orbital δ below the chemical potential is empty. A vacant orbital is sometimes known as a **hole**.

2. **Entropy of Mixing**
   Suppose that a system of N atoms of type A is placed in diffusive contact with a system of N atoms of type B at the same temperature and volume. Show that after diffusive equilibrium is reached, the entropy has increased by $2N \log 2$. This increase in entropy is called the *entropy of mixing*.

   If the atoms are identical ($A \equiv B$), show that there is no increase in entropy when diffusive contact is established. This difference in results is called the *Gibbs paradox*.

3. **Time for a large fluctuation**
   We ask how long it will take for a large fluctuation to occur in a macroscopic system. Consider a gas of ^{4}He atoms in a container of volume 0.1 liter at 300 K and a pressure of 1 atm. We ask how long it will take before the atoms are in a configuration in which they are all in one half of the container.

   (a) Estimate the number of states accessible to this system in this initial condition.
   Note: For this part, and other parts of this question which involve *huge* numbers (i.e. numbers of order e^N where $N \sim 10^{21}$), you should express your answer in the form 10^x, with the appropriate value for x, This gives an an intuitive appreciation of the result since x is the number of digits in the answer.

   (b) The gas is compressed isothermally to a volume of 0.05 liters (half of the initial volume). Estimate the number of states accessible now.

   (c) For the system in the 0.1 liter container estimate the value of the ratio
   $$\frac{\text{number of states for which all atoms are in one-half of the volume}}{\text{number of states for which the atoms are anywhere in the volume}}.$$

   (d) If the collision rate of an atom is $\approx 10^{10}$ s^{-1}, what is the total number of collisions of all atoms in the system in a year? Use this as the frequency with which the state of the system changes.

   (e) Estimate the number of years you would have to wait for all atoms to be in one half of the system, starting from the equilibrium configuration.
   Note: As stated in part (a), express your answer in the form 10^x and give the value of x.

4. **Gas of atoms with an internal degree of freedom**
Consider a classical ideal monatomic gas, where each atom has two internal energy states, one an energy Δ above the other. There are N atoms in volume V at temperature T. Find the

(a) chemical potential,

(b) free energy,

(c) entropy,

(d) pressure, and

(e) heat capacity at constant pressure.