

PHYSICS 116C

Homework 5

Due in class, Thursday October 29

1. The diffusion equation is

$$\frac{\partial u}{\partial t} = D\nabla^2 u,$$

where D is the diffusion constant.

- (a) *Verify* that

$$u(x, t) = \frac{1}{t^{1/2}} \exp\left(-\frac{x^2}{4Dt}\right)$$

is a solution to the one-dimensional diffusion equation.

Note: Verify means substitute the given solution into the equation and check that it works.

- (b) *Verify* that

$$u(\vec{r}, t) = \frac{1}{t^{3/2}} \exp\left(-\frac{r^2}{4Dt}\right)$$

is a solution to the diffusion equation in spherical polar coordinates.

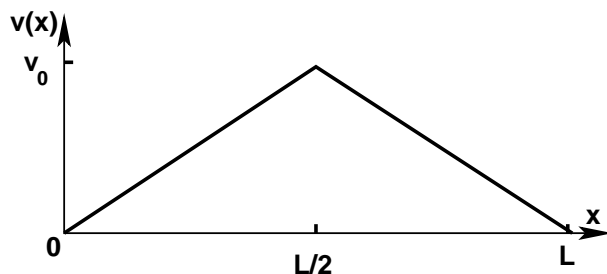
2. Solve the wave equation for a string of length L clamped at the ends given that the initial conditions are $y(x, 0) = y_0 \sin(3\pi x/L)$, $\partial_t y(x, t)|_{t=0} = 0$, where $y(x, t)$ is the displacement.

3. (a) Show that

$$\sin 3x = 3 \sin x - 4 \sin^3 x.$$

- (b) Solve the wave equation for a string of length L clamped at the ends given that the initial conditions are $y(x, 0) = y_0 \sin^3(\pi x/L)$, $\partial_t y(x, t)|_{t=0} = 0$.

4. A string of length L is initially stretched straight. At time $t = 0$, it is given an initial *velocity* $v(x) = \partial_t y(x, t)|_{t=0}$ as shown.



Find the displacement $y(x, t)$ at later times.

5. (a) We showed in class that the general solution of the one-dimensional wave equation is

$$u(x, t) = f(x - vt) + g(x + vt),$$

where f and g are arbitrary functions. Show that if we specify the initial conditions

$$u(x, 0) = \phi(x), \quad \partial_t u(x, t)|_{t=0} = \psi(x),$$

then the solution is

$$u(x, t) = \frac{1}{2} [\phi(x - vt) + \phi(x + vt)] + \frac{1}{2v} \int_{x-vt}^{x+vt} \psi(z) dz.$$

(This is known as d'Alembert's solution to the one dimensional wave equation.)

Hint: Look in the book.

- (b) Use d'Alembert's solution to find the solution to the wave equation that satisfies the initial conditions $u(x, 0) = \sin(\pi x/L)$, $\partial_t u(x, t)|_{t=0} = 0$ for $-\infty < x < \infty$. Express your answer as a standing wave.

6. Consider separation of variables of Laplace's equation in circular polars.

- (a) Show that the angular equation is

$$\frac{d^2\Theta}{d\theta^2} + n^2\Theta = 0 \quad (n = 0, 1, 2, \dots).$$

What is the solution of this equation (easy) and why must n be an integer?

- (b) Show that the radial equation is

$$r \frac{d}{dr} \left(r \frac{dR}{dr} \right) = n^2 R \quad (n = 0, 1, 2, \dots).$$

Look for solutions of the form $R = r^\lambda$ (i.e. just a single power of r) for a suitable choice of λ .
Note: You will need to treat the $n = 0$ case separately.

- (c) Write down the general solution of Laplace's equation in circular polars.