Tight Binding Density of States

Here are plots of densities of states for the tight-binding Hamiltonian for “cubic” lattices in several dimensions. In these figures I have set the minimum energy to be zero. For example, in three dimensions the energy is given by

$$\epsilon(k) = t[6 - 2(\cos k_x a + \cos k_y a + \cos k_z a)].$$  \hspace{1cm} (1)

There are analogous expressions for other dimensions.

Note the Van Hove singularities, which have the form of a square root divergence in one dimension, a discontinuity or log divergence in two dimensions, and a square root cusp in three dimensions.

The density of states in six dimensions is close to a Gaussian. In fact, one can show that the density of states becomes precisely a Gaussian when the dimension $d$ tends to infinity.
Also shown is the dispersion relation in some directions for the honeycomb lattice. (Here I have taken the center of the energy bands to be zero, rather than the minimum energy to be zero.) You are required to derive these results in a homework question.